

Mécanique, cours 12.1

<http://go.epfl.ch/traite-meca-1-12>



Video





- Définitions
- Quantité de mouvement conservée
- Energie cinétique, pas forcément
- Impacts

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Hello, welcome to the EPFL general physics course. In this lesson, I would like to show you how we can use the conservation of momentum and the conservation of energy to conduct a qualitative analysis of the collision between two material points. I will start by defining what I call in physics: "a collision", then, I will look at a particular case of collisions by applying the conservation of momentum. This will allow us to see that, in some cases, the kinetic energy is not conserved. I end with a model for collisions in the common sense, that means shocks with impacts that take place during a very short time.

Notes

Summary



0m 04s



Collision :

- interaction mutuelle entre deux objets,
- sans qu'il n'y ait nécessairement impact,
- interaction négligeable quand les objets sont éloignés.

Collision élastique :

- Même valeur de l'énergie cinétique avant et après ('conservée')

Collision inélastique :

- Énergie cinétique pas conservée

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I start with what I mean by "collision". I want to consider two material points interacting with each other for some time. I'm going to assume that there is not necessarily an impact, but simply that we have a force acting between the two material points. I assume that, when the material points are far enough from each other, the interaction between these two points is negligible. Therefore, I can define an initial state and a final state between the two, there is a moment when there is this mutual interaction taking place. I will distinguish two types of collisions, first, elastic collisions : these are the collisions, when the kinetic energy before and after is the same; I will say that the energy is conserved, and otherwise, we talk about an inelastic collision.

Notes

Summary



0m 57s

Conservation de la quantité de mouvement



Troisième loi de Newton, système isolé :

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}^{1 \rightarrow 2} + \mathbf{F}^{2 \rightarrow 1} = 0$$

$$\mathbf{p}_i = \mathbf{p}_f$$

- C'est une propriété fondamentale des forces.
- C'est une symétrie fondamentale.

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Let's apply the conservation of momentum. First, I make a reminder. You remember that Newton's third law said that if we have a system made of two objects, or two material points interacting with each other, the sum of these forces is zero. If now I have an isolated system, then the total momentum follows Newton's second law, if there are only the internal forces that cancel, we have dp over dt which is zero, this means that we have momentum which is a constant of motion. In our collision problem, this means that the initial momentum is equal to the final momentum. I started with conservation of momentum because it is a fundamental property of forces. It was announced, like Newton's third law, in a more advanced course, you may have the opportunity to see that this conservation of momentum results from a fundamental symmetry of nature; provided, of course, that the system is isolated.

Notes

Summary

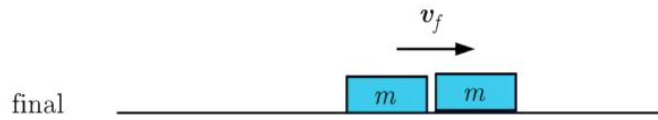


2m 03s

Exemple : choc totalement inélastique



$$p_i = mv + 0$$



$$p_f = (m + m)v_f$$

$$v_f = \frac{1}{2} v$$

$$T_i = \frac{1}{2}mv^2 \quad T_f = \frac{1}{2}(m + m) \left(\frac{1}{2}v\right)^2 = \frac{1}{4}mv^2$$

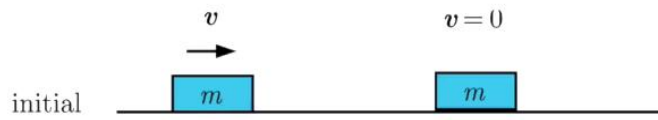
I will now take an example. You have, here, the experimental situation we have something like an air bench with two studs and I have an initial situation and the final situation. In the initial situation, I have a stud incident on a stud at rest in the reference frame. Afterwards, I assume that there is a mechanism, here, between the two studs, that makes the studs linked to each other, that they move together, with a speed that I note v_f . What does the principle of conservation of momentum tell us? First, we have conservation of momentum because in that direction, to a first approximation, there is no external force applied, we can neglect friction. So, we have an initial quantity of motion which is the one of the incident mass... afterwards, we have a double mass going at speed v_f , that is the momentum. And now we apply conservation of momentum, these two terms are equal, which gives us a final velocity that is half the speed of the incident particle. Let us now calculate the kinetic energy. In the initial state, we have the kinetic energy of the incident particle. In the final state, we have to calculate half the mass of the particle, times its velocity squared.

Notes

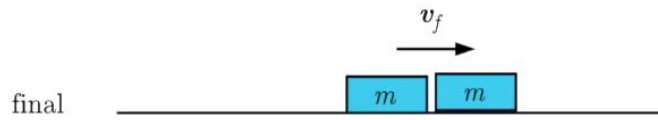
Summary



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That's the mass of that composite particle and the velocity, we had determined that was half of v . When you do the math, you just, here, the 2 simplifies with this 2. There's a quarter of mv^2 , whereas we started with half of mv^2 . So the final kinetic energy is not equal to the initial kinetic energy. In fact, there is some energy that must be in the mechanism that produced this coupling of the two masses.

Notes

Summary



Deux objets sont en collisions pendant un temps très court,
dans un champ de force extérieur.

$$\frac{d\mathbf{P}}{dt} = \mathbf{F}^{ext}$$

$$\int_{t-\varepsilon}^{t+\varepsilon} \frac{d\mathbf{P}}{dt} dt = \mathbf{P}_{final} - \mathbf{P}_{init} = \int_{t-\varepsilon}^{t+\varepsilon} \mathbf{F}^{ext} dt$$



Now, I propose a model for collision cases where there is a shock, with an impact, for a short time. In this case, I would like to look at what happens in the presence of external forces. I make a sketch of a situation that one could imagine, I have stretched a wire, a kind of receptacle, and I send a marble in there, assuming that the marble will stay stuck, here, in it. And I wonder how far this system will go up. What I can do in such a case, is that we have an interaction that takes place, for a very short time. When the ball arrives in there, it happens during a very short time, in relation to what? In relation to this time of evolution of the pendulum. So, I'll do the following thing. If t is the time of the collision, I'm going to consider my second law of Newton, where there are no interacting forces, so there can be all sorts of subtleties in the way the ball gets stuck in the receptacle, that belongs to the domain of the internal forces, I'm only looking at the external forces. And now, I integrate around that collision time, so I look just before and just after the collision, I integrate over time; obviously the integral of the derivative over time, that gives us the momentum, so we have the momentum just after, at time t plus *epsilon*, minus the momentum, just before.

Notes

Summary

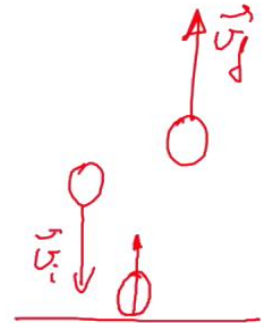


Deux objets sont en collisions pendant un temps très court, dans un champ de force extérieur.

$$\frac{d\mathbf{P}}{dt} = \mathbf{F}^{ext}$$

$$\int_{t-\varepsilon}^{t+\varepsilon} \frac{d\mathbf{P}}{dt} dt = \underbrace{\mathbf{P}_{final} - \mathbf{P}_{init}}_{2m\mathbf{v}} = \int_{t-\varepsilon}^{t+\varepsilon} \mathbf{F}^{ext} dt$$

Si l'impact est bref : $\varepsilon \rightarrow 0 \Rightarrow \int_{t-\varepsilon}^{t+\varepsilon} \mathbf{F}^{ext} dt \rightarrow 0$



pour autant que \mathbf{F}^{ext} soit en tout temps finie. Pas une force de contrainte!

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And that must be the integral of the external force in time, between t minus *epsilon* and t plus *epsilon*. Now, we will be able to make the following approximation, if the impact is brief, we will take the limit when *epsilon* tends to 0 (zero) and we will be able to say that this term tends to 0, so that we have conservation of momentum, just after, compared to just before the collision. This is only true if the external force, at any time, is finite. I show a counterexample. Imagine bouncing a ball on the ground, here is a ball that falls with an initial v velocity, that bounces, with a final v velocity... As the ball - so it bounces right up, I drew it next to - as the ball hits the ground, there's a reaction force here. Now, if I assume that, the impact lasts an extremely short time, I'm going to have a force, here n , that is extremely large. Why? Because that is always true. And that difference is essentially worth projected into the vertical $2mv$. You see, you have a downward velocity, after the upward velocity, the difference between the two, it's $2mv$.

Notes

Summary



7m 46s

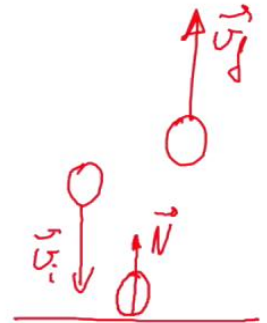
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So you have something finite here, this must be finite, the domain of integration time to ε is getting smaller and smaller, so this is the force that must be getting larger and larger. This n reaction force is singular. In such a case, we cannot apply this model. On the other hand, in the previous case, where I had a ball going into a pot attached to a wire, we only have forces of finite values. And at that point, we can take the limit when ε tends to 0, and that term is zero, and we have just at the collision, conservation of momentum.

Notes

Summary

