ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

http://go.epfl.ch/traite-meca-5-30

Résonance paramétrique

Mécanique, cours 28.1

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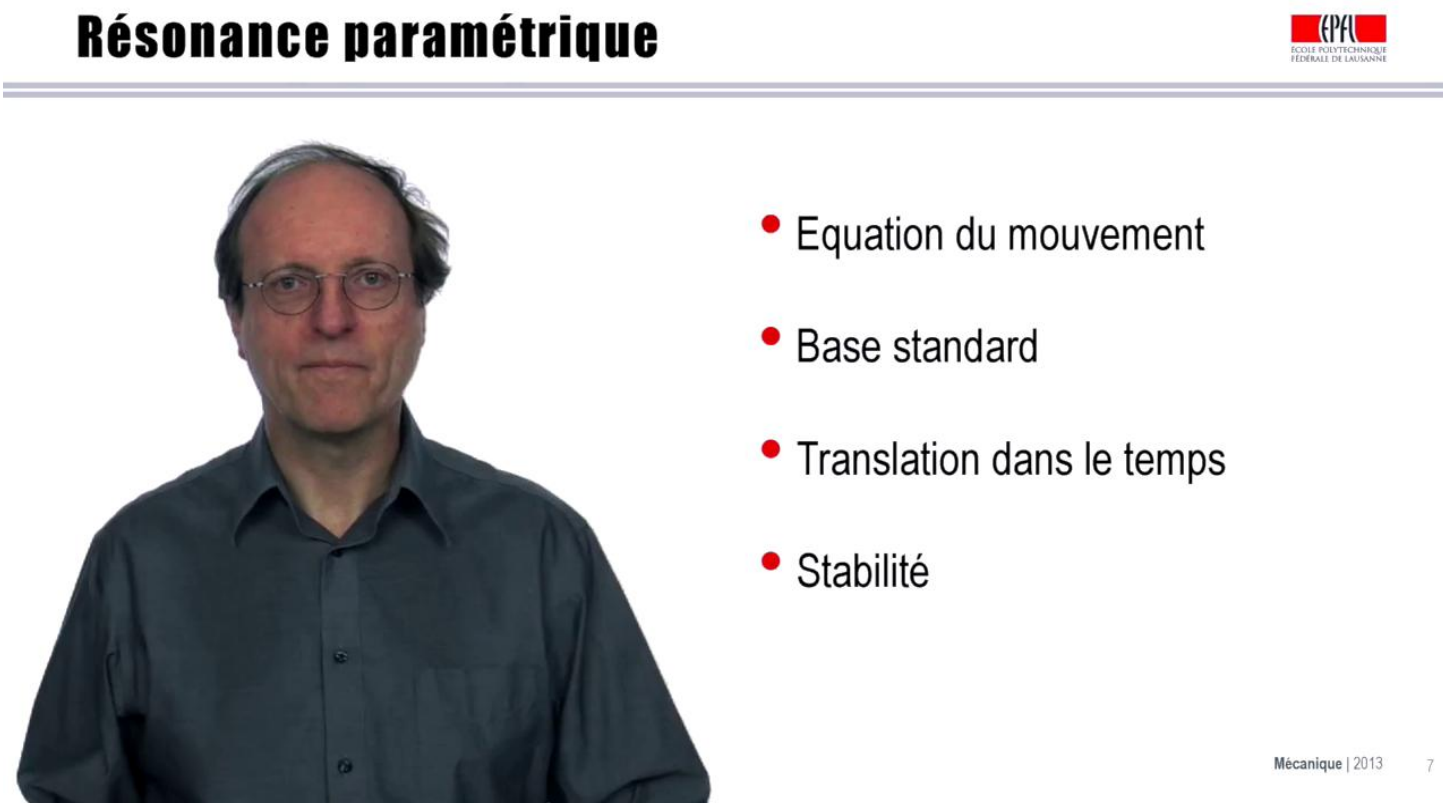
Video

EPFL



Hello. Welcome to the ÉPFL general physics course. In this lesson, I would like to analyze a class of dynamical systems in which the problem of the child on, who stands on a swing and bends his knees regularly and periodically to increase the amplitude of his swing is one of them. I will begin by defining an equation of motion that characterizes this class of dynamical systems. Then, I will define what is called a standard basis of solutions of this equation of motion. Then, we will see a time translation property of the solutions, and this will allow us to approach the question of the stability of these dynamical systems.

Notes

Summary

0m 03s

Exemple : équation de Mathieu

$$y'' + (p - 2q\cos(2\bar{t})\,)\,y = 0$$

$$\text{période : } \quad \bar{\tau} = \pi$$

Mécanique | 2013 14

I start by defining this class of motion by an equation which is as follows. You have a second derivative of a coordinate which of course is a function of time. If in the equation I wrote, G of t, and t a constant, we would have the equation of a harmonic oscillator. But here, we have a function of time, so it's like having a parameter characteristic of the dynamical system that becomes a function of time. That's why we'll talk about parametric resonance later. It is a parameter of the system which depends on time. It's not a force applied to the system, but it's a parameter that depends on time. We'll make the following assumption. We will assume that this function G of t is a periodic function of period tau. I'll give you an example of such a function, it's the example we'll see in the practical part of this lesson, it's the Mathieu equation which has the following form. Here, I wrote it with a t bar. It is a dimensionless independent variable which, the notation reminds us of time, because indeed, we, we will look at time dependent systems. This Mathieu equation is found in all sorts of other physical systems where then, the independent variable is perhaps not time; y second is the second derivative of this system with respect to time, t bar, and the period, since I wrote two t bars, the period tau bar, is pi.

Notes

Summary

1m 03s

Exemple : équation de Mathieu

$$y'' + (p - 2q\cos(2\bar{t})\,)\,y = 0$$

$$\text{période :}\quad \bar{\tau} = \pi$$

$$y'' = \frac{d^2y}{d\bar{t}^{\,2}}$$

Mécanique | 2013 15

So we have a system which corresponds to a Hill type equation, it's the equation which we proposed to analyse from the beginning with a particular form, here with this term in p and this term in q, the q giving the amplitude of the oscillation of the parameter.

Notes

Summary

3m 04s

Propriété : linéarité

$$\ddot{x}_1 + G(t)x_1 = 0 \qquad \ddot{x}_2 + G(t)x_2 = 0$$

$$x = \alpha x_1 + \beta x_2$$

$$\begin{aligned} \ddot{x} = \alpha \ddot{x}_1 + \beta \ddot{x}_2 &= -\alpha G(t) x_1 - \beta G(t) x_2 \\ &= -G(t)\left(\alpha x_1 + \beta x_2\right) \\ &= -G(t) x \end{aligned}$$

Mécanique | 2013 21

This Hill equation is a linear equation. Here, linear means the following. If x one is a solution of Hill's equation, and so is x two, then any linear combination of x one and x two is also a solution. This can be easily verified if we compute the second derivative of x. We can write it immediately in terms of the second derivative of x one and two. We substitute Hill's equation, finally the result of Hill's equation. We group the terms, and we can see that a term minus G times x appears, so it means that the equation x is also a solution of Hill's equation.

Notes

Summary

3m 28s

Définition : base standard

$$\begin{pmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{pmatrix} \begin{pmatrix} 1 \\ b \end{pmatrix} = c \begin{pmatrix} e_1 \\ \dot{e}_1 \end{pmatrix}$$

$e_1(t)$	$e_2(t)$
$e_1(0) = x_0$	$e_2(0) = 0$
$\dot{e}_1(0) = 0$	$\dot{e}_2(0) = v_0$

linéairement indépendantes :

$$e_1 + be_2 = ce_1 \implies b = 0 \text{ et } c = 1$$

Now, I will define a basis called standard. It is, it is here a basis of two linearly independent solutions, as we will see, which, which have, which will play a special role in our description. The first function e un of t, solution of this Hill equation has this property at time t equals zero. We have a non zero amplitude x zero and a zero velocity. For the second one, I'm going to take an initial condition which is that the position, the amplitude is zero, and the velocity is v zero, non zero. Now, I will convince you that these two solutions are linearly independent. What does it mean practically that they are linearly independent? Well it means for example, this: if I take a linear combination of e one and e two, and it happens that this combination is proportional to e one, it means necessarily, if the two solutions are linearly independent, it means necessarily that b must be zero. And at that point, c is one. Let's explain this. First, if I have this equation, I also have, for the derivatives with respect to time, an equation like this, with the derivatives with respect to time of the functions. Now, I'm going to write this system of equation here in a matrix way. Here it is.

Notes

Summary

4m 21s

Définition : base standard

$$\begin{pmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{pmatrix} \begin{pmatrix} 1 \\ b \end{pmatrix} = c \begin{pmatrix} e_1 \\ \dot{e}_1 \end{pmatrix}$$

$$\begin{pmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{pmatrix} \begin{pmatrix} 1-c \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$W = \begin{vmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{vmatrix} \neq 0$$

$$\dot{W} = \cancel{\dot{e}_1\dot{e}_2} + \overbrace{e_1\ddot{e}_2}^{-e_1 G e_2} - \overbrace{\ddot{e}_1 e_2}^{+e_1 G e_2} - \cancel{\dot{e}_1\dot{e}_2} = 0$$

$e_1(t)$	$e_2(t)$
$e_1(0) = x_0$	$e_2(0) = 0$
$\dot{e}_1(0) = 0$	$\dot{e}_2(0) = v_0$

linéairement indépendantes :

$$e_1 + b e_2 = c e_1 \implies b = 0 \text{ et } c = 1$$

And now, I can group the terms together and write it like this. Now, if this matrix here is invertible, then I'll have a minus c and b which is this, the inverse matrix times zero. And we know that this matrix is invertible if its determinant is non-zero, so we have to worry about the determinant. So luckily, one property of this determinant, is that it's time independent. So, we'll be able to compute it, especially by taking the values at t equals zero. Let's first prove that this determinant is independent of time. So, here I compute the derivative with respect to time of W. W which we call wronskian has a term in e one, e two point. So the derivative with respect to time will show an e one point, e two point, that's the term. An e one, e two point point, it's there, minus e one point point e two, it's there, and still minus e one, e two point, it's there. These two terms cancel each other out. I'm left with this term. But this one I can write e one with minus G times e two, and this term I can write plus e one times G, times e two. And so, these two terms cancel each other out. The derivative with respect to time of W is zero, so W is a constant.

Notes

Summary

6m 12s

Définition : base standard

$$\begin{pmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{pmatrix} \begin{pmatrix} 1 \\ b \end{pmatrix} = c \begin{pmatrix} e_1 \\ \dot{e}_1 \end{pmatrix}$$

$$\begin{pmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{pmatrix} \begin{pmatrix} 1-c \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$W = \begin{vmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{vmatrix} \neq 0$$

$$\dot{W} = \dot{e}_1\dot{e}_2 + e_1\ddot{e}_2 - \ddot{e}_1 e_2 - \dot{e}_1\dot{e}_2 = 0$$

$$W = x_0 v_0 \ \neq 0$$

$e_1(t)$	$e_2(t)$
$e_1(0) = x_0$	$e_2(0) = 0$
$\dot{e}_1(0) = 0$	$\dot{e}_2(0) = v_0$

linéairement indépendantes :

$$e_1 + be_2 = ce_1 \implies b = 0 \text{ et } c = 1$$

Mécanique | 2013 34

I can calculate it at t equals zero, it's this term times this term minus this one. So W is x zero, v zero. And we chose non-trivial solutions, so x zero and v zero, both are non-zero, and so the W, it's non-zero. Therefore, one minus c must be zero, and b equals zero, that's what we had expressed here. So, e one and e two are linearly independent solutions.

Notes

Summary

8m 04s

$$x(t) = ae_1(t) + be_2(t)$$

$$\mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Mécanique | 2013 37

I will now define the vector space of solutions. I do this in the following way. If x of t is a solution, then I can write x of t as a linear combination of the two functions e one and e two, which is what I have written here. Now, my vector space, I'm going to write it as a two-dimensional space where the vectors have components a and b. And all these vectors of type a and b represent linear combinations of e one and e two, thus solutions of our problem.

Notes

Summary

8m 44s

Translation dans le temps

$x(t)$ solution, alors

$x(t+\tau)$ aussi solution

Démonstration :

$$t' = t + \tau \qquad \frac{dx}{dt} = \frac{dx}{dt'}\frac{d(t+\tau)}{dt} = \frac{dx}{dt'} \qquad \frac{d^2x}{dt^2} = \frac{d^2x}{dt'^2}$$

$$\ddot{x} + G(t)x = 0 \qquad \frac{d^2x(t)}{dt^2} + G(t)x(t) = 0 \qquad \frac{d^2x(t+\tau)}{d(t+\tau)^2} + G(t+\tau)x(t+\tau) = 0$$

$$\frac{d^2x(t+\tau)}{dt^2} + G(t)x(t+\tau) = 0$$

Mécanique | 2013 48

Now I will prove a property which is the following. If x of t is a solution, then if we translate the solution of a time tau, I remind you that tau is the period of the function G of t which defines our problem, then, x of t plus tau is also a solution. Let's proceed to the demonstration. I consider t prime which is t plus tau. I consider x as a function of t prime, and t prime is a function of t. I apply the chain derivation rule, so d of x over dt, is d of x over d t prime, times the derivative of t prime with respect to t. This is worth one. So we have the derivative dx over dt which is worth dx over d t prime. Of course, for the second derivative, we have the same property. Now, I consider this foncti, this equation of motion, and I rewrite it explicitly like this so that x is clearly a function of t, G a function of t, x a function of t here, and now I'm going to express this function at another time. That's true for any time, I'm going to express it at time t plus tau. This gives me explicitly this equation. Now, when we drift with respect to t or t plus tau, we have the same thing. So this term, I can simplify it, like this. The G of t plus tau, because of the periodicity of G, it means G of t.

Notes

Summary

9m 28s

Translation dans le temps

$x(t)$ solution, alors

$x(t+\tau)$ aussi solution

Démonstration :

$$t' = t + \tau \qquad \frac{dx}{dt} = \frac{dx}{dt'}\frac{d(t+\tau)}{dt} = \frac{dx}{dt'} \qquad \frac{d^2x}{dt^2} = \frac{d^2x}{dt'^2}$$

$$\ddot{x} + G(t)x = 0 \qquad \frac{d^2x(t)}{dt^2} + G(t)x(t) = 0 \qquad \frac{d^2x(t+\tau)}{d(t+\tau)^2} + G(t+\tau)x(t+\tau) = 0$$

$$\frac{d^2x(t+\tau)}{dt^2} + G(t)x(t+\tau) = 0$$

cqfd

Mécanique | 2013 49

And I keep the x of t plus tau. What does this equation tell me? It tells me that this function, x of t plus tau, is a solution of the so-called Hill equation. So I have demonstrated what I wanted.

Notes

Summary

11m 29s

Matrice d'avancée d'une période

$$x(t) = ae_1(t) + be_2(t)$$
$$x(t+\tau) = a'e_1(t) + b'e_2(t)$$

$$e_1(t+\tau) = r_{11}e_1(t) + r_{21}e_2(t)$$

$$e_2(t+\tau) = r_{12}e_1(t) + r_{22}e_2(t)$$

$$x(t+\tau) = ae_1(t+\tau) + be_2(t+\tau) = a[r_{11}e_1(t) + r_{21}e_2(t)] + b[r_{12}e_1(t) + r_{22}e_2(t)]$$
$$= (ar_{11} + br_{12})e_1(t) + (ar_{21} + br_{22})e_2(t)$$

Mécanique | 2013 57

So now, I'm going to do something that allows me to calculate a solution at an advanced time according to a previous time. You'll see how this works and how it can be done in a matrix fashion. I have x of t which is a solution. I project it, on e one and e two. And now I consider the translate function of tau. I know it's a function that is a solution too, I just showed it. So I project it onto my standard basis, e one of t, e two of t. Now I'm going to do the same exercise with the basis, I consider the function e one translated from tau, and I'm going to write it with two coefficients, as here I had used a and b, I'm going to use coefficients r one, one and r two, one, because I'm going to define a matrix and I've put in the subscripts so that they match our usual convention of matrix product notation. For e two, I'm also going to put two coefficients, so e two translated from tau, I write it projected on e one and e two, thanks to that definition, now if I consider x of t plus tau, looking at that formula, I can put instead of t everywhere t plus tau, that gives me this, now e one of t plus tau, I've rewritten it as a function of e one of t and e two of t, likewise for e two of t plus tau, here the term proportional to a, the term proportional to b, now I group the terms into e one and e two, that gives me this.

Notes

Summary

11m 46s

Matrice d'avancée d'une période

$$x(t) = ae_1(t) + be_2(t)$$
$$x(t+\tau) = a'e_1(t) + b'e_2(t)$$
$$e_1(t+\tau) = r_{11}e_1(t) + r_{21}e_2(t)$$
$$e_2(t+\tau) = r_{12}e_1(t) + r_{22}e_2(t)$$

$$x(t+\tau) = ae_1(t+\tau) + be_2(t+\tau) = a[r_{11}e_1(t) + r_{21}e_2(t)] + b[r_{12}e_1(t) + r_{22}e_2(t)]$$
$$= (ar_{11} + br_{12})e_1(t) + (ar_{21} + br_{22})e_2(t)$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}}_{R} \begin{pmatrix} a \\ b \end{pmatrix} \qquad \mathbf{x}(t+\tau) = R\mathbf{x}(t)$$

And this must be my a prime, and this must be my b prime, which I had defined here. Then I can write this relation in a matrix way, I can write that a prime and b prime, it's this matrix r with the coefficients in this order, times ab. If I want to simplify the writings, the vector x which corresponds to a prime, b prime, I write function of t plus tau to remember that it is the solution x translated in time, it is this matrix r, times the vector ab, which is the solution at time t. So I have a relation here between the solution at time t plus tau and the solution at time t. It's this relationship that will allow me to analyze the stabilities of the system.

Notes

Summary

13m 54s

Matrice d'avancée d'une période

$$e_1(t+\tau) = r_{11}e_1(t) + r_{21}e_2(t) \qquad \dot{e}_1(t+\tau) = r_{11}\dot{e}_1(t) + r_{21}\dot{e}_2(t)$$

$$e_2(t+\tau) = r_{12}e_1(t) + r_{22}e_2(t) \qquad \dot{e}_2(t+\tau) = r_{12}\dot{e}_1(t) + r_{22}\dot{e}_2(t)$$

$$R = \begin{pmatrix} e_1(\tau)/x_0 & e_2(\tau)/x_0 \\ \dot{e}_1(\tau)/v_0 & \dot{e}_2(\tau)/v_0 \end{pmatrix}$$

$$W = \begin{vmatrix} e_1 & e_2 \\ \dot{e}_1 & \dot{e}_2 \end{vmatrix} = x_0 v_0 \qquad det(R) = \frac{e_1(\tau)\dot{e}_2(\tau) - \dot{e}_1(\tau)e_2(\tau)}{x_0 v_0} = 1$$

So, I'm going to start by expressing this matrix r as a function of the initial conditions. That I had given myself for e one and e two. Here, I recall the definition of the coefficients r one, one, r two one, r one, two and r two, two. I can derive these functions with respect to time, which gives me this. And now I take these relations at time t equals zero, and I see if I take for example here t equals zero, this e one of t is x zero, this is zero, and there we have e one of tau. So e one of tau divided by x zero is r one, one. That's what I wrote here. With the same reasoning, you find the other coefficients. Of the matrix. The other elements of the matrix. Now, we note that the determinant of r is this times this, minus this times this, I find here the term w, the wronskian, which, I remind you, was x zero, v zero, so the determinant of r is one, that's a property we're going to use later.

Notes

Summary

15m 03s

$$t = n\tau + t' \qquad \mathbf{x}(t) = R^n \mathbf{x}(t')$$

$$\text{Diagonalisation : } \; R = U^{-1} R_D U$$

$$\mathbf{x}(t) = R^n \mathbf{x}(t') = U^{-1} R_D^n U \mathbf{x}(t')$$

$$R_D = \begin{pmatrix} \rho_+ & 0 \\ 0 & \rho_- \end{pmatrix}$$

Now, to understand the parametric resonance, you just have to do the following reasoning. I imagine that I know my function x of t for a time t that I have noted t prime, small. And now I would like to know what happens much later in time. So a time t. I can write t as a number of periods, plus t prime. So the solution at time t is R to the power n, you apply R n times, on the solution x at time t prime. So now we're going to analyze what this matrix R to the power n is worth. We're going to diagonalize R, so we're going to assume, and we're going to find it, this, we're going to do this diagonalization, we've got R which we can write as a product U minus one, R diagonal times U, at this time, x, it's R to the power n, and there it's easy to see, we can do it like this, R to the power n, it's R times R, times R, et cetera, we can write it as U minus one, RD U, U minus one, RDU, et cetera, eh, I just added the matrix one here, and so we're left with the U minus one at the beginning, the U at the end and RD to the nth power, if RD is a diagonal matrix, just put the elements on the diagonal to the nth power. And I will write these diagonal elements, rhô plus and rhô minus.

Notes

Summary

16m 32s

Résonance paramétrique

$$\begin{pmatrix} e_1(\tau)/x_0 & e_2(\tau)/x_0 \\ \dot{e}_1(\tau)/v_0 & \dot{e}_2(\tau)/v_0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \rho \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\left(\frac{e_1(\tau)}{x_0} - \rho\right)\left(\frac{\dot{e}_2(\tau)}{v_0} - \rho\right) - \frac{\dot{e}_1(\tau)e_2(\tau)}{x_0 v_0} = 0$$

$$\rho^2 - \rho\left(\frac{e_1(\tau)}{x_0} + \frac{\dot{e}_2(\tau)}{v_0}\right) + 1 = 0$$

$$T = Tr(R) = \frac{e_1(\tau)}{x_0} + \frac{\dot{e}_2(\tau)}{v_0} = \rho_+ + \rho_-$$

$$\rho_\pm = \frac{T \pm \sqrt{T^2 - 4}}{2}$$

$$t = n\tau + t' \qquad \mathbf{x}(t) = R^n \mathbf{x}(t')$$

$$\text{Diagonalisation :} \quad R = U^{-1} R_D U$$

$$\mathbf{x}(t) = R^n \mathbf{x}(t') = U^{-1} R_D^n U \mathbf{x}(t')$$

$$R_D = \begin{pmatrix} \rho_+ & 0 \\ 0 & \rho_- \end{pmatrix}$$

$$det(R) = \rho_+ \rho_- = 1$$

Remember that the determinant of R, is one, so rhô plus times rhô minus equals one. This is a property that will be very important for this question of stability, as you will see. So now, I'm going to calculate rhô plus and rhô minus, so I'm basically looking for, you see, I have to calculate the eigenvalue problem and eigenvectors, rhô is the eigenvalue, I'm looking for the vector a and b to be associated, I have this matrix times the vector which is worth a number times the vector, so it's an eigenvalue problem. When I do the detailed calculation, so I pass these terms on the other side of the equal sign, and I look for a zero determinant so that I have ab non-trivial solutions, then I have these two terms that appear. In this term, I have, in this product, these two elements, I have rhô square which appears, I have the term of mixed product, which is here, and at the end, I have an e one times e two point, or that, it's our w again, which was x, zero, v, zero, so there is the one. This term is the sum of these two terms, so it's what we call the trace of the matrix, and it's worth rhô plus, plus rhô minus. Now, if I rewrite T here and look for the solutions rhô plus and rhô minus, I have T plus or minus root of T square minus four, all divided by two. So now, we're going to see right away how solutions that diverge, unstable solutions, appear.

Notes

Summary

18m 35s

Solutions instables

$$\rho_\pm = \frac{T \pm \sqrt{T^2 - 4}}{2}$$

$$T^2 > 4$$

$$\mathbf{x}(t) = R^n \mathbf{x}(t')$$

$$n \to \infty \implies \rho_i^n \to \infty \quad (i = + \text{ ou } -)$$

Mécanique | 2013 84

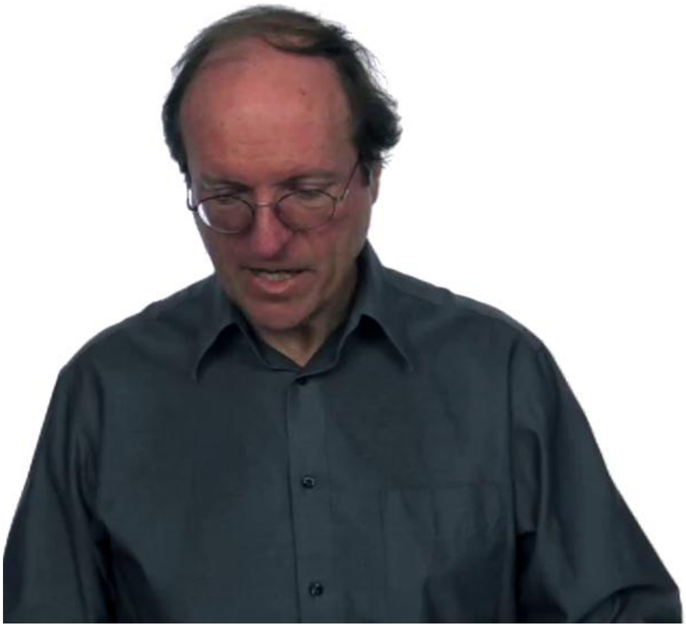
Indeed, if we consider these so, solutions with a T square which is greater than four, rhô plus and rhô minus real, so, I remind this formula, when we are going to look for a very large T, n is very large, we are going to have rhô plus to the power n and rhô minus to the power n which intervene. Now, since rhô plus times rhô minus is one, you have rhô plus, which is one over rhô minus. If a root is greater than one, we'll see afterwards the case of the root that is one, but if one root is worth, is greater than one, the other is smaller than one. Of the two, there is always one that diverges. So we have a system that becomes unstable.

Notes

Summary

20m 45s

Solutions stables



$$\rho_{\pm} = \frac{T \pm \sqrt{T^2 - 4}}{2}$$

$$T^2 < 4$$

$$\rho_{\pm} = \frac{T \pm i\sqrt{4 - T^2}}{2}$$

$$|\rho\pm| = 1$$

$$\rho_{\pm} = e^{\pm i\phi}$$

Mécanique | 2013 90

Now I look at the stable cases. I consider this formula for rhô plus and minus and I take the case T square is smaller than four. At this point, my rhô plus, I can write it as T plus or minus i times the root of four minus T square, that's a real number. You can see that the modulus of rhô plus and rhô minus must be calculated T square plus four minus T square, divided by four, that makes one, the modulus is one. So I can write my rhô plus, minus as e to the power plus or minus i, phi. Now, there are some special properties involved.

Notes

Summary

21m 42s

Solutions stables

$$\rho_\pm = e^{\pm i\phi} \qquad \phi \neq 0 \text{ et } \phi \neq \pi$$

changement de base pour diagonaliser R :

$$\{e_1(t), e_2(t)\} \to \{\bar{e}_1(t), \bar{e}_2(t)\}$$

$$\bar{e}_k(t+\tau) = e^{\pm i\phi}\bar{e}_k(t) \quad (k = 1, 2)$$

$$\bar{e}_k(t) = e^{\pm i\phi t}u_k(t) \quad (k = 1, 2)$$

$u_k(t)$ est une fonction périodique, de période τ

Démonstration :

$$u_k(t) = e^{\mp i\phi t/\tau}\bar{e}_k(t)$$

$$u_k(t+\tau) = e^{\mp i\phi t/\tau}e^{\mp\phi}\bar{e}_k(t+\tau)$$

I will consider the case where rhô plus, minus is e to the power plus or minus i, phi and phi is different from zero and pi, we will deal with these two cases at the end. In this case, I imagine that my change of basis goes from the standard basis to the basis of the eigenvectors of the matrix R, and so I have this property that if I take the vector of basis e, k translated from tau, I have an e power plus or minus i, phi depending on whether I take one or the other of the solutions, times the basis function at time t. Now, I choose to write this function as e power i, phi of t times u, k of t, this is what Floquet had done and this theory is known as Floquet theory. If we do this, we can convince ourselves that u, k of t is a periodic function. In another theoretical framework, this is called Bloch's theorem. I will demonstrate it, the fact that u, k is a periodic function of period tau, it suffices to write the definition of u, k of t like this. This is true for all t. I'll take for t the value t plus tau, so I write this, here I put a t plus tau and t plus tau. Now, I observe the term that I can deduce from this relation.

Notes

Summary

22m 30s

Solutions stables

$$\rho_\pm = e^{\pm i\phi} \qquad \phi \neq 0 \text{ et } \phi \neq \pi$$

changement de base pour diagonaliser R :

$$\{e_1(t), e_2(t)\} \to \{\bar{e}_1(t), \bar{e}_2(t)\}$$

$$\bar{e}_k(t+\tau) = e^{\pm i\phi}\bar{e}_k(t) \quad (k = 1, 2)$$

$$\bar{e}_k(t) = e^{\pm i\phi t}u_k(t) \quad (k = 1, 2)$$

$u_k(t)$ est une fonction périodique, de période τ

Démonstration :

$$u_k(t) = e^{\mp i\phi t/\tau}\bar{e}_k(t)$$

$$u_k(t+\tau) = e^{\mp i\phi t/\tau}e^{\mp\phi}\bar{e}_k(t+\tau) = e^{\mp i\phi t/\tau}\bar{e}_k(t) = u_k(t)$$

So I have an e, k of t, which appears here, and so I have this term which remains, this which simplifies as e, k of t and now e power minus or plus i, phi of t times e, k, that makes u, k of t. So I just said that u, k of t plus tau is equal to u, k of t, so that's saying that u, k is a periodic function of period tau. So here we have a whole series of bounded solutions.

Notes

Summary

24m 15s

Solutions stables

$\phi = 0$ $\qquad$ $\bar{e}_k(t+\tau) = e^{\pm i\phi}\bar{e}_k(t)$

$\bar{e}_k(t+\tau) = \bar{e}_k(t)$

les fonctions $\bar{e}_k(t)$ sont des ***fonctions de période*** τ

$\phi = \pi$

$\bar{e}_k(t+2\tau) = (-1)^2\bar{e}_k(t)$

les fonctions $\bar{e}_k(t)$ sont des ***fonctions de période*** 2τ

Mécanique | 2013 107

I still have to deal with the case where phi equals zero. If phi equals zero, well, we can see right away that we have e, k translated from tau which is equal to e, k. This means that we have functions of period tau. I remind you that tau is the period of this function g of t which characterizes how the parameter of my dynamic system oscillates. If I now take phi equals pi, I have an e power minus i, pi which appears, which is worth minus 1, so we have to go forward two tau to find the function. So we have a periodic function of period two tau. So, as we'll see in the next module, the analysis of a particular case can become quite heavy but in the case I'm going to show, these functions, those which correspond to phi equals zero and phi equals pi, they are what we call eigenfunctions of the Mathieu equation, they delimit the, the, the stable regime from the unstable regime. This is what we will see in the next module.

Notes

Summary

24m 49s