



- Puissance d'une force
- Travail d'une force
- Energie cinétique
- Théorème de l'énergie cinétique
- Unités

Mécanique | 2013 7

Hello, welcome to the EPFL General Physics course. In this lesson I will introduce the notion of work and energy. Until now we have examined the dynamics of the material point using Newton's second law. We have obtained equations of motion which are differential equations with obviously second derivatives with respect to time. When we want to study the dynamics of a system, it is useful to be able to find the constants of motion, these are quantities which depend on first derivatives with respect to time and which happen to be constants. This allows to better analyze the system, first because instead of having second derivatives, we have first derivatives, so we have already made a step towards the integration of the dynamical system. To introduce the notion of energy, I start with the notion of power of a force, from there I can define the work of a force, then I define the kinetic energy and I will show the close link between the kinetic energy and the work of a force. I end this lesson with a reminder about the units of magnitude of mechanics.

Notes

Summary



0m 04s

Définition : puissance instantanée d'une force



Point matériel :
-Vitesse \mathbf{v}
-Force exercée \mathbf{F}

$$P = \mathbf{F} \cdot \mathbf{v}$$

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I start with the definition of what I call the instantaneous power of a force. Imagine that we have a material point of velocity \mathbf{v} which is subjected to a force \mathbf{F} . Then by definition I will call the instantaneous power of the force, the scalar product \mathbf{F} times \mathbf{v} . So here I have P which is a scalar quantity that results from the scalar product of the force and the velocity.

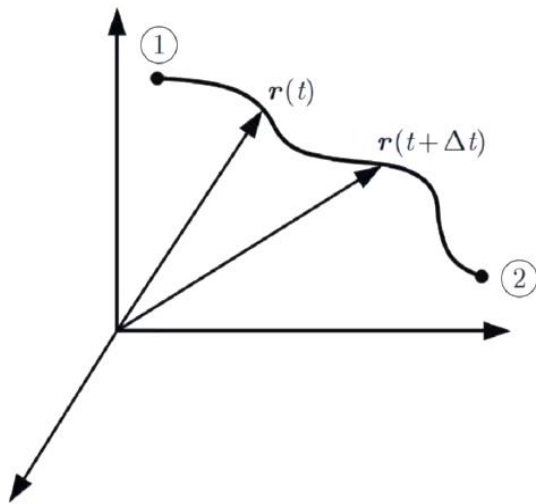
Notes

Summary



1m 32s

Définition : travail d'une force



$$W_{12} = \int_{t_1}^{t_2} P(t) dt$$

$$= \int_{t_1}^{t_2} \mathbf{F}(t) \cdot \mathbf{v} dt$$

$$\delta W = \mathbf{F} \cdot d\mathbf{r}$$

I now move to the definition of the work of a force. I imagine that I have a reference frame materialized here by a system of axes, I suppose that I already know the trajectory of the material point, and I suppose that I go from point one to point two, I am at time T_1 at point one and at time T_2 at point two and I call the work of the force the integral in time of the instantaneous power. If now I use my definition of my instantaneous power, it is the force times the velocity, I see appearing a $\mathbf{v} dt$, it is a term that I can call $d\mathbf{r}$, it is thus the limit of the $\Delta \mathbf{r}$ that I have here, this is a $\Delta \mathbf{r}$ since here I have given a Δt , it is therefore the limit of this term when Δt tends towards zero and, as we have seen, the speed is tangent to the trajectory so the $d\mathbf{r}$ is tangent too. So one way of thinking about the notion of work of a force, is to say that in a displacement $d\mathbf{r}$ the work of the force \mathbf{F} is this term, the one that we have here, that I'm rewriting here, that we often call, that we often denote δW so if we want to calculate the work to go from one to two, we have to sum up the δW along the trajectory. Now let's look at this infinitesimal work δW , it's the scalar product of \mathbf{F} times $d\mathbf{r}$.

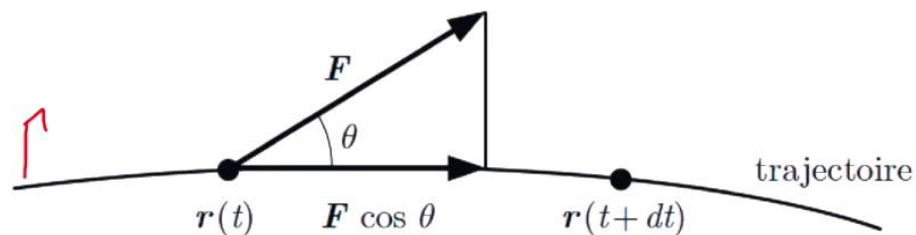
Notes

Summary



$$\delta W = \mathbf{F} \cdot d\mathbf{r} = |\mathbf{F}| |d\mathbf{r}| \cos \theta$$

$$W = \int_{\Gamma} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$$



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If we make a geometric representation of what's going on here's an element of trajectory, I represented \mathbf{r} of t and \mathbf{r} of t plus dt so I have a vector $d\mathbf{r}$ which is like this, and this scalar product corresponds to the displacement $d\mathbf{r}$, which we have here, times the projection of the force, the $F \cos \theta$ gives us the projection of the force which I indicated here, which I noted $F \cos \theta$. So it's the component of the force tangent to the trajectory that works. This component, I could have drawn it like this, this component is perpendicular to $d\mathbf{r}$ so it gives zero. There is only the component projected on the tangent that contributes to the work. Now if we want to calculate the work to go to two points, with a finite displacement along the trajectory, one way to express the work is to write it like this: I have an integration sign with a gamma, I'll agree that I'll call the trajectory gamma, and so I mean I have to compute the $\int \mathbf{F} \cdot d\mathbf{r}$ where the $d\mathbf{r}$ is tangent to the trajectory and I make a sum of $\mathbf{F} \cdot d\mathbf{r}$ all along the trajectory. That gives me the work.

Notes

Summary



4m 09s

Définition : énergie cinétique



Point matériel :

-Vitesse \mathbf{v}

-Masse m

$$T = \frac{1}{2} m \mathbf{v}^2$$

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Now I introduce the notion of kinetic energy. So I take a material point whose velocity is given, \mathbf{v} , mass m , by definition I call kinetic energy which I note T half of $m \mathbf{v}^2$. If we have several material points, it is enough to sum all the contributions of type one half of $m \mathbf{v}^2$. This is the definition of my kinetic energy.

Notes

Summary



5m 53s

Propriété : théorème de l'énergie cinétique

$$T_2 - T_1 = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} dt$$

$$\int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} dt = \int_{t_1}^{t_2} m \mathbf{a} \cdot \mathbf{v} dt = \int_{t_1}^{t_2} m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v} dt$$

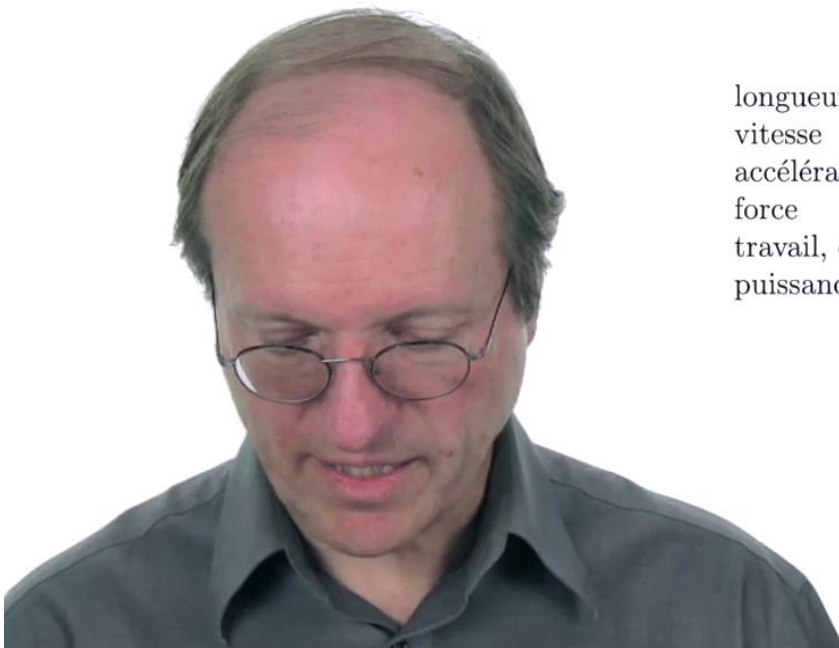
$$\int_{t_1}^{t_2} \underbrace{m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v}}_{\text{red underline}} dt = \int_{t_1}^{t_2} \frac{d}{dt} \left(\frac{1}{2} m v^2 \right) dt = \frac{1}{2} m v^2 \Big|_1^2 = T_2 - T_1$$

Now I'll move on to what I'll call the kinetic energy theorem. It says this: T_2 minus T_1 this is the change in kinetic energy when the material point goes from position one to position two and this theorem says this is worth the work so the integral of the instantaneous power between time T_1 and time T_2 . I now prove this theorem. So I start with this right-hand term, the equal sign $\mathbf{F} \cdot \mathbf{v} dt$ and for \mathbf{F} I use Newton's second law. I have \mathbf{F} equals $m\mathbf{a}$, that's what I wrote here. The $m\mathbf{a}$ I'm going to write $m \frac{d\mathbf{v}}{dt}$ and now this term I want to recognize in it a derivative with respect to time of one half of $m v^2$. This is what I have written here below. This term I write as the derivative with respect to the time of half a $m v^2$ squared, so we have the integral of the derivative. So this function, what I have written here, taken at time T_2 minus the value at time T_1 . Half a $m v^2$ squared is the kinetic energy so I have T_2 minus T_1 . So I have the work to go from one to two which is equal to the change of kinetic energy when we go from one to two.

Notes

Summary





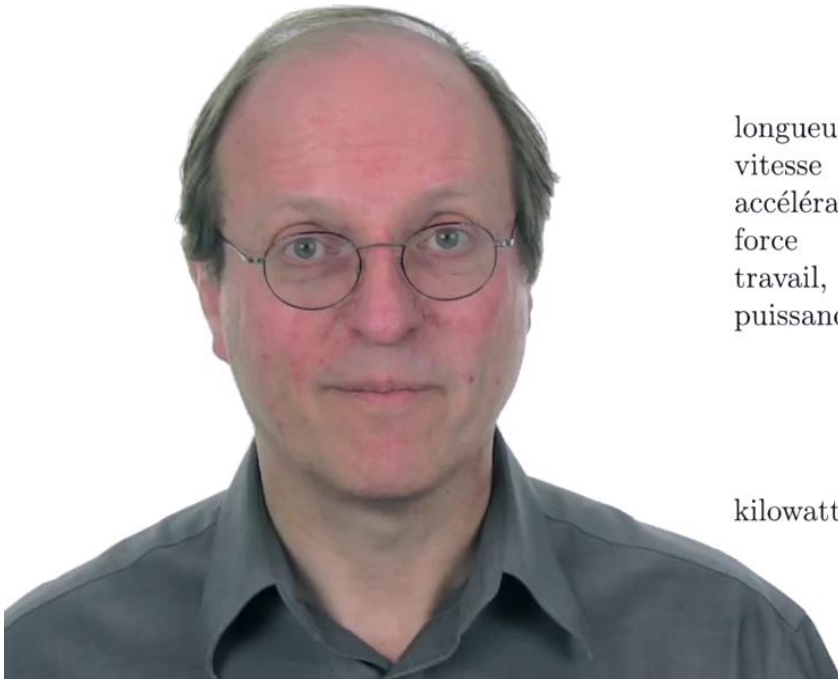
longueur	m	
vitesse	m s^{-1}	
accélération	m s^{-2}	
force	kg m s^{-2}	<i>newton</i>
travail, énergie	$\text{kg m}^2 \text{s}^{-2}$	<i>joule</i>
puissance	$\text{kg m}^2 \text{s}^{-3}$	<i>watt</i>

This lesson is already coming to an end, the subject is extremely important, we're going to apply it in the next module to an extremely important phenomenon, the resonance phenomenon but before leaving this lesson I want to review with you the units of the quantities we use in mechanics. So we're going to use the international system, for length, I use the meter, a speed is a displacement divided by a time so it's meters per second, what I've indicated here, an acceleration is a change of speed per unit of time so it's a speed per unit of time so it's meters per second minus two, a force is, we can think of F equals ma , it's a mass times an acceleration, so I take the units of acceleration here, and I multiply by a unit of mass, the kilo, the kilogram meter second minus two, we call it the newton, the newton is a unit of force, to calculate a work or an energy, we just saw with the kinetic energy theorem that the variation of energy was equal to a work, both have the same units. So we can find the energy by thinking about the kinetic energy, we take the speed squared and multiply by the mass. So we have kilograms square meter second minus two, that's what I wrote here.

Notes

Summary





longueur	m	
vitesse	m s^{-1}	
accélération	m s^{-2}	
force	kg m s^{-2}	<i>newton</i>
travail, énergie	$\text{kg m}^2 \text{s}^{-2}$	<i>joule</i>
puissance	$\text{kg m}^2 \text{s}^{-3}$	<i>watt</i>

$$\text{kilowattheure} = (\text{puissance}) \times (\text{temps}) = \text{énergie}$$

And this, this unit is called joules. The joule is a unit of work or energy. Now we saw that energy or work is the integral of power in time. So we had a $p \, d t$. So $p \, d t$ gives a unit of energy so the unit of power is an energy per unit of time. So we take our unit of energy, we divide by a time, so we have kilograms square meter second minus three. That's called watts. One last little remark, from now on, if you didn't know you shouldn't have any doubt about the meaning of this unit, the kilowatt hour. The kilowatt hour means kilowatts times hours. It is power times time, it is energy. When you get an electricity bill in kilowatt hours it tells you how much energy you have used. It is in no way kilowatts per hour. I'd like to make sure you're clear on that term because it's often used in technology.

Notes

Summary

