Expériences : oscillateurs harmoniques couplés

Mécanique, cours 27.exp

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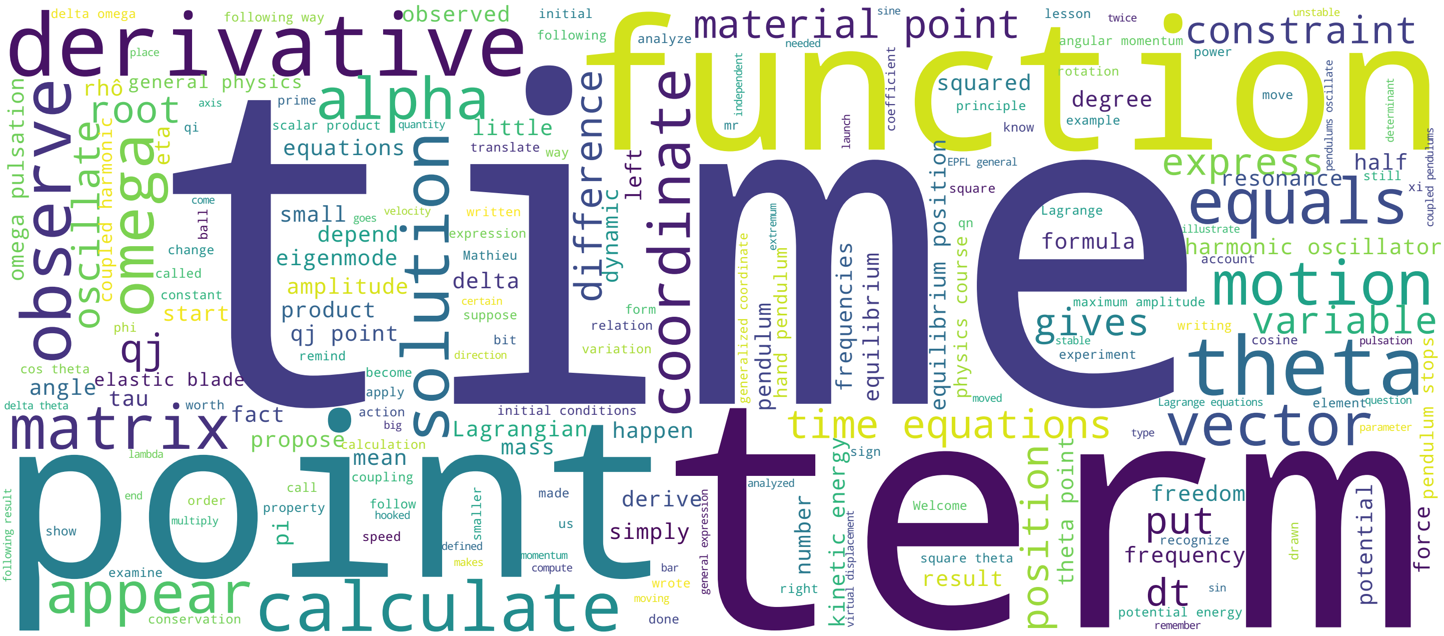
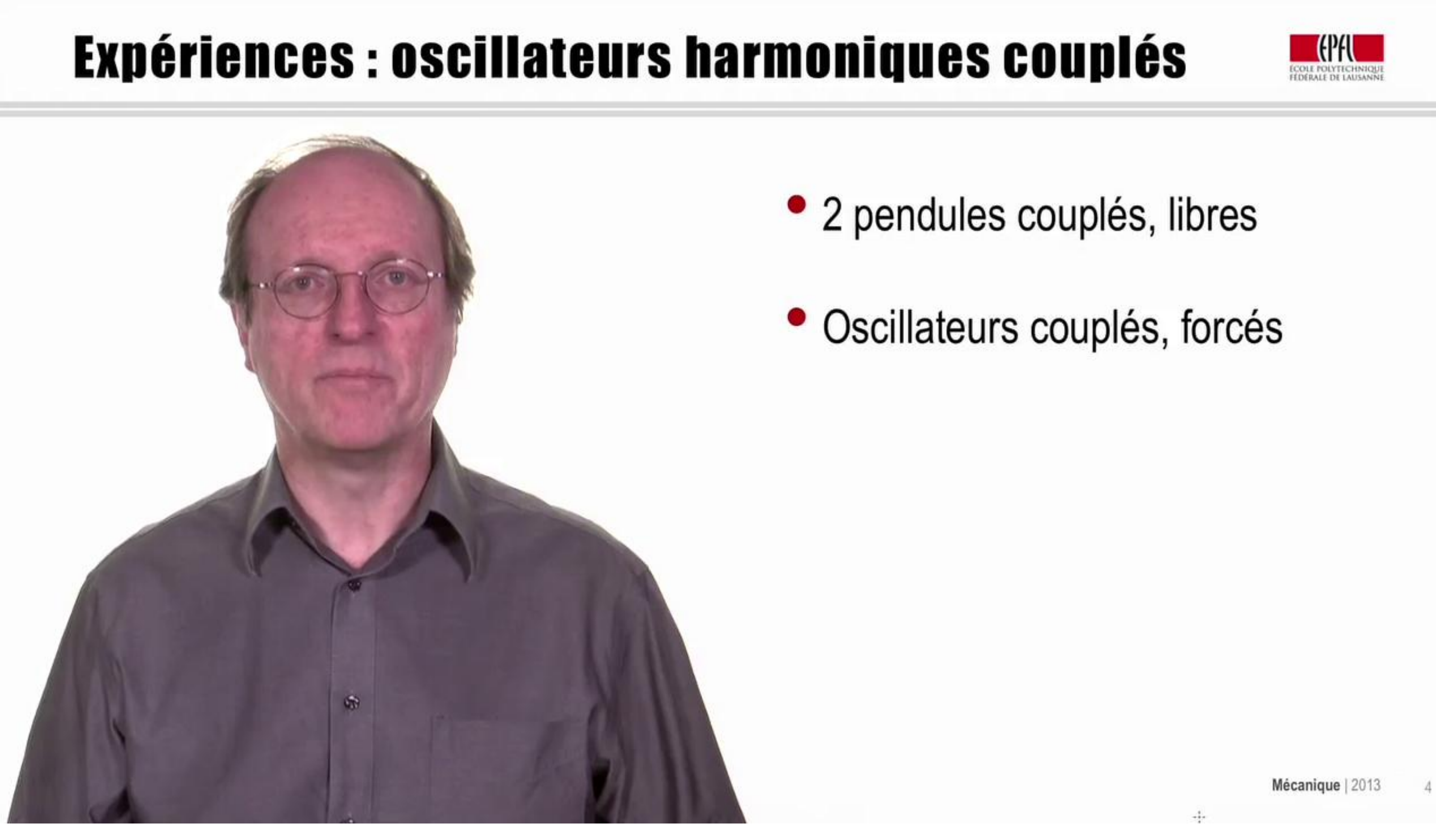


Video

EPFL

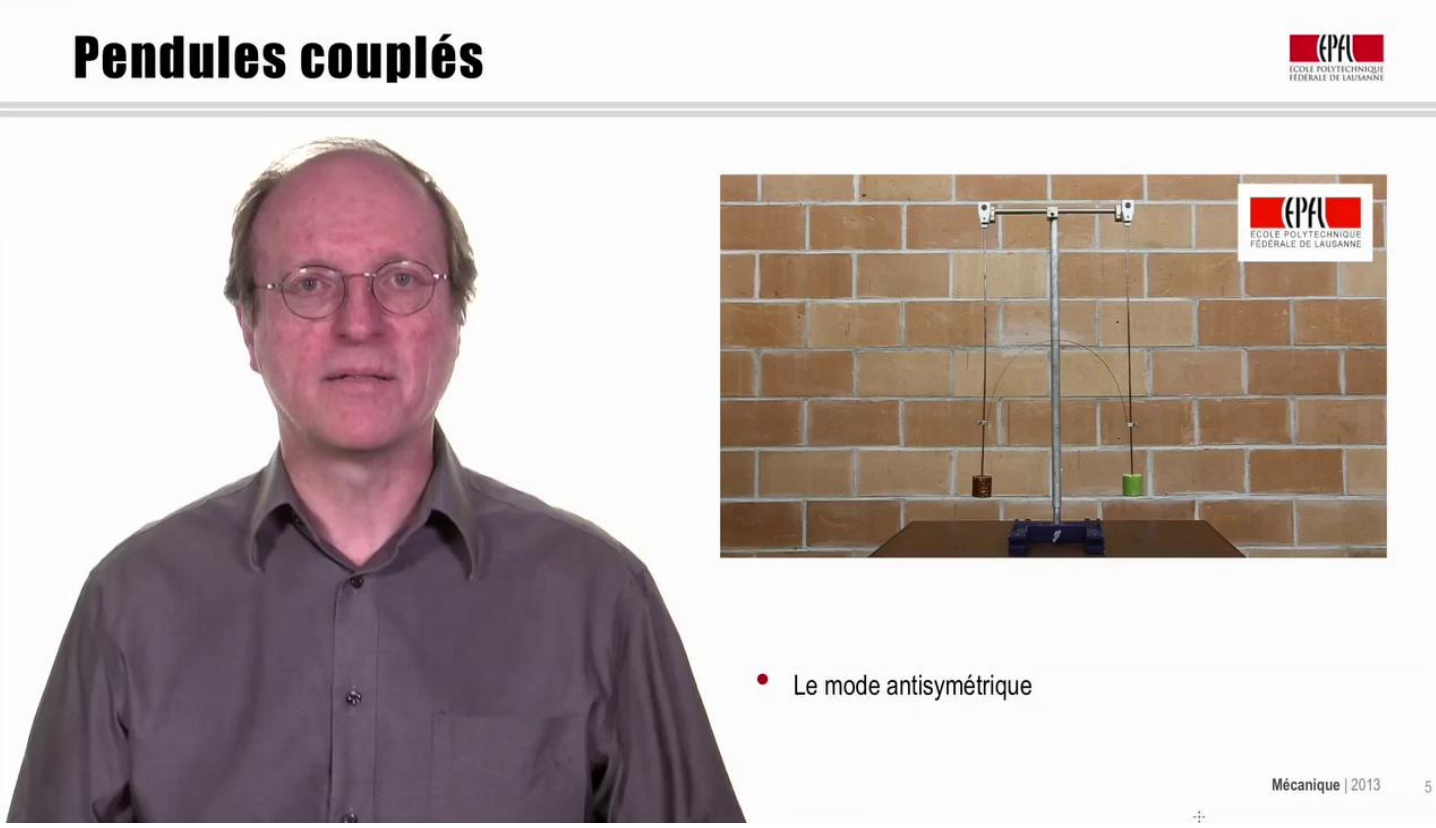


Welcome to the EPFL general physics course. In this lesson, we analyzed the dynamics of a system of coupled harmonic oscillators. Here, we will illustrate our results with two experiments: In the first, we will examine two coupled pendulums and observe and analyze the beating phenomenon. In the second, we will have two coupled harmonic oscillators. The system is hooked to a vibrating pot and we will look for the resonances of the system. We will observe that the resonances take place at the frequencies of the eigenmodes of this system.

Notes

Summary

0m 03s

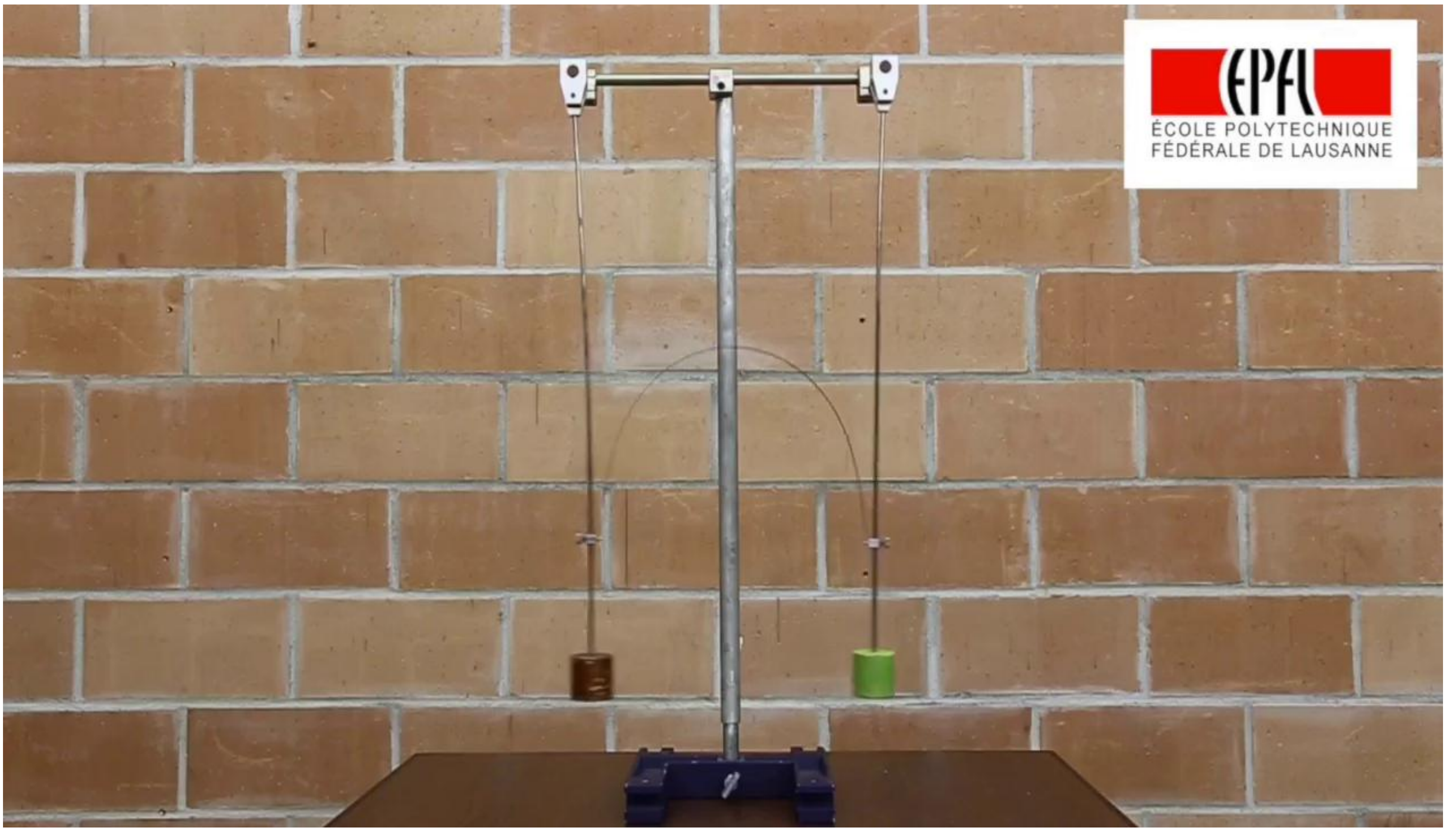


I start with a system made of two identical pendulums connected by an elastic blade.

Notes

Summary

0m 46s

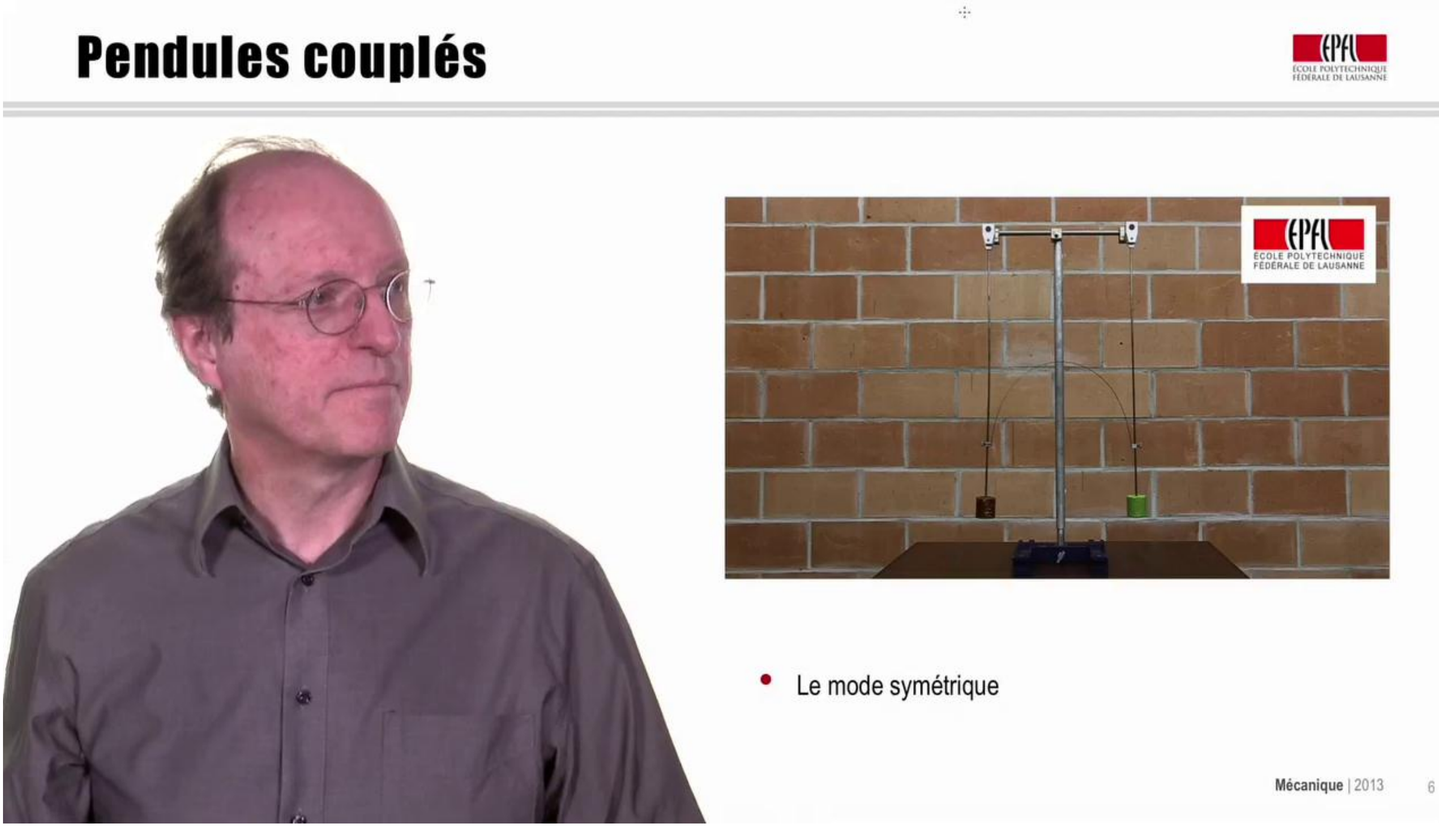


Let's first observe the antisymmetric mode. We observe that the two pendulums oscillate at the same frequency, they are simply in phase opposition.

Notes

Summary

0m 52s

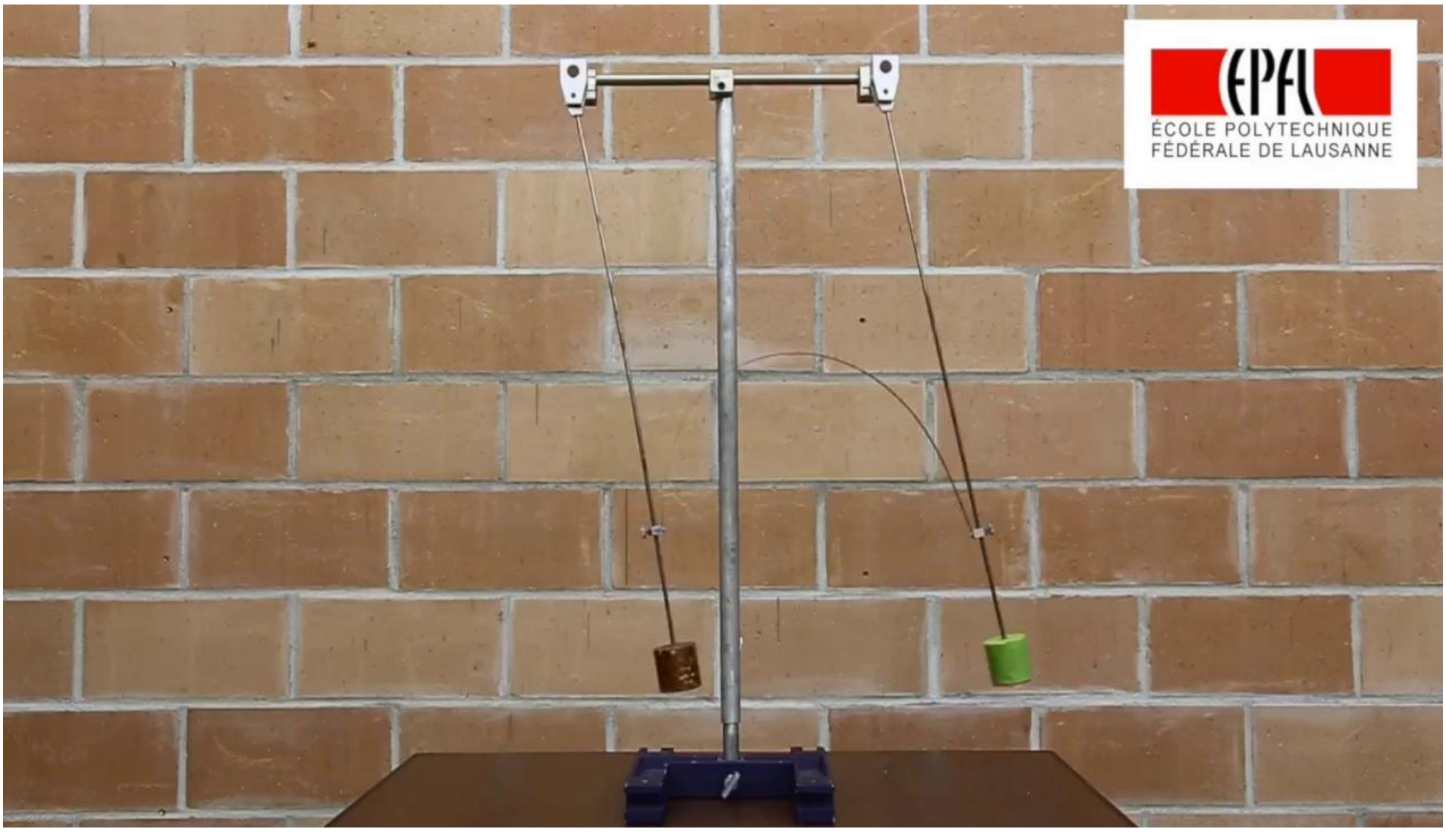


Let's now look at the symmetric mode.

Notes

Summary

1m 09s

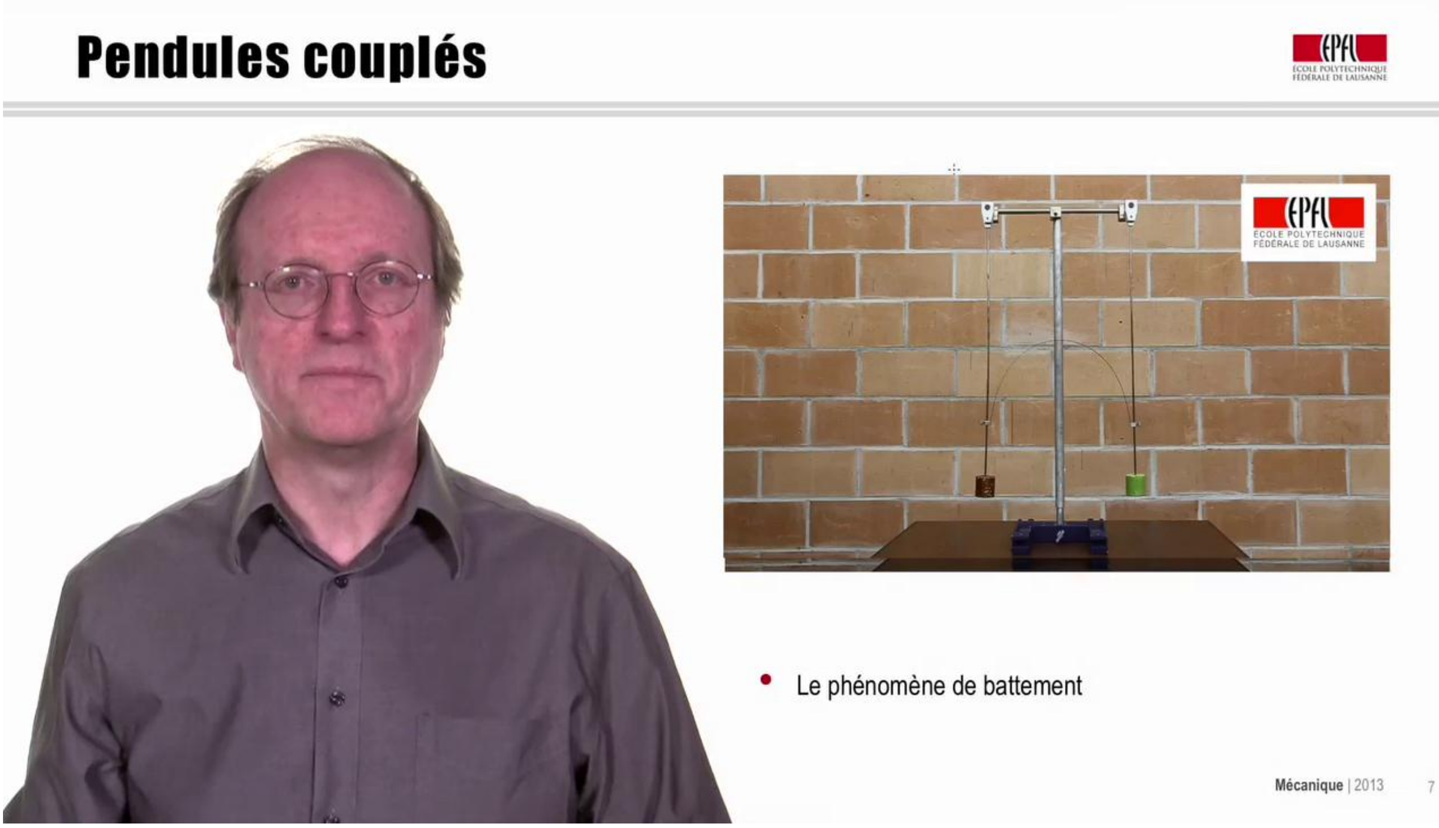


Note that the elastic blade hardly deforms since the pendulums have a small amplitude; the blade only translates.

Notes

Summary

1m 21s

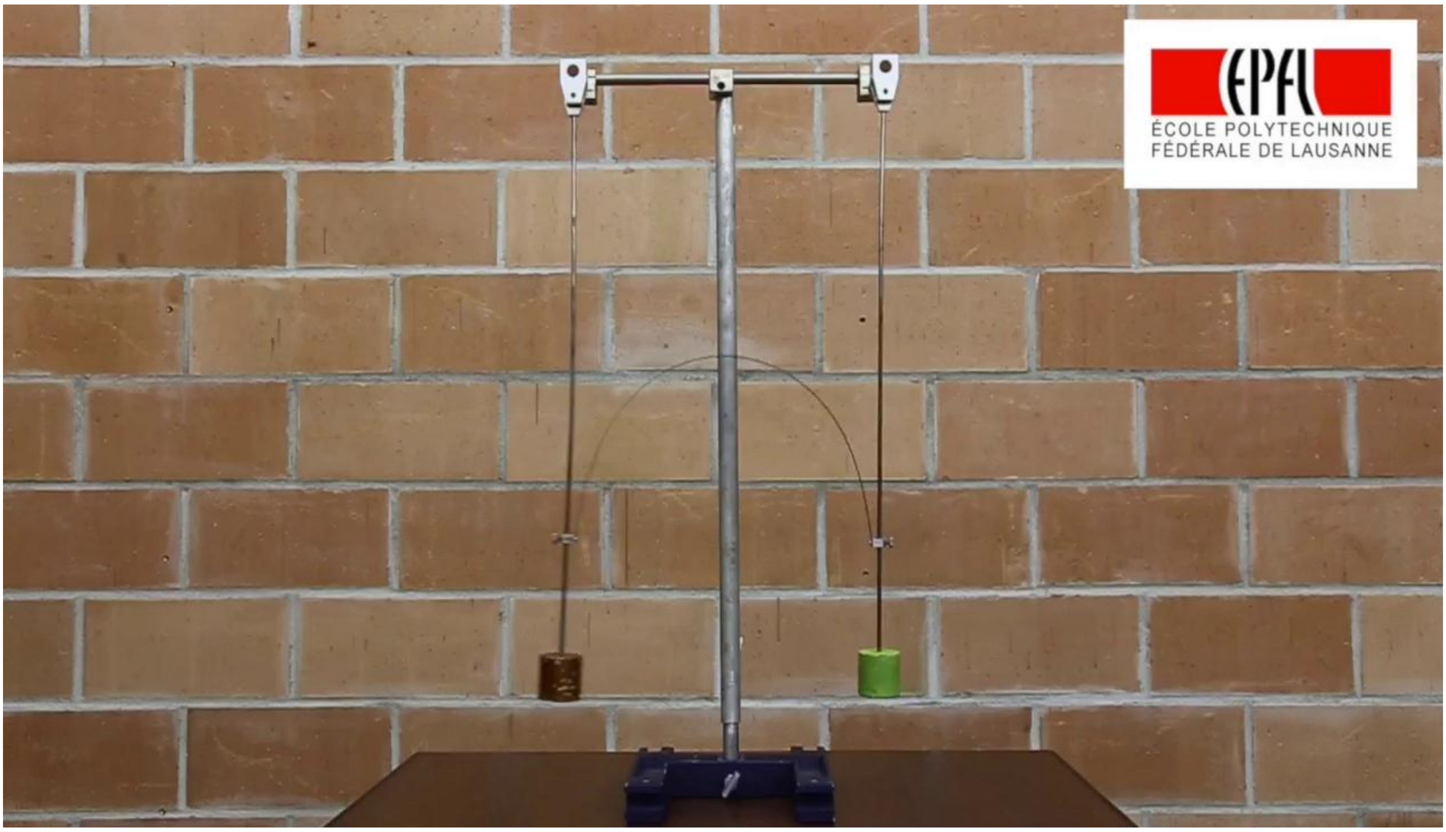


Let's observe what happens now if we launch the system by moving a single pendulum from its equilibrium position.

Notes

Summary

1m 33s



You will see that, little by little, the left-hand pendulum stops the right-hand pendulum has the maximum amplitude. And if you wait a bit more, the right pendulum stops. So now we have the maximum amplitude on the right. The left pad starts to oscillate again And the amplitude on the right decreases.

Notes

Summary

1m 52s

Pendules couplés : battements

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 \cos(\omega_1 t + \phi_1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \cos(\omega_2 t + \phi_2) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Conditions initiales : $x_1 = A,\ x_2 = 0$

$\dot{x}_1 = 0,\ \dot{x}_2 = 0$

$$x_1 = \frac{1}{2} A(\cos\omega_1 t + \cos\omega_2 t)$$
$$x_2 = \frac{1}{2} A(\cos\omega_1 t - \cos\omega_2 t)$$

$$x_1 = A\cos\left(\frac{\omega_1 + \omega_2}{2} t\right) \cos\left(\frac{\omega_1 - \omega_2}{2} t\right)$$
$$x_2 = A\sin\left(\frac{\omega_1 + \omega_2}{2} t\right) \sin\left(\frac{\omega_1 - \omega_2}{2} t\right)$$

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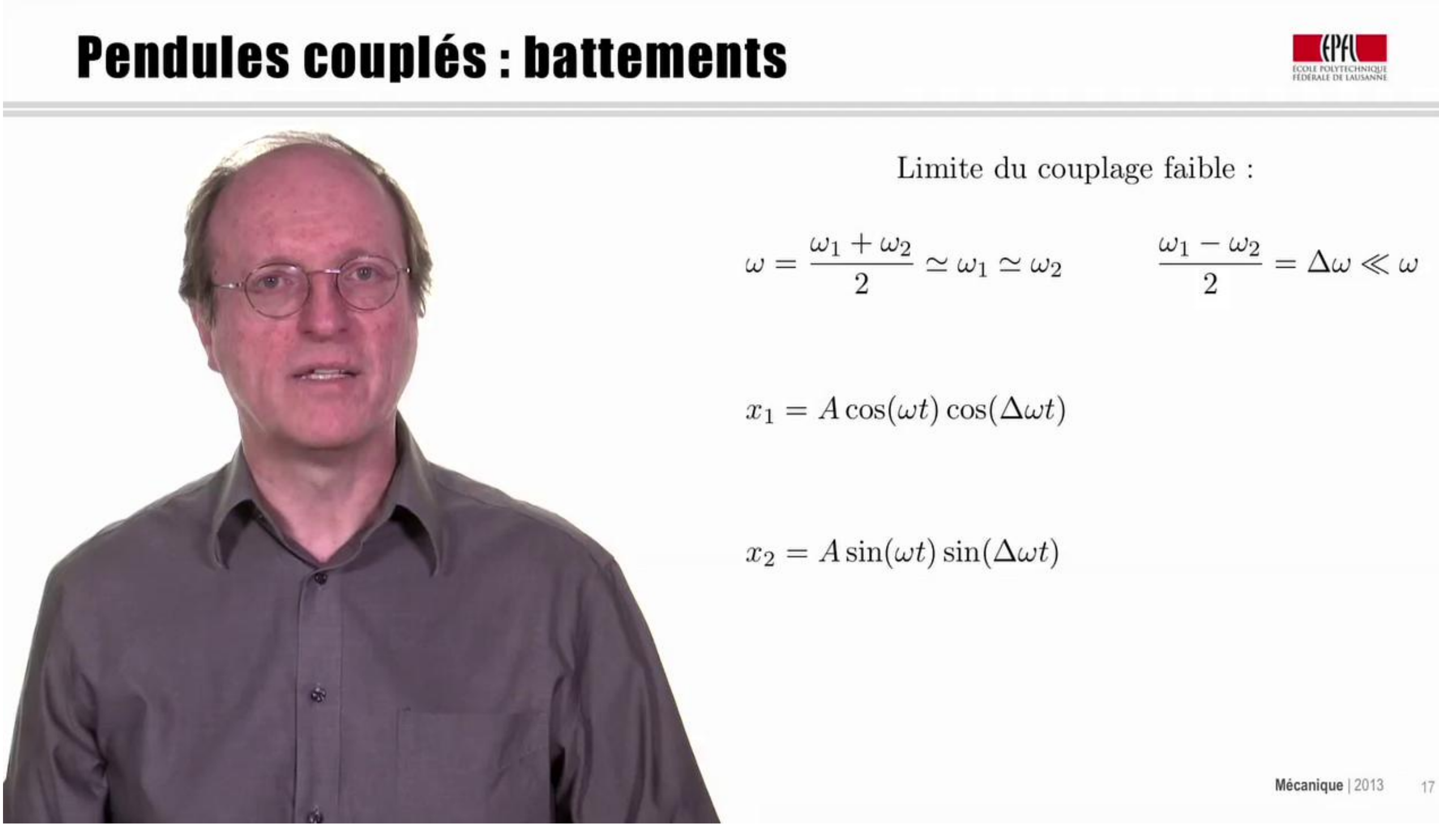
Let's see how to account for the flapping phenomenon using our general results. First, here are the time equations for the two pendulums in their general expression. So what have we done? We moved one stud away from its equilibrium position. So we have an initial condition that we will express as follows: we'll say that at time $t=0$, one stud has been moved away by an amplitude A the other has remained stationary. We also have a condition on the speeds : we have both initial velocities zero. If we put these initial conditions into our time equations, we get the following particular time equations; here they are. As it often happens in physics, we have a mathematical expression which is correct but does not immediately show what we observed. Some algebraic manipulations are needed. Here, I propose to transform these cosine sums into products of trigonometric functions using standard formulas. We get the following result, where now appears the average pulse and the difference of the pulses. If you want, the average frequency and the difference of the frequencies.

Notes

Summary

2m 28s



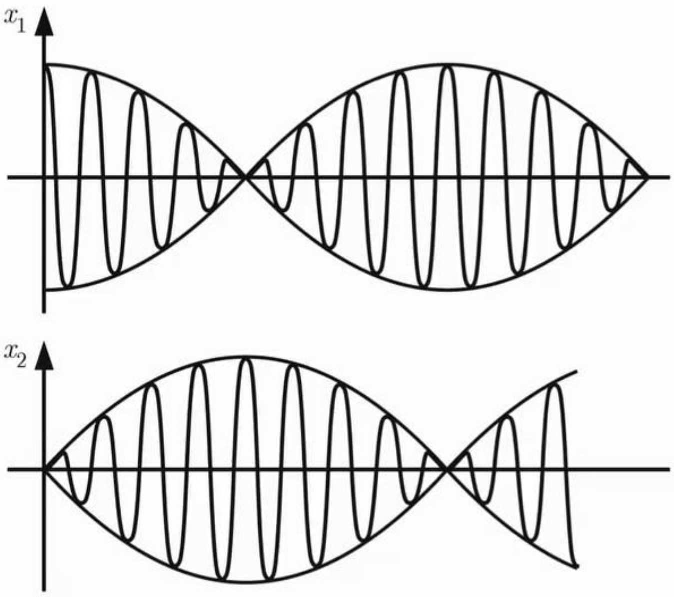
We still have to express the fact that the coupling between the two pendulums is weak. I propose to do it in the following way: we will note omega, the average pulsation and delta omega, the half of the difference of the pulsations of the two eigenmodes of the system. By doing so, the time equations take the following shape and we now see functions that oscillate at the omega pulsation, whose amplitude is modulated at the delta omega pulsation which is much smaller than omega.

Notes

Summary

4m 08s

Pendules couplés : battements



Limite du couplage faible :

$$\omega = \frac{\omega_1 + \omega_2}{2} \simeq \omega_1 \simeq \omega_2 \qquad \frac{\omega_1 - \omega_2}{2} = \Delta\omega \ll \omega$$

$$x_1 = A\cos(\omega t)\cos(\Delta\omega t)$$

$$x_2 = A\sin(\omega t)\sin(\Delta\omega t)$$

This gives the following hourly equations where I have drawn this oscillating function and the envelope of these functions. As we have observed, we have a maximum for one plot when we have a minimum for the other.

Notes

Summary

4m 46s