ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Expériences : résonance paramétrique

Mécanique, cours 28.exp

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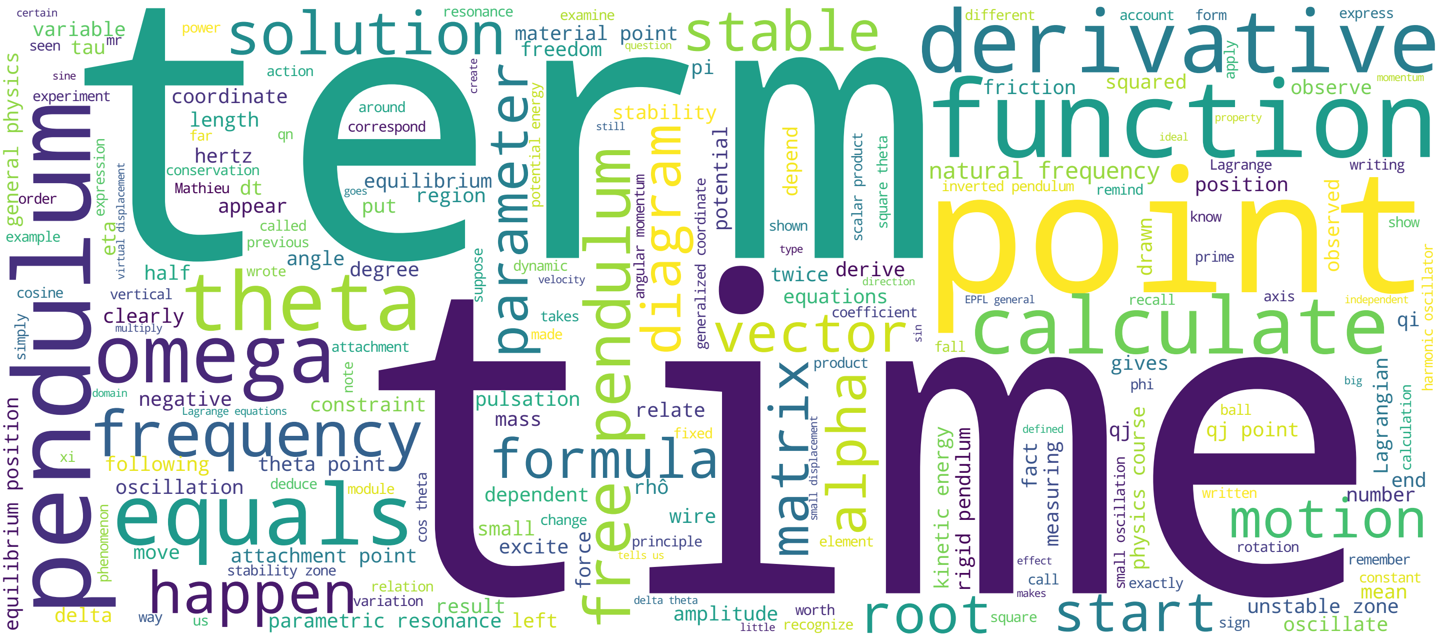
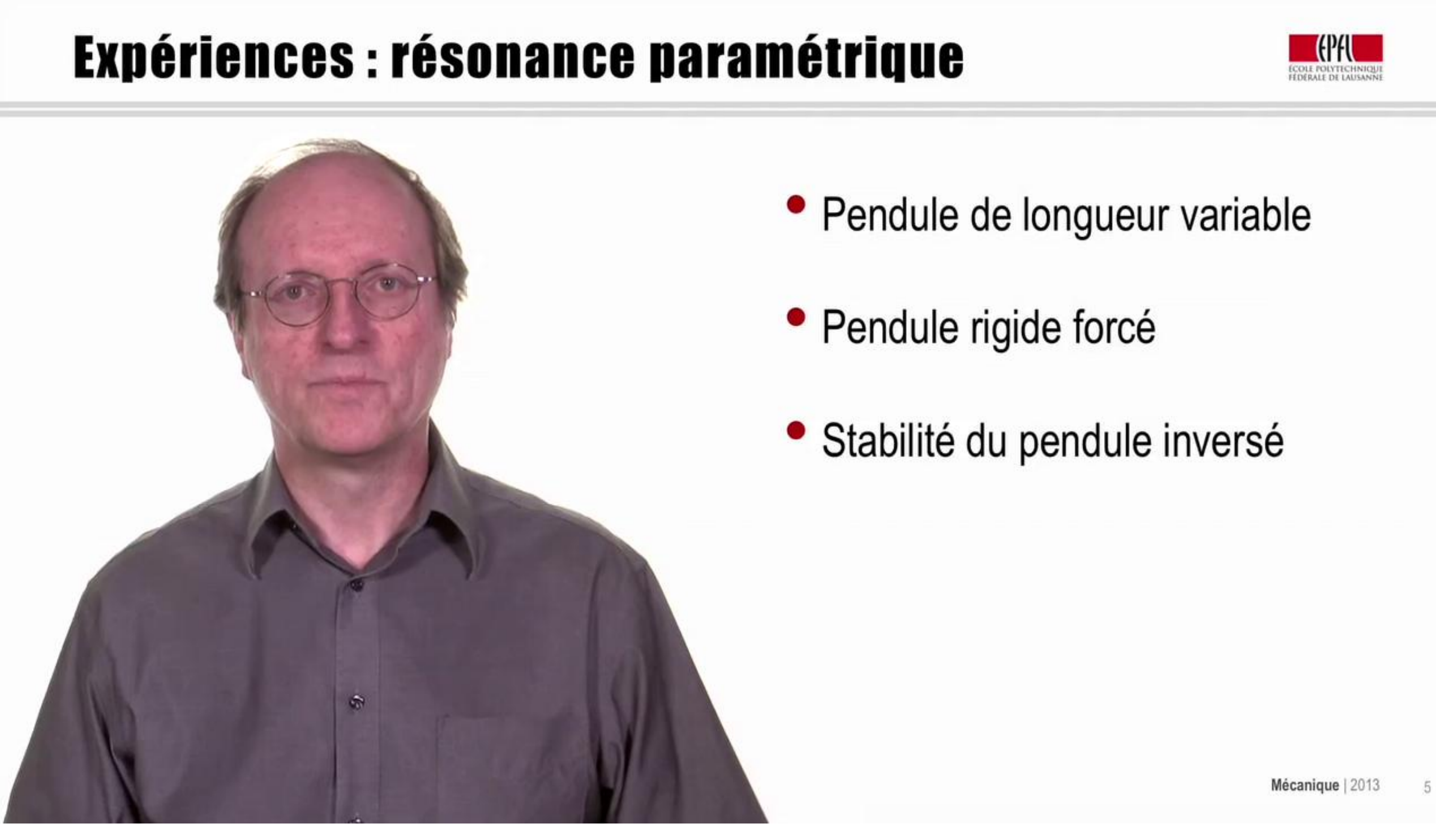


EPFL

Video



Welcome to the EPFL general physics course. This last lesson of mechanics gave an opening on a very rich world, the one of the physics of nonlinear systems. We could appreciate the complexity of the domain by trying to analyze simply the parametric resonances. Here, I give two illustrations of parametric resonances: in the first one, we will consider a pendulum whose length is made to vary, periodically in time, in the first one. In the second one, we have a rigid pendulum and this time it is the attachment point that oscillates periodically along a vertical line. We'll end with an amusing example: a stable inverted pendulum.

Notes

Summary

0m 04s

I start with a pendulum formed simply by a wire and a mass. To excite the parametric resonance, we will vary the length of the wire. Let's start by measuring the natural frequency of the free pendulum.

Notes

Summary

0m 59s

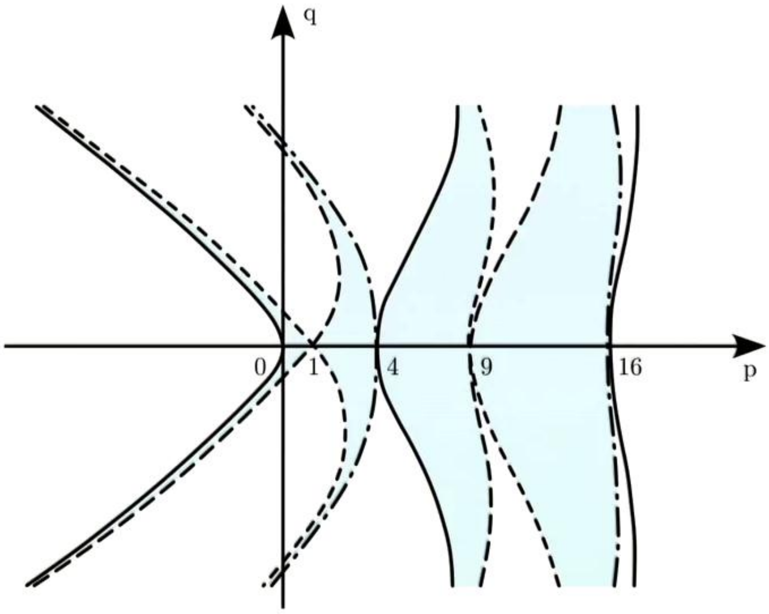
Remember that this frequency is about 0.5 hertz.

Notes

Summary

1m 29s

Relation de dispersion p(q) des fonctions propres



$$y'' + (p - 2q\cos 2\bar{t}\,)\, y = 0$$

$$q = \frac{-2d_0}{L}$$

$$p = \frac{g}{L\Omega^2}$$

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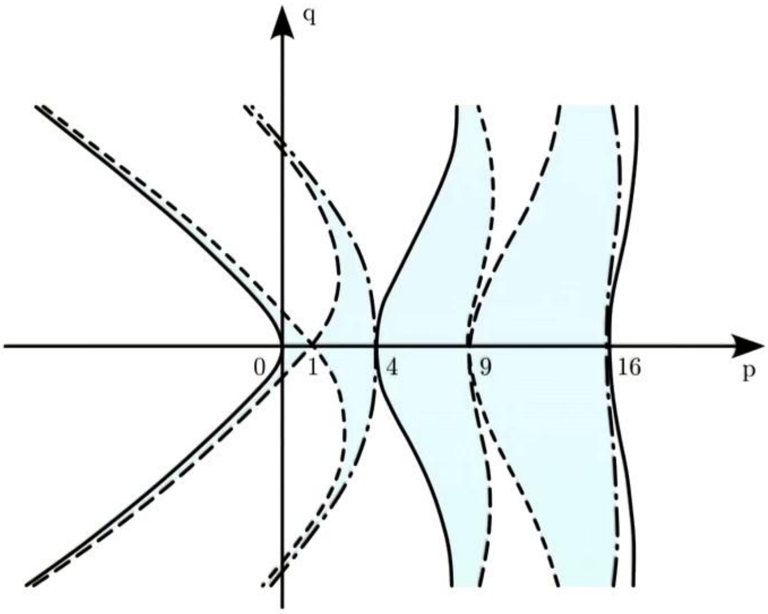
In the example treated theoretically, the diagram of the stable and unstable zones was drawn, as recalled here. The quantities *p* and *q* correspond to the parameters of the oscillator. We see that the point *p* equals 1, on the axis, and is very close to the unstable zones. I recall that the blue marked areas are the stable areas, the white areas are the unstable areas. So, the point *p* equals 1 has a special property: it only takes a very small displacement *q* to fall into the unstable zone. We note that *p* equals 1 means that omega is worth root of *g* on L, this is the pulsation of the free pendulum. But remember that we agreed that the parameter oscillates at two omega, so we have to go to twice the frequency of the free pendulum to observe this effect here.

Notes

Summary

1m 35s

Relation de dispersion p(q) des fonctions propres



$$y'' + (p - 2q\cos 2\bar{t}\,)\,y = 0$$

$$q = \frac{-2d_0}{L}$$

$$p = \frac{g}{L\Omega^2}$$

$\frac{1}{2}\sqrt{\frac{g}{L}}$

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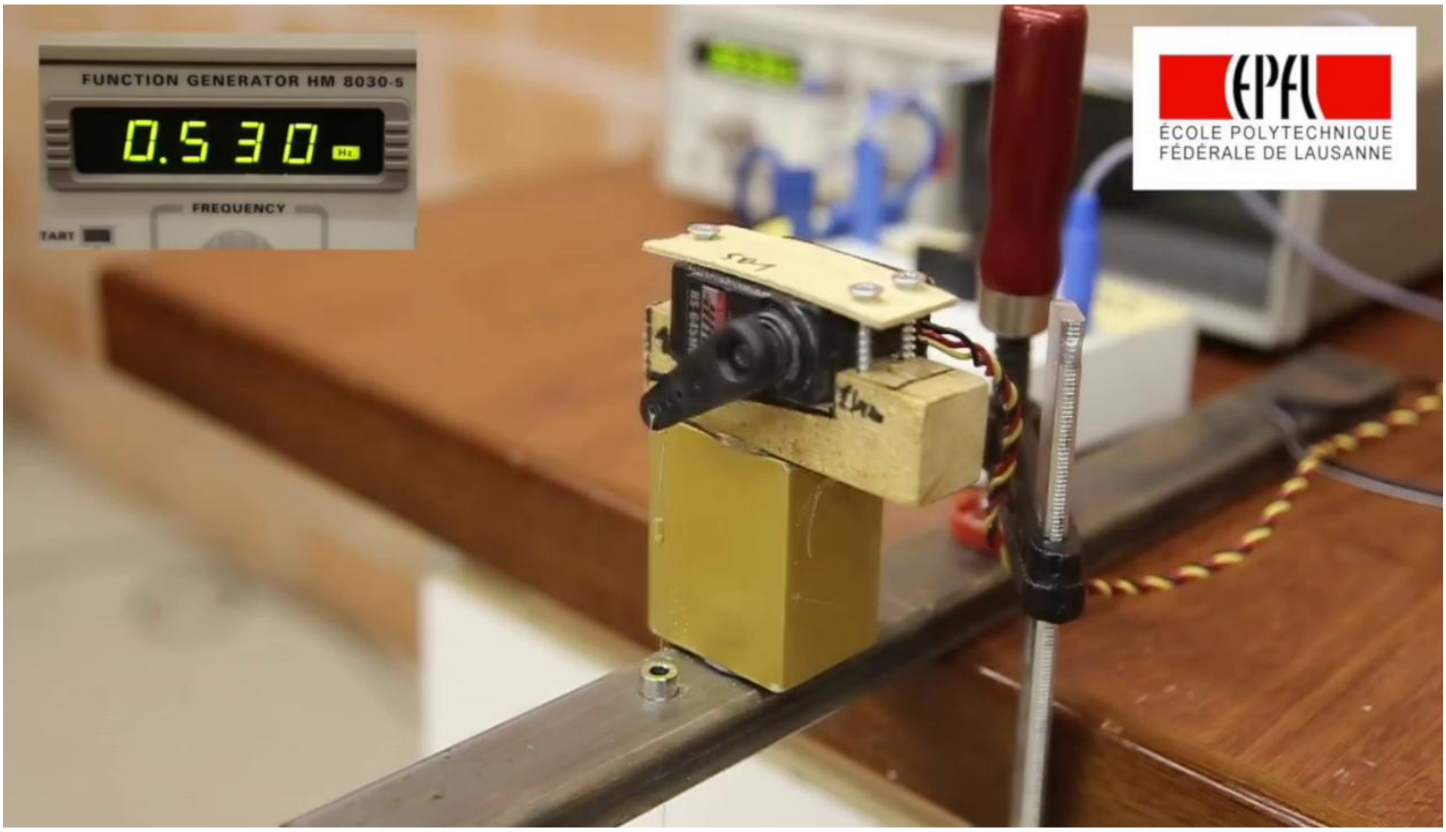
Let's see what happens with this pendulum made of a wire and a mass: We can now examine the stability when the oscillation frequency is exactly that of the resonance of the free pendulum. So we want to have 1/2 root of *g* on L This means that *p* is 4. So we're in that region of the diagram. The diagram was drawn for the ideal case where there is no friction. If there is friction, as our intuition tells us, the stability zones increase. So this region becomes stable. You have to have enough amplitude to enter the instability zone.

Notes

Summary

2m 42s



Let's see what happens in the experiment: the length of the pendulum now oscillates with a frequency of 0,5 hertz.

Notes

Summary

4m 21s

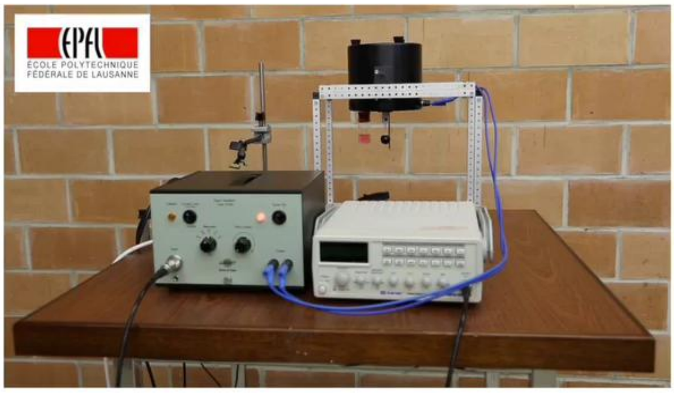
The pendulum is dependent on friction and we do get a small oscillation, but not much. The behaviour here is very different from what we observed at twice the natural frequency.

Notes

Summary

4m 40s

Stabilité du pendule rigide forcé veticalement



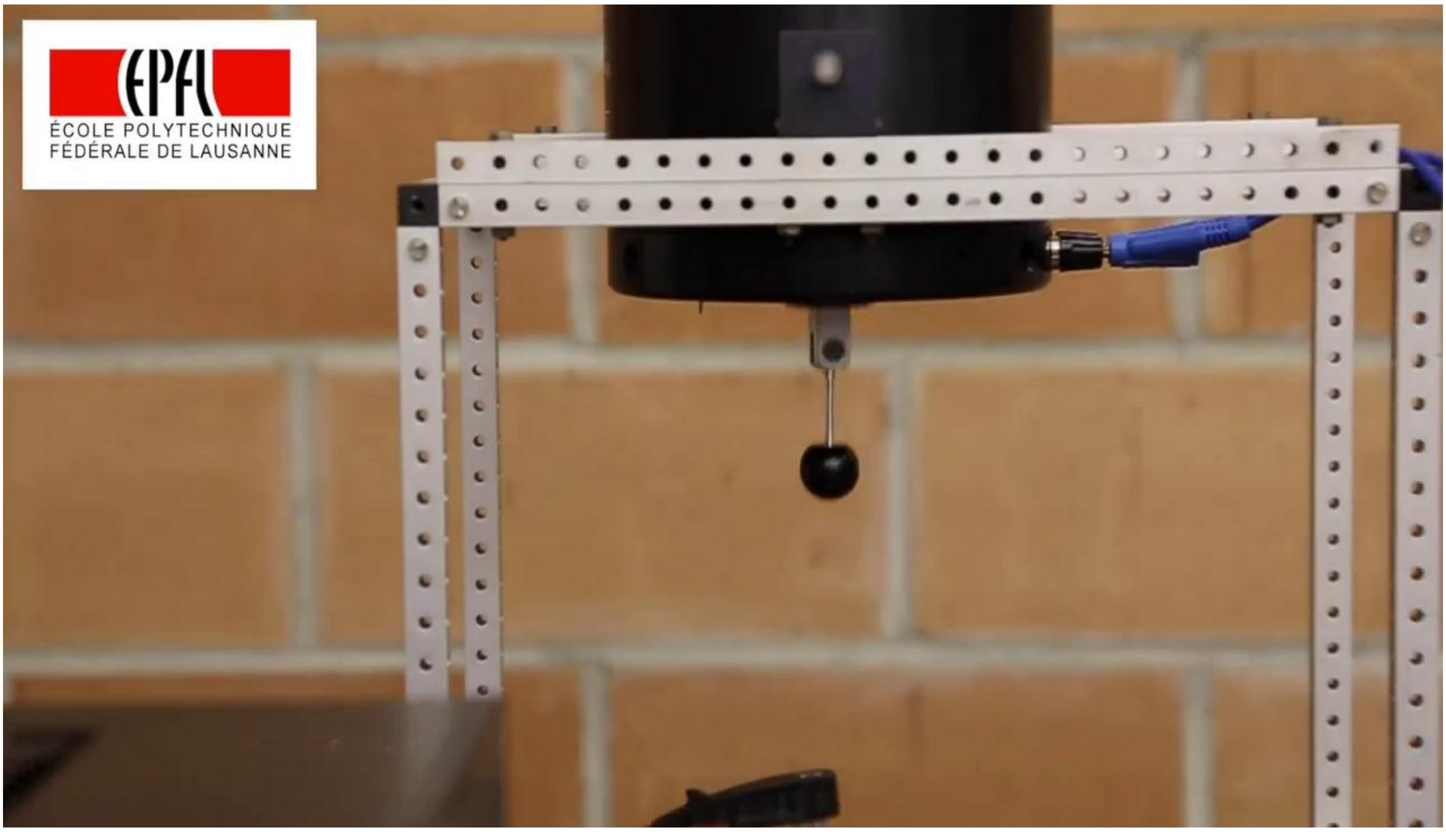
- Le pendule rigide dirigé vers le bas
- Mesure de la fréquence d'oscillation

Let's move on to a rigid pendulum: its length is fixed and it is the attachment point that we will make oscillate with a vibrating point along a vertical. Let's start by measuring the natural frequency of the free pendulum.

Notes

Summary

4m 53s

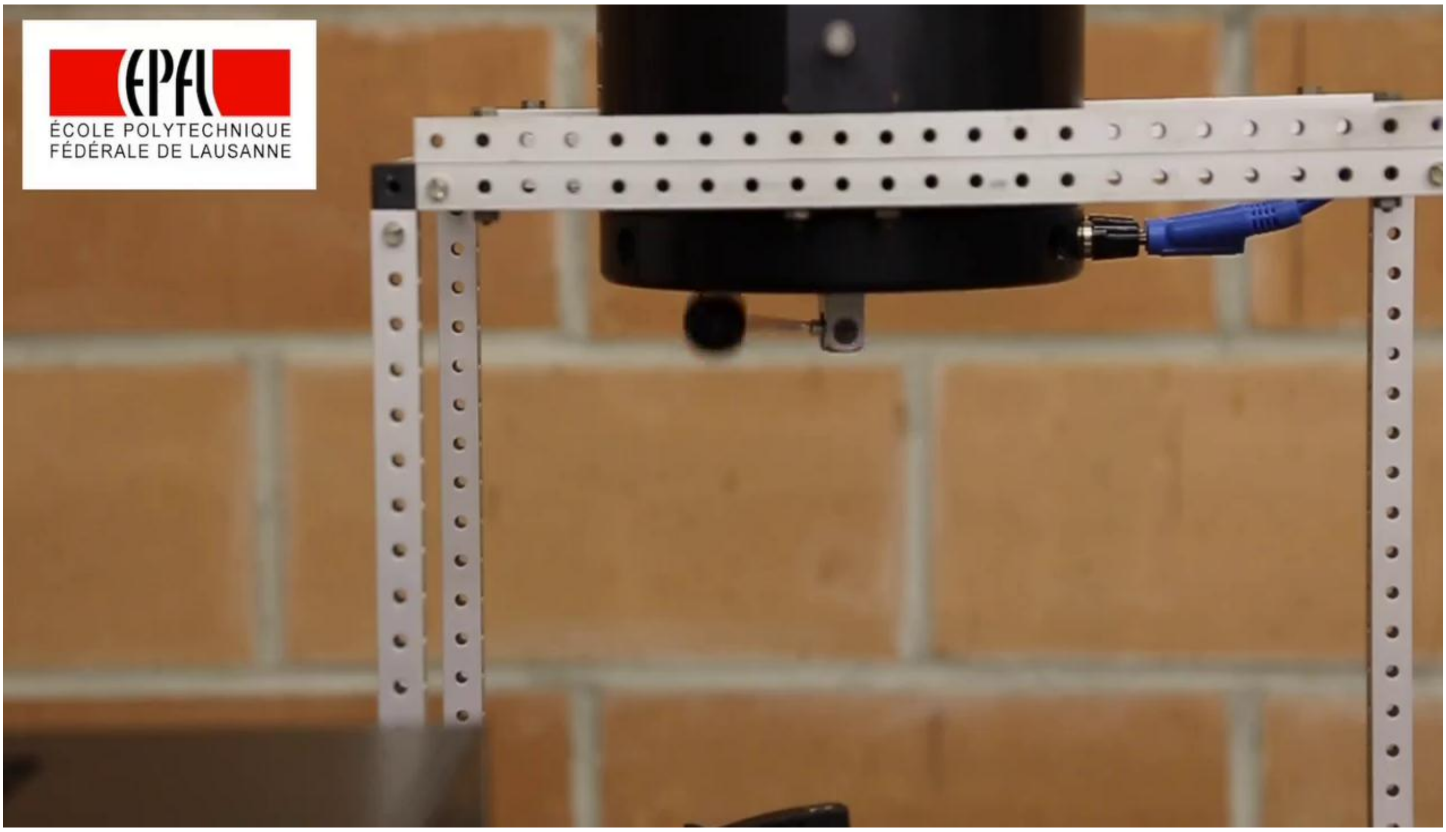


Let's observe what happens if the attachment point oscillates at the frequency of the free pendulum: we have to create a perturbation to observe hardly an oscillation which is very quickly damped. This system is much more dependent on friction than the previous system, so, at one time the frequency, we have a stable system.

Notes

Summary

5m 20s

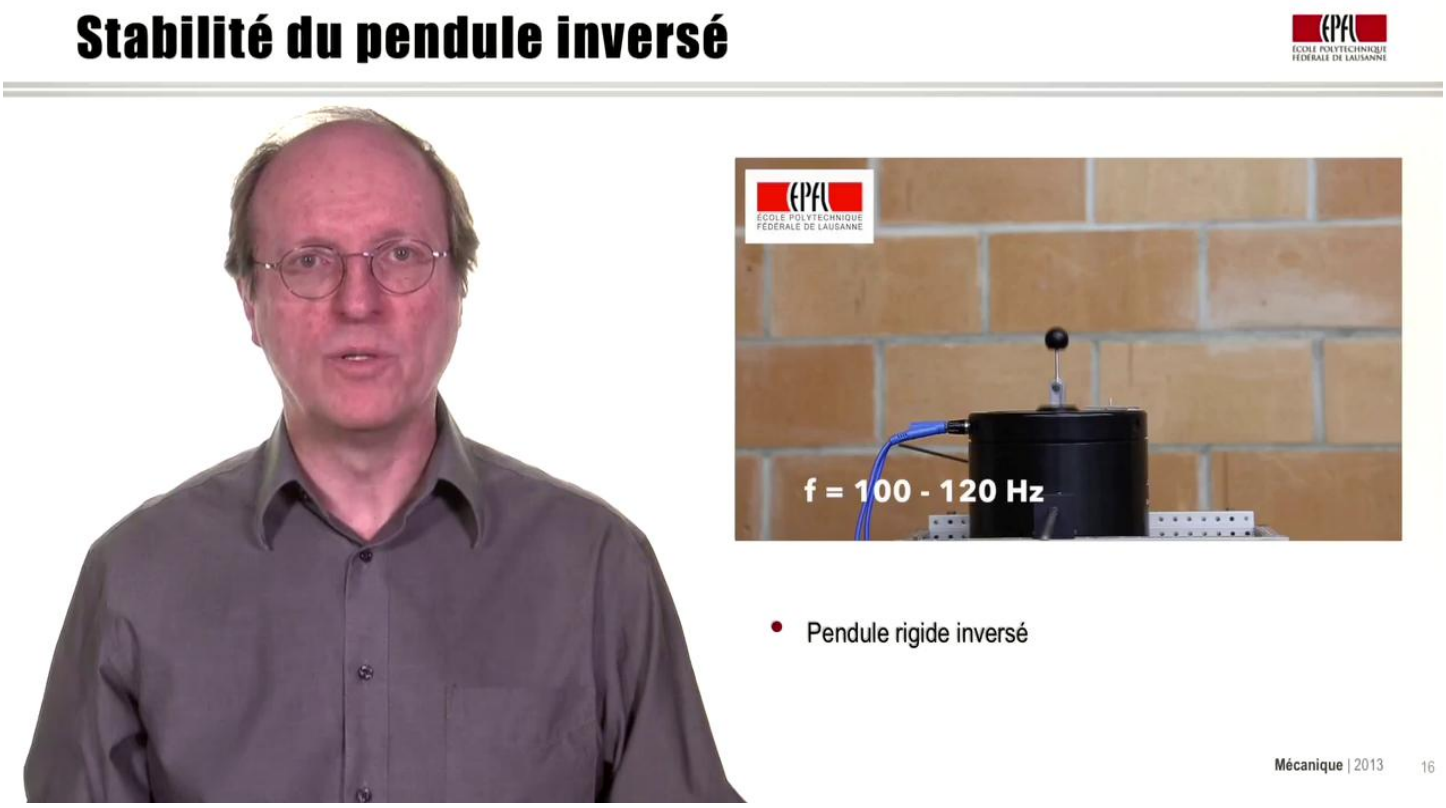


Let's observe what happens at twice the natural frequency: the system is stable, in this equilibrium position. All we need is a small perturbation and we excite the parametric resonance very clearly.

Notes

Summary

5m 45s

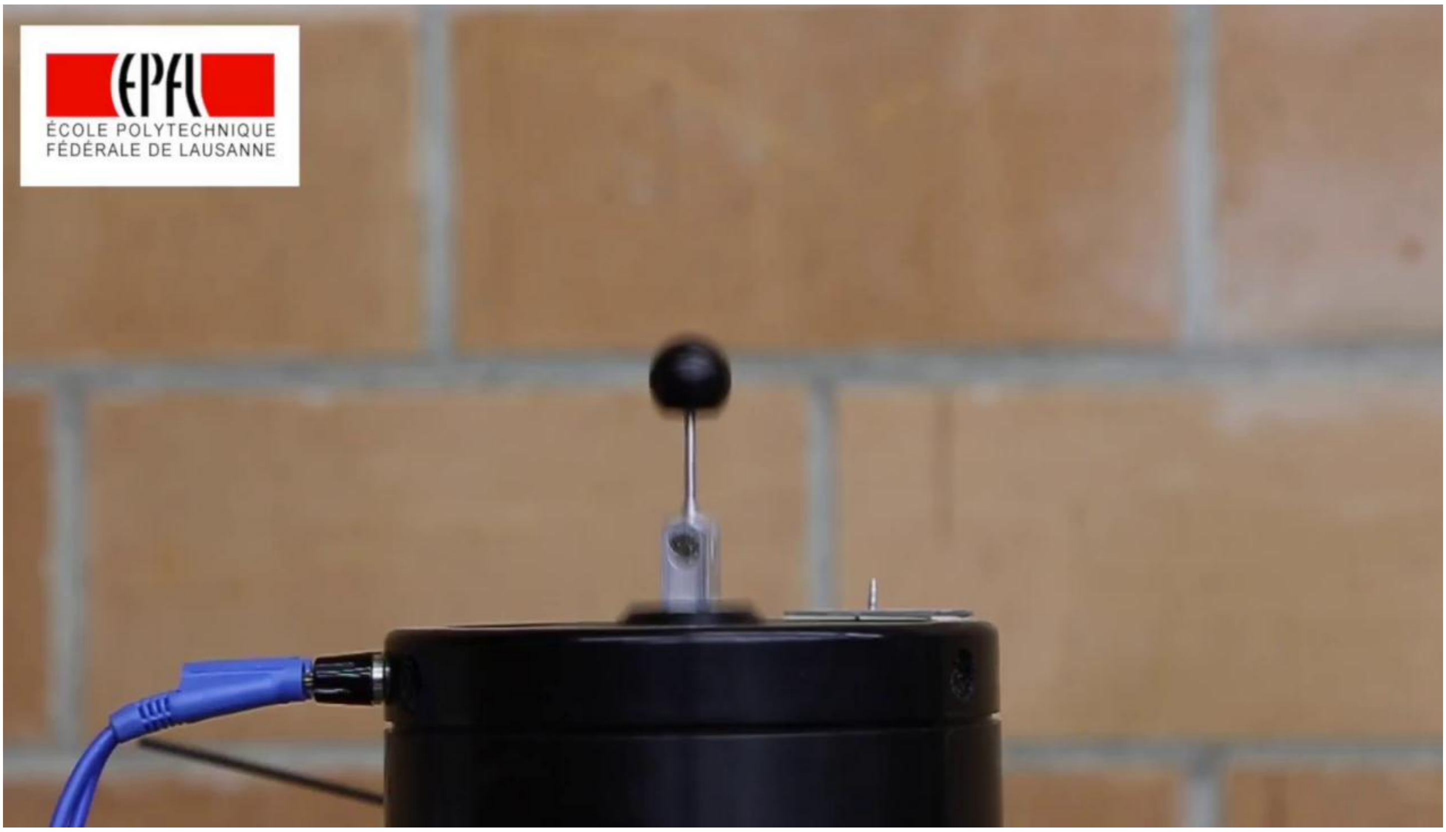


We end this module with a funny experiment: we will make the pendulum vibrate, but with the attachment point down and the pendulum directed upwards. We will see that there is a frequency range where, for a sufficient amplitude, the pendulum is stable, vibrating upwards.

Notes

Summary

6m 11s



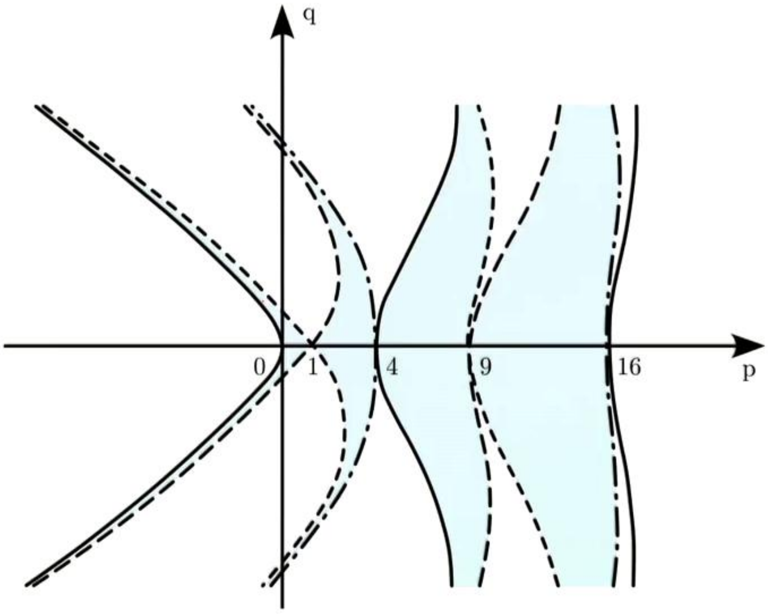
Only in the range between 100 and 120 hertz this phenomenon can be observed. And we can test the stability of this inverted pendulum. Once again... Clearly, the pendulum is stable.

Notes

Summary

6m 37s

Relation de dispersion p(q) des fonctions propres



$$y'' + (p - 2q\cos 2\bar{t}\,)\,y = 0$$

$$q = \frac{-2d_0}{L}$$

$$p = \frac{g}{L\Omega^2}$$

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Here, to use the theoretical development we have seen, we must consider g negative to take into account the fact that the pendulum is inverted. So we are in that region of the diagram. And we see that for a negative g, if the amplitude is large enough, q being related to the amplitude of the oscillation of the point of attachment, we can be in a stability zone.

Notes

Summary

7m 12s

Stabilité du pendule inversé : ordre de grandeur

$$p = \frac{g}{L\Omega^2} \qquad q = \frac{-2d_0}{L}$$

$$-\frac{p}{2} = \cfrac{q^2}{4 - p - \cfrac{q^2}{16 - p - \cfrac{q^2}{36 - p - \cfrac{q^2}{(\ldots)}}}}$$

Ici : $2\,\Omega/2\pi = 110\,\mathrm{Hz}$

$$p = \frac{g}{L\Omega^2} = \frac{-10}{0.05(2\pi\,55)^2} \approx -1.7\,10^{-3}$$

$$-\frac{p}{2} \approx \frac{q^2}{4} \implies q \approx 0.06$$

amplitude de l'oscillation du point d'attache :

$$2d_0 \approx 3 \text{ mm}$$

Here are the equations that relate the parameters p and q to the pendulum parameters. If we take for the frequency 110 hertz, we find that the parameter p has the value shown here, which is small. So, in this formula which relates p to q, we can be satisfied with the first term of this continued fraction, which gives the value of q and from q, with this formula, we deduce the amplitude of the oscillation, we deduce 3 millimeters, which is not far from what we could observe.

Notes

Summary

7m 42s