ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

http://go.epfl.ch/traite-meca-1-9

# Contraintes géométriques

## Mécanique, cours 9.1

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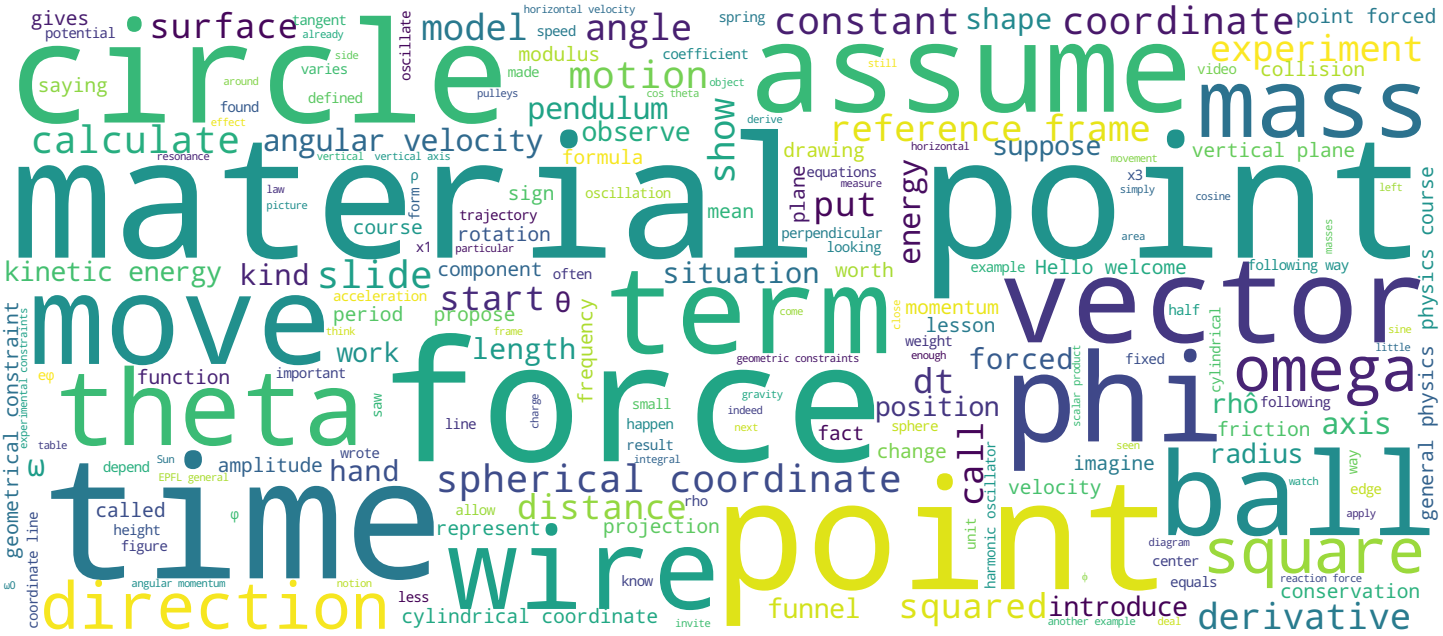


Video

# Contraintes géométriques

- Contraintes géométriques
- Modélisation
- Force

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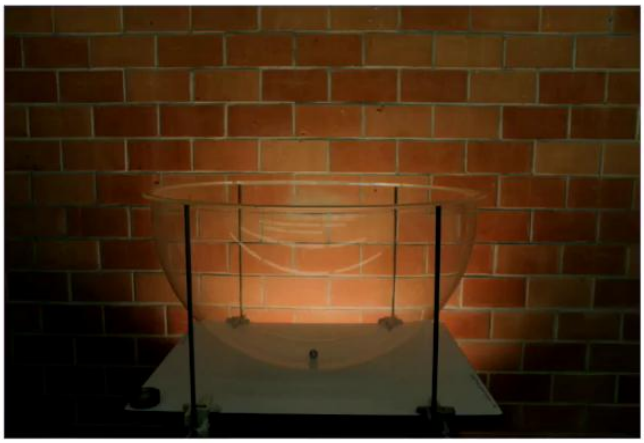
Hello, welcome to the ÉPFL general physics course. In this lesson I would like to show how to deal with a problem of material point mechanics when this material point is constrained to move on a given surface or line belonging to the reference frames. We saw in the previous lessons, the cylindrical and spherical coordinates, and I justified this passage, by saying that often in physics we had problems or situations with a particular symmetry and that it was important to write this system with coordinates which easily account for the symmetries of the problem. This is exactly the kind of situation we're going to have, here. So, first I want to show examples of mechanical systems with geometric constraints. I'm going to show how we model with conditions on the coordinates, these geometrical constraints and finally, I'm going to make, make you aware of the fact that we have forces that intervene to account for the geometrical constraints.

Notes

Summary

0m 04s

# Contraintes géométriques ou 'liaisons'





So, I start with the idea of having a material point, forced to move on a surface, which introduces a geometrical constraint or what others call links. Take this example: You have a marble rolling in a big bowl. So, first, I will assume that the rolling of the ball in the bowl causes the ball to rotate on itself and I will assume that this effect can be neglected. So I will suppose, that I can represent my ball by a material point. On the other hand, I will assume that the bowl has a spherical shape. And so, to deal with this problem, I will say that I have a material point forced to move on a sphere.

Notes

Summary

1m 24s

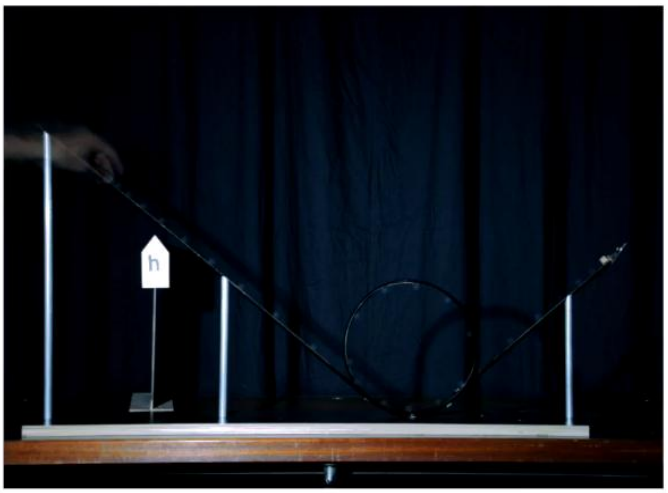
The second problem is close to the first one. I have a kind of funnel which has a shape, with a cylindrical symmetry, with the vertical axis of symmetry, passing through the center of the funnel. The shape of the funnel is probably chosen so that the height of a point on the surface is inversely proportional to the distance of this point from the axis of symmetry of this figure. In this case, it is obvious that the use of cylindrical coordinates with z for the height and rho for the distance to the axis will allow us to define this constraint. We will simply assume that z of rho is the shape of this curve, the shape of the funnel.

Notes

Summary

2m 16s

# Contraintes géométriques ou 'liaisons'





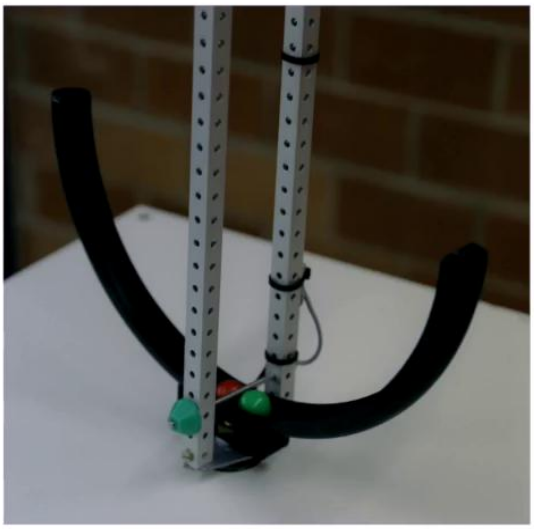

I'm looking at, now another device. Now, instead of being forced to move on a surface we're going to have a ball on a looping track. We'll suppose, once again that we can approximate the material point and we don't have a motion constrained to a surface but to a line. So we have a straight line followed by the loop which we will assume to be a circle.

Notes

Summary

3m 13s

# Contraintes géométriques ou 'liaisons'



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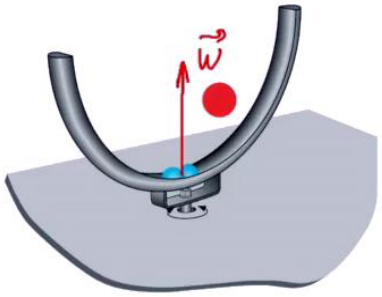
Now I'll take another example here is a slide that allows... to roll balls by holding them in the slide. So, the slide has edges that can introduce friction, I will neglect them. This slide is in a vertical plane, the vertical plane rotates, thanks to a motor placed under the table. I will assume that the angular velocity is constant, and I have a system that I will model in the following way: here's a diagram of the system.

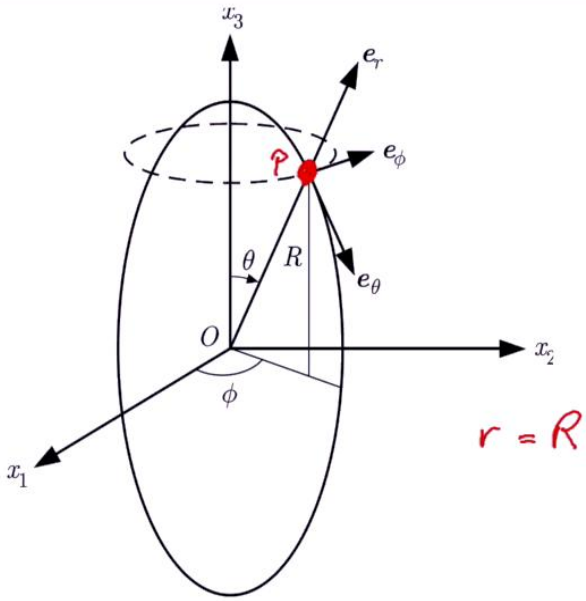
Notes

Summary

3m 46s

# Modélisation géométrique



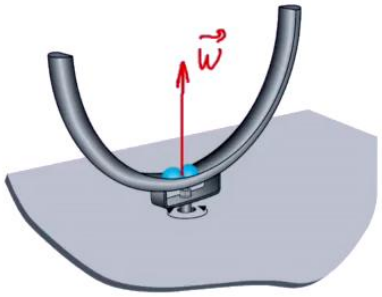


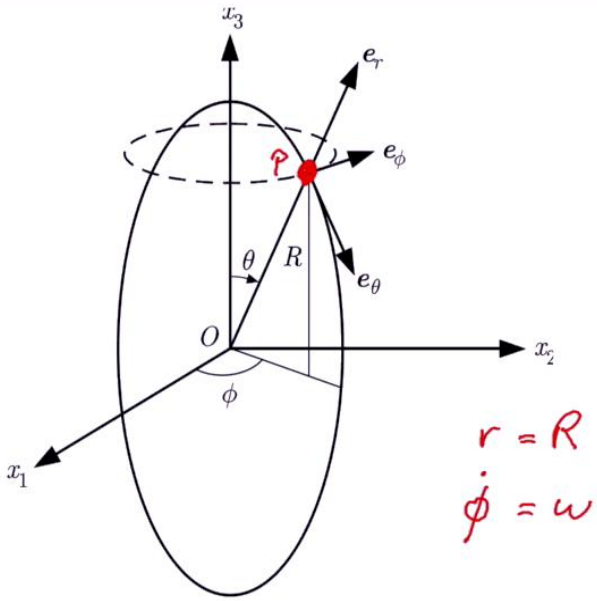

So, first, I'd like to show you a diagram of the experiment and next to it, I've made a geometric model of the experiment. First, we give ourselves a rotation speed with a vertical axis. So, we must assume an omega angular velocity, like this. I'm going to go from reality to, to the model assuming that I have a full circle that the balls can go all over the circle, practically I have a closure here but I want to ignore that. And now, what I propose to do is to consider the spherical coordinates with distance r, angle theta and angle phi. I have the material point presumed to be here, here is my p point I put it there on top, why? Because I want to reproduce the figure that I had, when I defined the r, theta, and phi coordinates, the spherical coordinates. So, my ring here, my slide, becomes the coordinate line, or theta varies with phi fixed, and r fixed. The experimental constraints become geometric constraints. It is thus the geometrical modeling of the experimental constraints. On the one hand, I have the coordinate r, of the spherical coordinates which is the radius of the circle, and on the other hand, we have this condition of constant angular speed.

Notes

Summary

4m 27s






To this angular velocity corresponds this angle and I have therefore, as a condition that phi point will drift with respect to time of the spherical coordinate phi is omega, which is given by the experiment. So you see, here, how we go from an experimental reality, with all kinds of details. I had mentioned the friction of the ball on the edge of the slide to a model, with a material point. The material point is forced to move on the circle. We don't care what keeps the material point up there. Often students ask if the material point is on the slide or in the slide. At this point in the modeling, the material point is on this circle.

Notes

Summary

6m 20s

## Un pendule formé d'un fil et d'une masse

Hypothèses :

- Masse ponctuelle
- Fil sans masse
- Pas de frottement au point d'attache
- Pas de frottement de l'air

Modèle :

point matériel astreint à se déplacer sur un cercle

… ou une sphère.



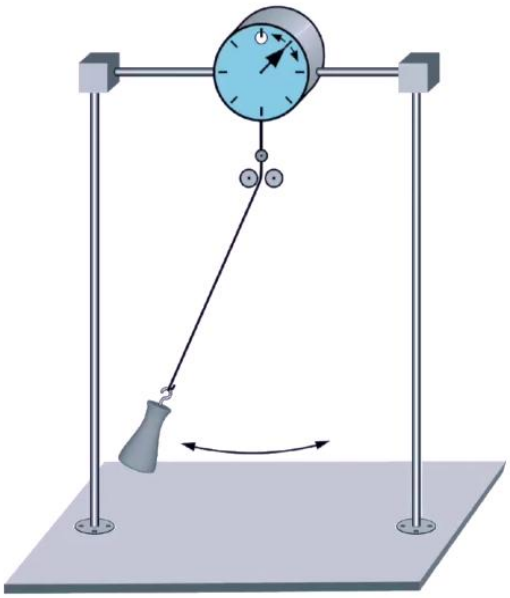
I now consider another example: Imagine a pendulum formed by a wire and a mass hanging on the wire, and you hang the wire somewhere, at a point of the reference frame. I make the following approximations: I assume I can treat this mass as a material point or if you like, as a point mass. So it is small compared to the length of the wire. I assume that the mass of the wire plays no role in what I want to observe. So, I'll assume the wire has no mass. I assume there is no friction effect, of any kind, at the point of attachment. And then I'll neglect air friction. At that point, I can model this mechanical system in the following way: I can say that I have a material point constrained to move on a circle. Then, I will say that it is a circle if I assume that in my initial conditions, I have no horizontal velocity. At that point, we're guaranteed to have motion in a plane. And so, we can say that the mass is forced to move on a circle. If I give a horizontal velocity I have a three dimensional motion, and I will be obliged to say that the material point moves on a sphere.

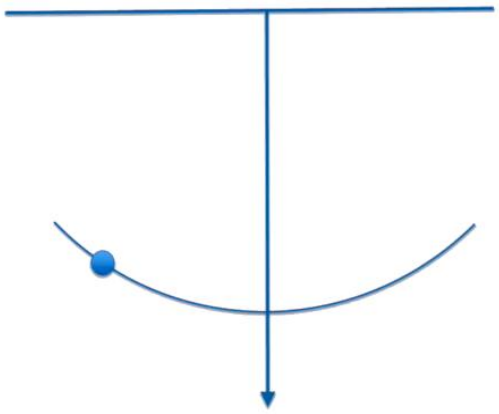
Notes

Summary

7m 21s

# Force de liaison ou force de contrainte





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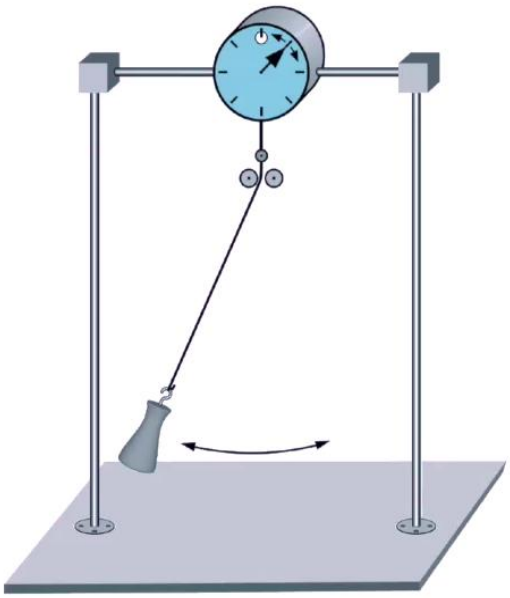
I would now like to introduce the notion of the force associated to a link. To do this, I suggest you imagine the following device: you have a weight suspended from a wire. The wire is mounted here through two pulleys which mark the position of the wire at this point but thanks to the pulleys we can measure, with a dynamometer, the force that the wire exerts on the weight. Of course this force is not zero, our intuition is enough to say so. If you go and watch the video, you will see, moreover, that when the mass oscillates, the force, here, oscillates too. I can make a model of this pendulum which consists in saying that, here I have a material point. We'll suppose a movement, in a vertical plane. And so, the material point is on a circle. So, I can make this drawing to represent the situation. I say, now, that I have a material point forced to move on a circle. Here is the center of the circle, and the radius of the circle is equal to the length of the wire. Now, when dealing with a problem with a mass attached to a wire, if the problem, explicitly says that, I think most students will have no trouble remembering that you have to associate a force with the action of the wire, on the weight.

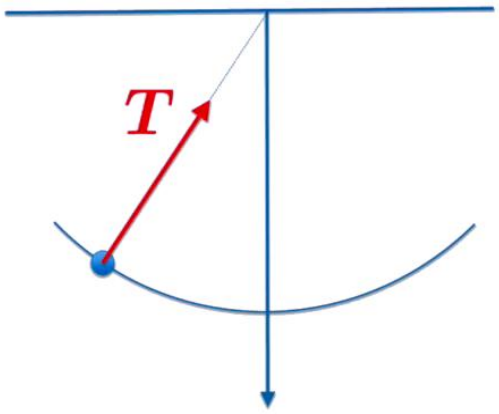
Notes

Summary

8m 51s

# Force de liaison ou force de contrainte








On the other hand, here, we have to be careful and not forget that if the material point is forced to move on this circle, there must be a force which basically plays a reaction force of the circle, on the material point and so, I must have this force, which will be the same force as the one I measured, here. This force is called a binding force, or a constraining force.

Notes

Summary

10m 29s