ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

# Analyse d'une collision élastique

## Mécanique, cours 12.2

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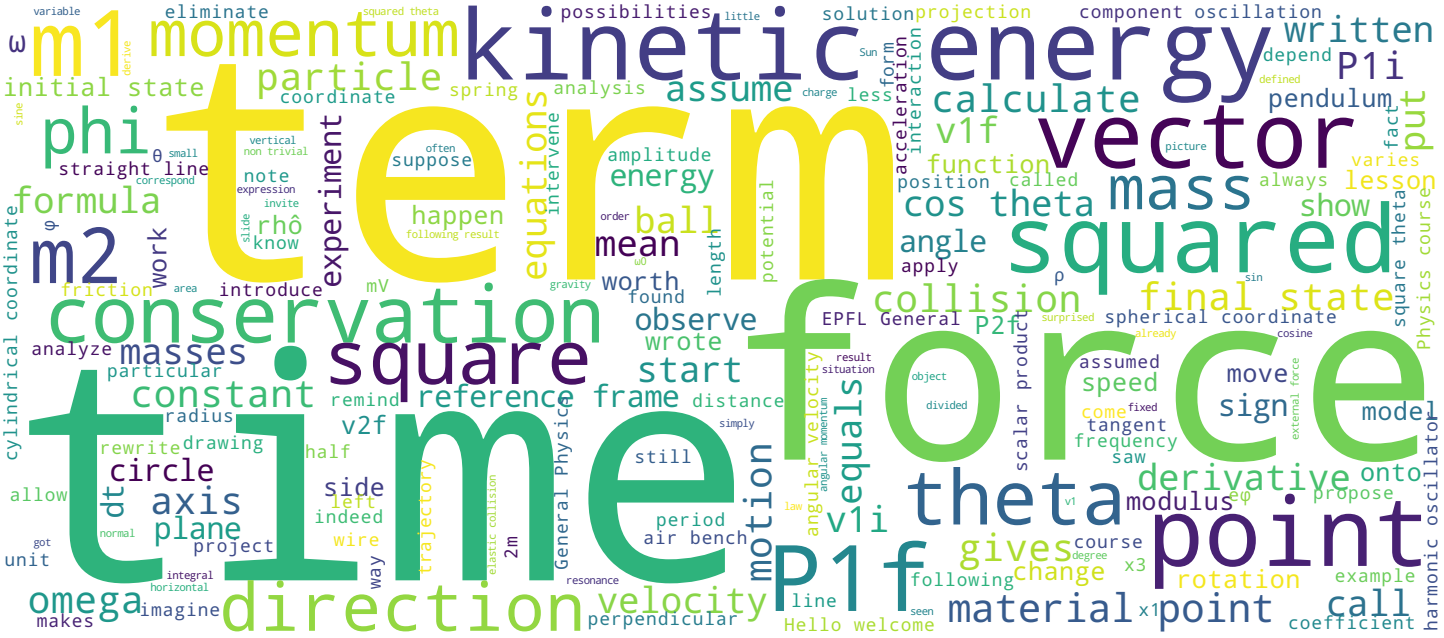
http://go.epfl.ch/traite-meca-2-12




EPFL

Video

# Analyse d'une collision élastique

- Principes de conservation
- Dans un plan
- Masses égales
- Sur une ligne droite

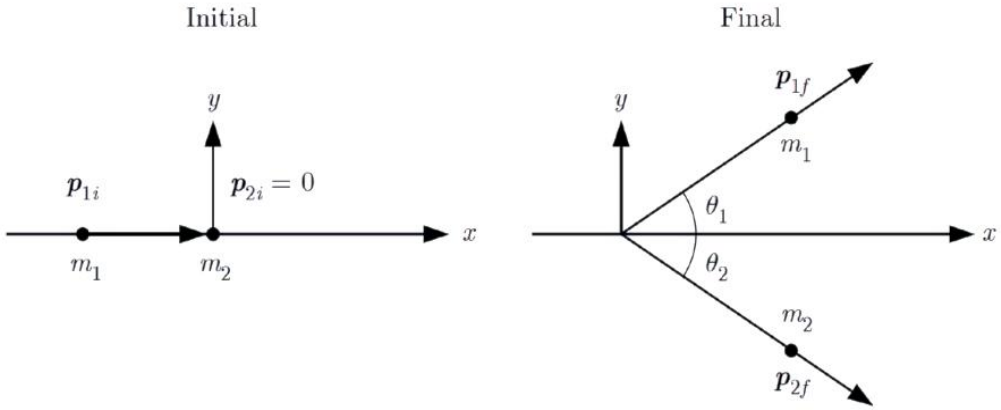


Hello, welcome to the EPFL General Physics course. In this lesson, we learn how to analyze collisions between material points using the conservation of momentum and if necessary, the conservation of kinetic energy. In this module, I will look at exactly the second case, the one where we also have the conservation of kinetic energy. We're going to look at a problem in the plane, we're going to apply the two conservation principles, we're going to look at what consequences it has when we have a collision that takes place in the plane, as opposed to on a straight line, like an air bench, we're going to look rather what happens for an air table. Then, we'll look at the particular case of equal masses, there is a very nice result that comes out when the masses are equal. And finally, we're going to analyze what we saw on the videos of experiments, what happens on a straight line.

Notes

Summary

0m 03s





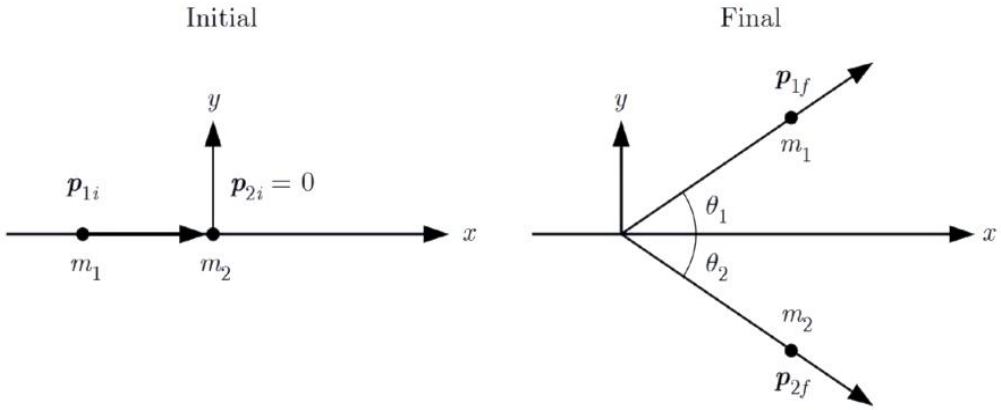

So, here's a drawing, so in the logic of the analysis of a collision, we look at the initial state, the final state. I have two material points, a mass *m1*, a mass *m2*, here is *m1* and *m2* in the final state. In the initial state, I assume that mass *m2* is at rest in the reference frame, and mass *m1* collides with *m2*. I give myself a system of coordinate axes, found before and after the collision, I align the *x* axis in the direction of the momentum of the incident particle. I note *P1i* the initial momentum of particle 1, *P2i* so that is zero, the momentum of particle 2, in the final state I have *P1f*, *P2f*, and I remind you that it is assumed that in the final state, the mutual interaction between the two particles is negligible, so these two material points have a uniform rectilinear motion and I give this direction of motion, by the angle theta 1 for the mass *m1*, *theta 2* for the mass *m2*. So I have, conservation of momentum because it is assumed that there is no external force acting on these two masses, and I have conservation of kinetic energy by assumption.

Notes

Summary

1m 09s

# Principes de conservation




$$\boldsymbol{p}_{1i} + \boldsymbol{p}_{2i} = \boldsymbol{p}_{1f} + \boldsymbol{p}_{2f}$$
$$T_{1i} + T_{2i} = T_{1f} + T_{2f}$$



Conservation of momentum is written like this, and here I have written conservation of kinetic energy where *T* is the kinetic energy for particle 1, for particle 2, in the initial state, in the final state. I'm going to now, here I have two equations because I have a vector equation, I'm going to project that equation there onto the *x* axis and onto the *y* axis.

Notes

Summary

2m 50s

# Choc élastique : calculs

$$p_{1i} = p_{1f}\cos\theta_1 + p_{2f}\cos\theta_2$$
$$0 = p_{1f}\sin\theta_1 - p_{2f}\sin\theta_2$$
$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2}$$



When I project *P1f* onto the *x* axis, I'll have a *cos theta 1* that intervenes, when I project *P1f* onto the *y* axis, I'll have a *sinus theta 1* that intervenes, it gives the following result, here, you have the two equations that correspond to the conservation of momentum, and here, I wrote the conservation of kinetic energy, I wrote *P* square over *2m* for kinetic energy, because half of *mV* square, with *P* being *mV*, this kinetic energy can also be written *P* square over *2m*. We can assume that *P1i* is experimental data, and then, what we would like to know, is *P1f*, which is the modulus of the momentum *theta 1* in the direction of the particle, and the same for the second. So we have four unknowns and here we have written three equations. We won't be able to solve completely, but that's normal, because, the existence of constants of motion, restricts the possibilities of the final state compared to the initial state, but if we don't say anything about the interaction, we shouldn't be surprised that we can't say everything about the final state, we would still need to have a model of the interaction forces in order to be able to reach a conclusion about the final state, for given initial conditions.

Notes

Summary

3m 18s

# Choc élastique : calculs

$$p_{1i} = p_{1f}\cos\theta_1 + p_{2f}\cos\theta_2$$
$$0 = p_{1f}\sin\theta_1 - p_{2f}\sin\theta_2$$
$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2}$$

$$(p_{1i} - p_{1f}\cos\theta_1)^2 = p_{2f}^2\cos^2\theta_2$$
$$p_{1f}^2\sin^2\theta_1 = p_{2f}^2(1 - \cos^2\theta_2)$$

$$(p_{1i} - p_{1f}\cos\theta_1)^2 + p_{1f}^2\sin^2\theta_1 = p_{2f}^2$$



So, we are not surprised by this. So, what we're going to do is we're going to eliminate some variables, I'm going to start by eliminating *cosinus theta 2*, so here I have a term in *cos theta 2*, I'm going to square it, I'm passing that term on the other side of the equal sign, I square it; here I have a *P2f cos theta 2* squared; here I can pass that term to the other side of the equal sign, squared, and replace *sin square theta 2*, with *1 minus cos square theta 2*, that's what I did here. Now, I can sum these two equations here, it gives me this, indeed this term here, when I do the addition, cancels out with this one, this term here, I find it here, this term, it's here, and the square bracket, it's here. Now, I'm going to use that equation there and that one, and eliminate *P2f* between the two.

Notes

Summary

4m 57s

# Choc élastique : calculs

$$p_{1i} = p_{1f}\cos\theta_1 + p_{2f}\cos\theta_2 \quad \longrightarrow \quad (p_{1i} - p_{1f}\cos\theta_1)^2 = p_{2f}^2\cos^2\theta_2$$

$$0 = p_{1f}\sin\theta_1 - p_{2f}\sin\theta_2 \quad \longrightarrow \quad p_{1f}^2\sin^2\theta_1 = p_{2f}^2(1-\cos^2\theta_2)$$

$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2}$$

$$(p_{1i} - p_{1f}\cos\theta_1)^2 + p_{1f}^2\sin^2\theta_1 = p_{2f}^2$$

$$p_{1f}^2\left(1+\frac{m_2}{m_1}\right) - 2p_{1i}p_{1f}\cos\theta_1 + p_{1i}^2\left(1-\frac{m_2}{m_1}\right) = 0$$

$$\frac{p_{1f}}{p_{1i}} = \frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1+m_2}\left\{\cos\theta_1 \pm \left[\cos^2\theta_1 - \left(1-\frac{m_2^2}{m_1^2}\right)\right]^{1/2}\right\}$$



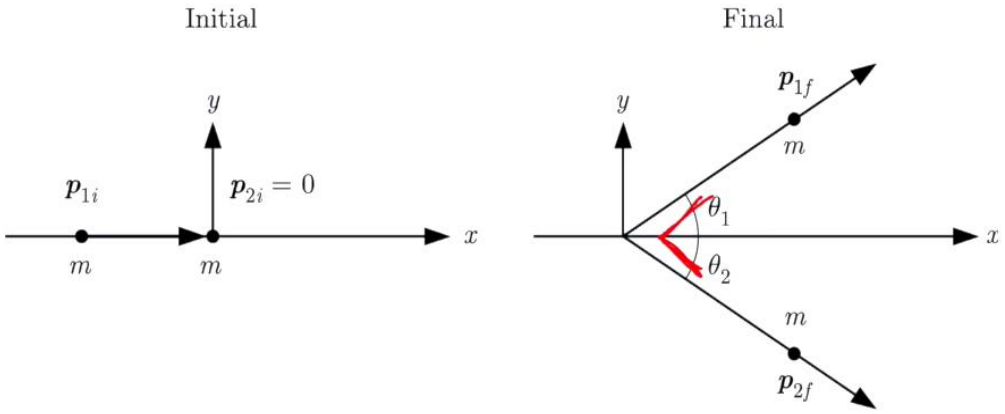
So, I'll expand this square, it gives me this, we'll see...I have this term that gives me the *P1i* squared here, there I have a *P1f* squared with a *cos squared theta*, and there's the *sin squared theta*, so I have *P1f* squared, it's that term, I have the double product, here, of *P1i P1f cos theta 1*, which I have written here, and now I will take the *P2f* squared, I will put it on the other side of the equal sign, and I observe from that equation there, that I have, minus *P2f* squared, which is worth *m2*, over *m1*, and there is more *P1f* squared, and there is less *P1i* squared. This term here is this one, and this term here is this one. So I have accounted for all the terms. I observe now, that I have here an equation in *P1f* square, *P1f*, term independent of *P1f*. Incidentally, I can divide by *P1i* squared, to have here *P1f* over *P1i*, the ratio squared; here I would also have *P1f* over *P1i* and there an independent term of *P1f* over *P1i*, I use the second degree binomial formula, and voila, I find *P1f* over *P1i* which is therefore *v1f* over *v1i*, which is written like this.

Notes

Summary

6m 05s

# Choc élastique : masses égales




$$\boldsymbol{v}_{1i} = \boldsymbol{v}_{1f} + \boldsymbol{v}_{2f} \longrightarrow \boldsymbol{v}_{1i}^2 = \boldsymbol{v}_{1f}^2 + \boldsymbol{v}_{2f}^2 + 2\boldsymbol{v}_{1f} \cdot \boldsymbol{v}_{2f}$$

$$\boldsymbol{v}_{1i}^2 = \boldsymbol{v}_{1f}^2 + \boldsymbol{v}_{2f}^2$$

$$\boldsymbol{v}_{1f} \cdot \boldsymbol{v}_{2f} = 0$$



Now I'd like to show an interesting case, because it's simpler mathematically, it's the case of equal masses. So, I take again, my same scheme of analysis of a collision, I have an initial state, a final state, I have the same notation, simply I suppose I have equal masses. Now, conservation of momentum is simply expressed like this, since we have *m* in front of all *v*, so I can simplify by *m*, so that's the expression for conservation of momentum. And for the conservation of kinetic energy, which is supposed to be the case, that we assume we have an elastic collision, we have here, the kinetic energy before, times 2 and divided by *m*, same thing there, after; and now if I take that term there, square it, I have all these terms. And now I appeal to the conservation of kinetic energy, which allows me to eliminate those three terms, I'm left with just that term, so I have the following result: *v1f* and *v2f* are perpendicular, scalar product is zero, that means the two vectors are perpendicular. So contrary to the way my drawing looks, actually, what I just found, is that there, there is a right angle. The next time you play pool, you'll see that the two balls, after the collision, escape at about 90 degrees.

Notes

Summary

8m 13s

# Choc élastique : sur une ligne droite

$$\frac{p_{1f}}{p_{1i}} = \frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{\cos\theta_1 \pm \left[\cos^2\theta_1 - \left(1 - \frac{m_2^2}{m_1^2}\right)\right]^{1/2}\right\}$$

$$\theta_1 = 0 \implies$$

$$\frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{1 \pm \sqrt{1 - \frac{m_1^2 - m_2^2}{m_1^2}}\right\} = \frac{m_1 \pm m_2}{m_1 + m_2} \quad \begin{matrix} 1 \\ \dfrac{m_1 - m_2}{m_1 + m_2} \end{matrix}$$



Now, I'm going to analyze the elastic shock when we're on a straight line, I take the formula we got in the planar case, where we had *costheta 1*, now we assume we're on an air bench, so there's a constraint, which makes *theta 1* always zero. So I rewrite the formula with *cos theta 1* which is 1, it immediately gives me this, here, if I put at the same, this term there, I can write it as *m2* squared, over *m1* squared, and I have to take the square root of it, so in fact I have *m2*, over *m1*, and then I have 1 plus or minus that, which I can write as *m1*, plus or minus *m2*, over *m1*, and now this *m1* in the denominator and that one cancel each other out, and I'm left with what I'm announcing here. So I have two possibilities, if I take sin plus, I have 1, 1 means *v1f* equals *v1i*, so it means that the particle 1, goes through the collision, without anything happening to it, it basically means that there was no collision, it's all our algebraic manipulations that introduced this solution, which is not excluded, on the basis of the principles we gave ourselves, without knowing anything about the form of the interaction.

Notes

Summary

9m 55s

# Choc élastique : sur une ligne droite

$$\frac{p_{1f}}{p_{1i}} = \frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{\cos\theta_1 \pm \left[\cos^2\theta_1 - \left(1 - \frac{m_2^2}{m_1^2}\right)\right]^{1/2}\right\}$$

$$\theta_1 = 0 \implies$$

$$\frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{1 \pm \sqrt{1 - \frac{m_1^2 - m_2^2}{m_1^2}}\right\} = \frac{m_1 \pm m_2}{m_1 + m_2} \quad \begin{cases} 1 \\ \dfrac{m_1 - m_2}{m_1 + m_2} \end{cases}$$

$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2} \qquad \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$v_{2f}^2 = \frac{m_1}{m_2}v_{1i}^2\left(1 - \frac{v_{1f}^2}{v_{1i}^2}\right) = \frac{m_1}{m_2}v_{1i}^2\left(1 - \frac{(m_1 \pm m_2)^2}{(m_1 + m_2)^2}\right) \quad \begin{cases} 0 \\ v_{2f} = \dfrac{2m_1}{(m_1 + m_2)}v_{1i} \end{cases}$$



And the second solution, the non-trivial solution, is this, now I have therefore *v1f*as a function of the *v1i*, I would like to have *v2f*, to do this I can use conservation of kinetic energy, which I rewrite here in the usual way, and I will write *v2f*, as a function of *v1f*over *v1i*, like this. I can write *v2f*, there's a *m1* over *m2*, times *v1i* squared, I highlight it and I have this ratio that's there, that was given to me there. I substitute this value in there, it gives me this, there are two possibilities, because there I have plus or minus, the plus gives me the 0, so at *v1* at the velocity of particle 1 that doesn't change, I have particle 2 that doesn't change either, so it's really the case of 0 collision; and otherwise for the non-trivial case, in the case of velocity 1, for velocity 2, I find this formula there, which is particularly interesting if we look at the case *m1* equals *m2*, if *m1* equals *m2*, here we'll have 0, so we'll find *v1f* is zero, and here, if *m1* equals *m2*, we'll have *v2f* equal *v1i*.

Notes

Summary

11m 46s

# Choc élastique : sur une ligne droite

$$\frac{p_{1f}}{p_{1i}} = \frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{\cos\theta_1 \pm \left[\cos^2\theta_1 - \left(1 - \frac{m_2^2}{m_1^2}\right)\right]^{1/2}\right\}$$

$$\theta_1 = 0 \implies$$

$$\frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{1 \pm \sqrt{1 - \frac{m_1^2 - m_2^2}{m_1^2}}\right\} = \frac{m_1 \pm m_2}{m_1 + m_2} \quad \begin{cases} 1 \\ \dfrac{m_1 - m_2}{m_1 + m_2} \end{cases}$$

$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2} \qquad \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$v_{2f}^2 = \frac{m_1}{m_2}v_{1i}^2\left(1 - \frac{v_{1f}^2}{v_{1i}^2}\right) = \frac{m_1}{m_2}v_{1i}^2\left(1 - \frac{(m_1 \pm m_2)^2}{(m_1 + m_2)^2}\right) \quad \begin{cases} 0 \\ v_{2f} = \dfrac{2m_1}{(m_1 + m_2)}v_{1i} \end{cases}$$



That's what we see on the experiment videos, indeed, if we arrange to have an elastic collision with equal masses, what we observe is that the incident pad stops, and it's the pad that was stopped that starts again with the speed of the incident plane, that's that formula.

Notes

Summary

13m 21s