



- Modèle 3D des coordonnées sphériques
- Glissière hémisphérique

Mécanique | 2013 2

Hello, welcome to the EPFL general physics course. In this lesson I have introduced cylindrical and spherical coordinates. Some of you may have difficulty imagining the definitions of spherical coordinates. For this reason, the preparers have built a three-dimensional model and I hope this will help you. Next I will show you an experiment that lends itself particularly well to the use of spherical coordinates.

Notes

Summary



0m 04s

Modèle 3D des coordonnées sphériques



- Sphère, on ouvre et on voit un quadrant
- Les angles et distances.

Mécanique | 2013 3

Here is a sphere in which we have cut a dial to define the spherical coordinates.

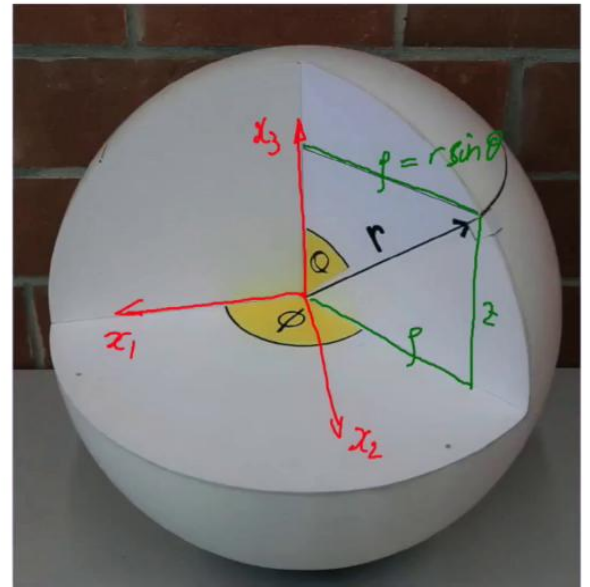
Notes

Summary



0m 37s

Modèle 3D des coordonnées sphériques



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So in the background, from this picture of a three-dimensional object I will now draw the axes that are missing on this model. So to be consistent with the definition we adopted in the form I have to take the x_1 axis like this, the x_2 axis, it is perpendicular to it about like this, and the x_3 , vertical here. Note again... Yes we can have fun redefining the cylindrical coordinates on this picture. For the cylindrical coordinates it's this length that comes into play which I'll call ρ . And we see that ρ is $r \sin \theta$ sorry, $r \sin \theta$. And then for the coordinates... cylindrical you still have to define the height z above the plane that contains x_1 and x_2 . So you have once again the ρ that appears here. There you have it all in one drawing.

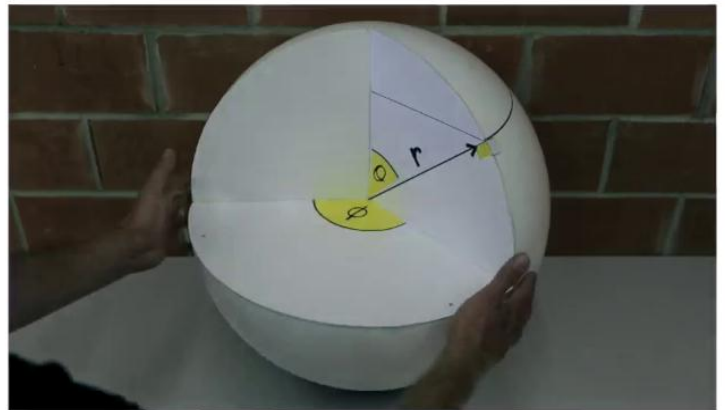
Notes

Summary



0m 48s

Modèle 3D des coordonnées sphériques



Expression en coordonnées sphériques
d'un élément de volume.

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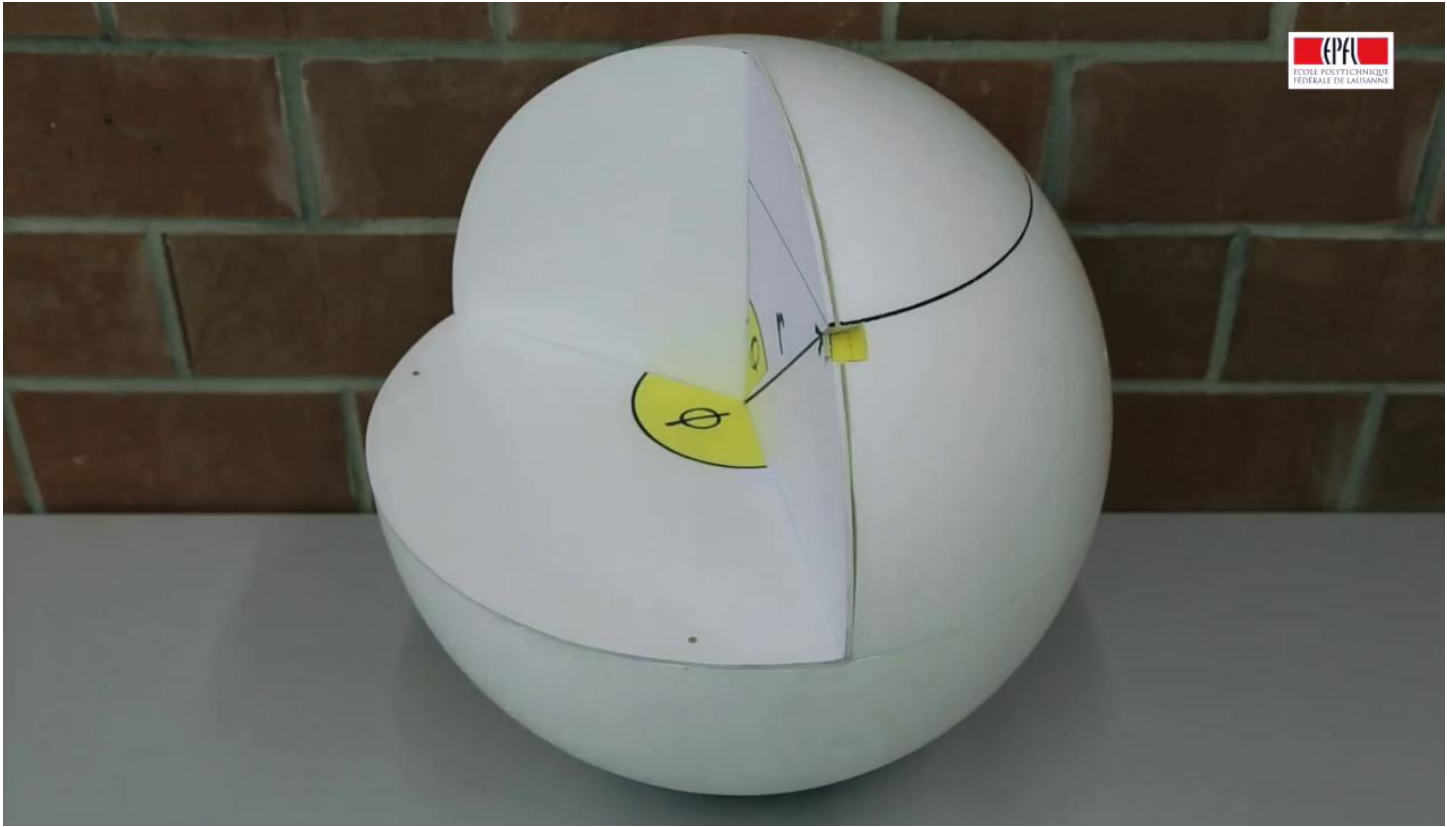
I'd like to take advantage of this passage on spherical coordinates with a three dimensional model to express a volume element in spherical coordinates. Watch the video.

Notes

Summary



2m 14s



It is this small volume element that I would like to express in spherical coordinates.

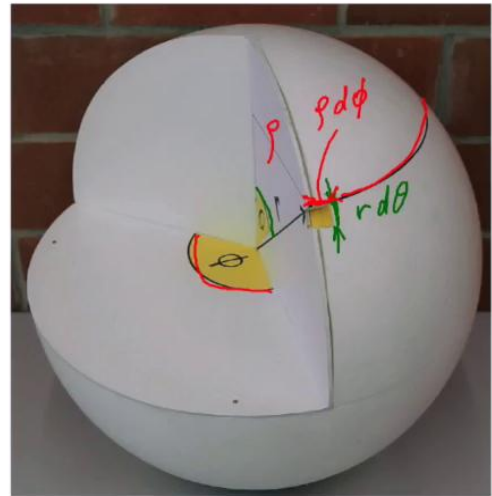
Notes

Summary



2m 29s

Modèle 3D des coordonnées sphériques



$$dV = \underline{\rho d\phi} \underline{r d\theta} \underline{dr} = \sin \theta d\theta d\phi r^2 dr$$

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Here is a picture of our little volume element. You agree that in this direction there I have a distance which is $r d\theta$. θ is the angle I have there. When θ varies by $d\theta$ we travel on the sphere this distance $r d\theta$. When ϕ varies... When ϕ varies by $d\phi$ we will vary by this magnitude there and this magnitude there will be our ρ if we call ρ the radius of this circle here and so here we have a distance that is $\rho d\phi$. And then obviously in the radial direction we have a dimension dr . So we end up with a volume element. So this is the volume of this little cube, we have $\rho d\phi$ in one direction, we have $r d\theta$ in the other direction and dr in the radial direction. And with ρ being $r \sin \theta$ we end up with this formula there $r^2 \sin \theta d\phi$. Now I move on to an experiment.

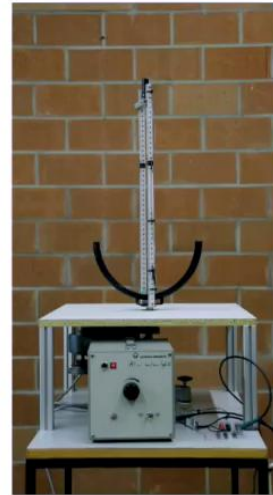
Notes

Summary



2m 36s

Glissière hémisphérique



- Un tel système appelle naturellement à l'usage des coordonnées sphériques pour exprimer de façon simple le mouvement de la bille dans la glissière.

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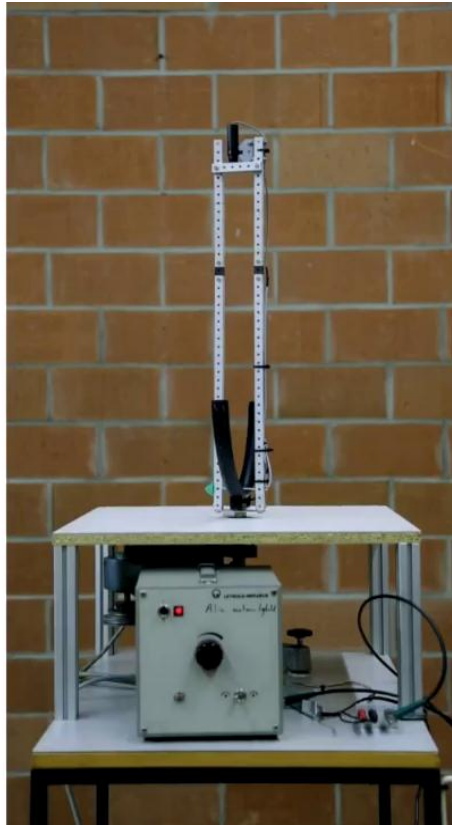
This is a ball in a hemispherical slide. This black half-arc is a slide. There are two balls, a red one, a black one at the bottom of the slide and you will see on the film that we have the possibility to make this slide rotate in a controlled way with a constant angular speed.

Notes

Summary



4m 12s



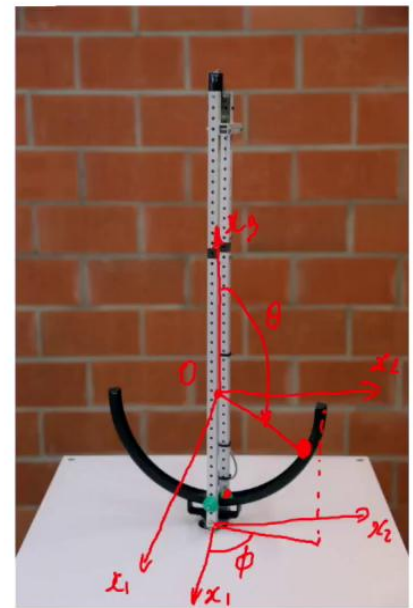
And we can change the angular speed and you may have noticed that the red and green balls are now sideways. They are no longer at the bottom of the slide.

Notes

Summary



Glissière hémisphérique



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So how do you make use of the spherical coordinates in such a situation? You have your marble which is here right now. We'd like to represent its position when it's like this sideways. So we could take coordinate axes like this along the edges of the table x_1 x_2 . We might have a projection like this so it would have the angle ϕ that's there. And what are we going to do with θ ? So, obviously we could say, "Well, here's the center of the circle". I'm going to define that as the θ angle. And if I do that I have a different definition of θ angles than I usually do so we're going to avoid that way of working because we'd like to be able to use the form that we've established. So what I'm going to do is I'm going to define my reference frame origin here at the center of the circle. So this, this will be my O point. I'm going to take x_1 from there up like this here's x_1 x_2 over there and I'm going to take x_3 up to define the θ angle. So I have to make an auxiliary line here and my θ angle is this one. This is my θ angle. Now I have r which is given by the radius of the slide. θ and ϕ , the spherical coordinates which allow me to represent very well the fact that this material point is forced to move in a glitier that rotates with a given speed $\dot{\phi}$.

Notes

Summary



4m 53s