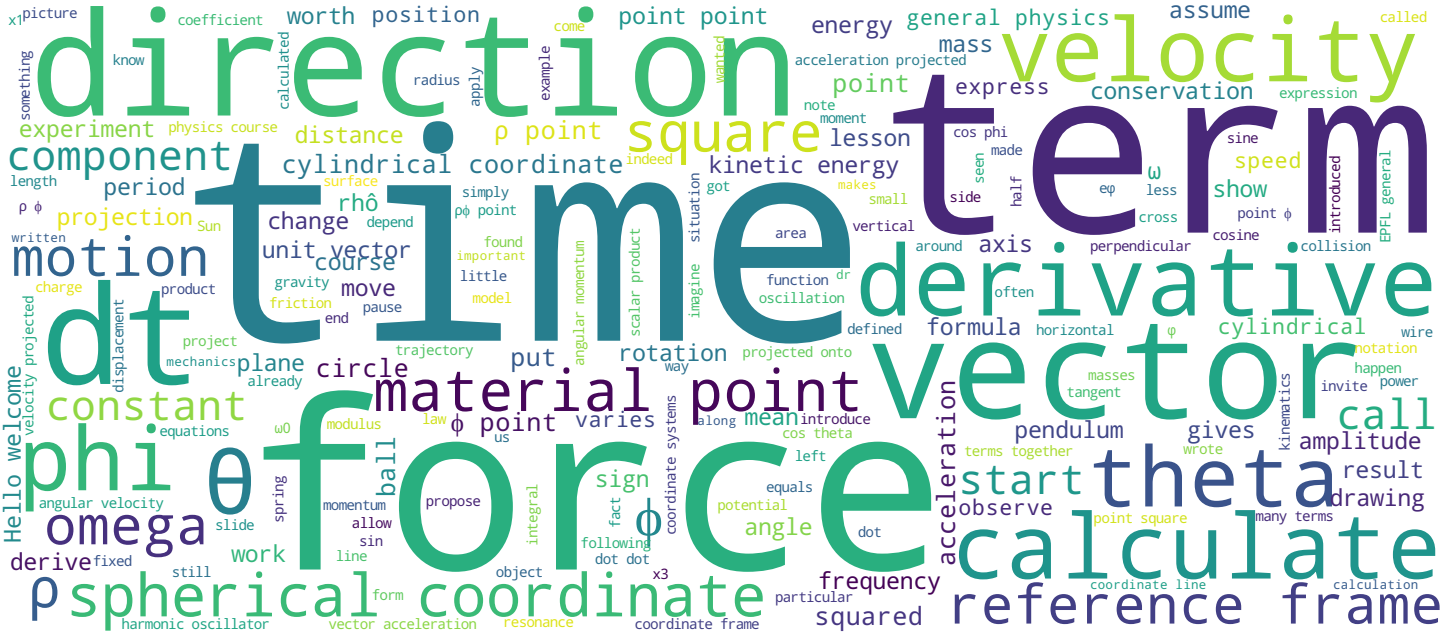
Accélération en coord. cylindriques et sphériques

Mécanique, cours 7.3

Jean-Philippe Ansermet

http://go.epfl.ch/traite-meca-2-7



Video

Accélération en coord. cylindriques et sphériques

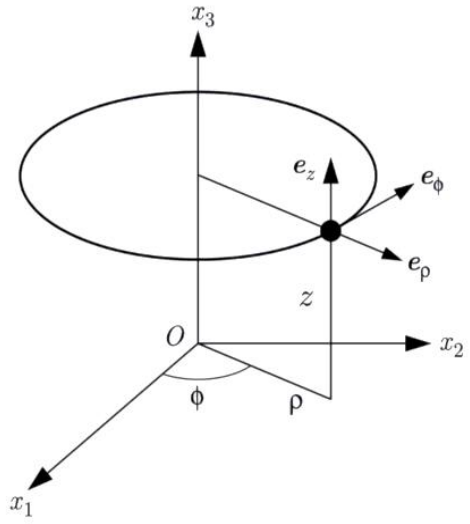
Hello, welcome to the ÉPFL general physics course. In this lesson, I introduced the cylindrical and spherical coordinates, and I wanted to show how we will use these coordinate systems to express the kinematics of the material point. We have seen in the previous module how to express the vector velocity projected on the reference frames associated to these 2 coordinate systems. Now we have to project the acceleration of the material point, the vector acceleration projected on these reference frames. We will consider the acceleration as the derivative of the velocity, expressed in cylindrical and spherical coordinates.

Notes

Summary

0m 04s

Accélération projetée sur le repère, c. cylindriques



$$\boldsymbol{v} = \dot{\rho}\boldsymbol{e}_\rho + \rho\dot{\phi}\boldsymbol{e}_\phi + \dot{z}\boldsymbol{e}_z$$

$$\frac{d\boldsymbol{e}_\rho}{dt} = \dot{\phi}\,\boldsymbol{e}_\phi$$

$$\frac{d\boldsymbol{e}_\phi}{dt} = -\dot{\phi}\,\boldsymbol{e}_\rho$$

$$\frac{d\boldsymbol{v}}{dt} = \ddot{\rho}\boldsymbol{e}_\rho + \rho\ddot{\phi}\boldsymbol{e}_\phi + \dot{\rho}\dot{\phi}\boldsymbol{e}_\phi + \ddot{z}\boldsymbol{e}_z$$

$$+\dot{\rho}\frac{d\boldsymbol{e}_\rho}{dt} + \rho\dot{\phi}\frac{d\boldsymbol{e}_\phi}{dt}$$

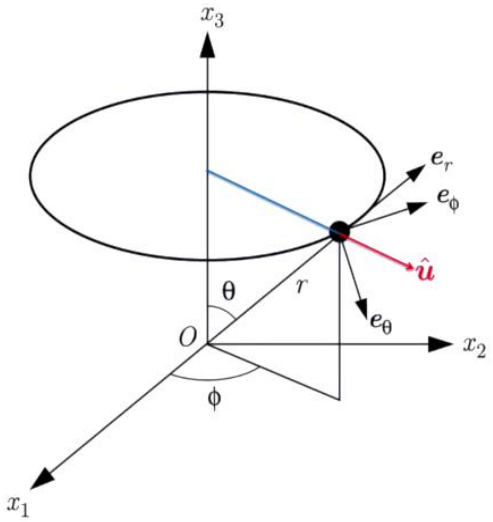
I start with the cylindrical coordinates. Here is the drawing that summarizes the definition of the cylindrical coordinates, *ρ*, *ϕ*, *z*, and the associated unit vectors, ***e****ρ*, ***e****ϕ*, ***e****z*. We got this for the velocity. When we derive the velocity with respect to time, to get the acceleration, we will, of course, have the components that we derive with respect to time; but we will also have to derive ***e****ρ* and ***e****ϕ* with respect to time. ***e****z*, there is no worry, ***e****z* always remains vertical, so it has zero derivative with respect to time. So, we already got *d* from ***e****ρ* on *dt*, and *d* from ***e****ϕ* on *dt*. Therefore, when I want to calculate the acceleration, I calculate the derivative of the velocity, I have a *ρ point point*, these 2 terms will give me *ρ point*, *ϕ point*, and *ρϕ point*, there they are. This term in *z dot* will give me *z dot*. And now there's the derivative of the unit vectors, like this, with *d* of ***e****ρ* on *dt*, which is given here, and *d* of ***e****ϕ* on *dt*, here.

Notes

Summary

0m 56s

Accélération projetée sur le repère, c. sphériques



$$\boldsymbol{v} = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\boldsymbol{e}_\theta + r\dot{\phi}\sin\theta\boldsymbol{e}_\phi$$

$$\frac{d\boldsymbol{e}_r}{dt} = \dot{\theta}\boldsymbol{e}_\theta + \dot{\phi}\sin\theta\boldsymbol{e}_\phi$$

$$\frac{d\boldsymbol{e}_\phi}{dt} = -\dot{\phi}\hat{\boldsymbol{u}}$$

$$\hat{\boldsymbol{u}} = \sin\theta\boldsymbol{e}_r + \cos\theta\boldsymbol{e}_\theta$$

$$\frac{d\boldsymbol{e}_\theta}{dt} = -\dot{\theta}\boldsymbol{e}_r + \dot{\phi}\cos\theta\boldsymbol{e}_\phi$$

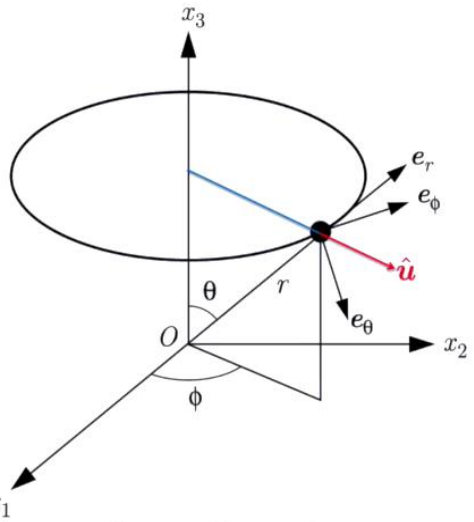
So I have one term less *ρϕ point square*, in the ***e**ρ* direction, I have a *ρ point ϕ point*, here, and I also have a *ρ point ϕ point*, there. I put the terms together, here they are. In the ***e**ρ* direction, in that direction there, I have a *ρ point point* and a minus *ρ ϕ point square*. In the direction tangent to the circle, I have *ρ ϕ point point*, but I also have *2 ρ point ϕ point*. And simply *z dot dot* in the *z* direction. I now move to the acceleration projected onto the spherical coordinate frame, the *r*, *θ*, *ϕ* coordinates. I will project the vector acceleration onto ***e**r*, ***e**θ*, ***e**ϕ*. I start from the expression for the velocity projected onto the spherical coordinate frame, ***e**r*, ***e**θ*, ***e**ϕ*. Again, when I calculate the derivative of the velocity, I will have the derivative of the components, but I will also have the derivative of the unit vectors. We calculated it, I repeat our results: here is *d* of ***e**r* on *dt*, here is *d* of ***e**ϕ* on *dt*, with ***u***, here, which has these components there, *d* of ***e**θ* on *dt*, we had gotten that too.

Notes

Summary

2m 21s

Accélération projetée sur le repère, c. sphériques



$$\boldsymbol{v} = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\boldsymbol{e}_\theta + r\dot{\phi}\sin\theta\boldsymbol{e}_\phi$$

$$\frac{d\boldsymbol{e}_r}{dt} = \dot{\theta}\boldsymbol{e}_\theta + \dot{\phi}\sin\theta\boldsymbol{e}_\phi$$

$$\frac{d\boldsymbol{e}_\phi}{dt} = -\dot{\phi}\hat{\boldsymbol{u}}$$

$$\hat{\boldsymbol{u}} = \sin\theta\boldsymbol{e}_r + \cos\theta\boldsymbol{e}_\theta$$

$$\frac{d\boldsymbol{e}_\theta}{dt} = -\dot{\theta}\boldsymbol{e}_r + \dot{\phi}\cos\theta\boldsymbol{e}_\phi$$

$$\frac{d\boldsymbol{v}}{dt} = \ddot{r}\boldsymbol{e}_r + \dot{r}\dot{\theta}\boldsymbol{e}_\theta + r\ddot{\theta}\boldsymbol{e}_\theta + \dot{r}\dot{\phi}\sin\theta\boldsymbol{e}_\phi + r\ddot{\phi}\sin\theta\boldsymbol{e}_\phi + r\dot{\phi}\dot{\theta}\cos\theta\boldsymbol{e}_\phi + \dot{r}\frac{d}{dt}\boldsymbol{e}_r + r\dot{\theta}\frac{d}{dt}\boldsymbol{e}_\theta + r\dot{\phi}\sin\theta\frac{d}{dt}\boldsymbol{e}_\phi$$

And now, if I want to calculate the acceleration, I take the velocity, I drift with respect to time. We catch our breath, because there are many terms. Here we go. We must have an *r point point*, it's there. Here we have 2 terms, we must get 2 terms, they're there. Here we have 3 terms, so we'll get the sum of 3 terms : one, two, three, there they are. *t* of ***er*** over *2 θ*, we have it here. *d* of ***eθ*** on *dt*, there it is. and the *d* of ***eϕ*** on *dt*, we got it like this. I invite you to pause, and try to put the terms together on your own.

Notes

Summary

3m 54s

Vitesse et accélération, composantes

Coordonnées cylindriques :

$$\boldsymbol{v} = \dot{\rho}\boldsymbol{e}_\rho + \rho\dot{\phi}\boldsymbol{e}_\phi + \dot{z}\boldsymbol{e}_z$$

$$\boldsymbol{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\boldsymbol{e}_\rho + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\boldsymbol{e}_\phi + \ddot{z}\boldsymbol{e}_z$$

Coordonnées sphériques

$$\boldsymbol{v} = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\boldsymbol{e}_\theta + r\dot{\phi}\sin\theta\boldsymbol{e}_\phi$$

$$a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\cos\theta\sin\theta$$

$$a_\phi = r\ddot{\phi}\sin\theta + 2r\dot{\phi}\dot{\theta}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta$$

The summary, here it is: it's a page that will give you an important form for the continuation. In cylindrical coordinates, we had the velocity and acceleration projected on the cylindrical coordinates reference frame, here. For the spherical coordinates, The velocity, we had obtained, and now, for the acceleration there are many terms, I note them thus: ***a**r* for the component, the acceleration in the *r* direction, ***a**θ* in the *θ* direction, and ***a**ϕ* in the *ϕ* direction. With this, we are ready to do mechanics using cylindrical or spherical coordinates.

Notes

Summary

4m 42s