Expérience : minimisation

Mécanique, cours 26.exp

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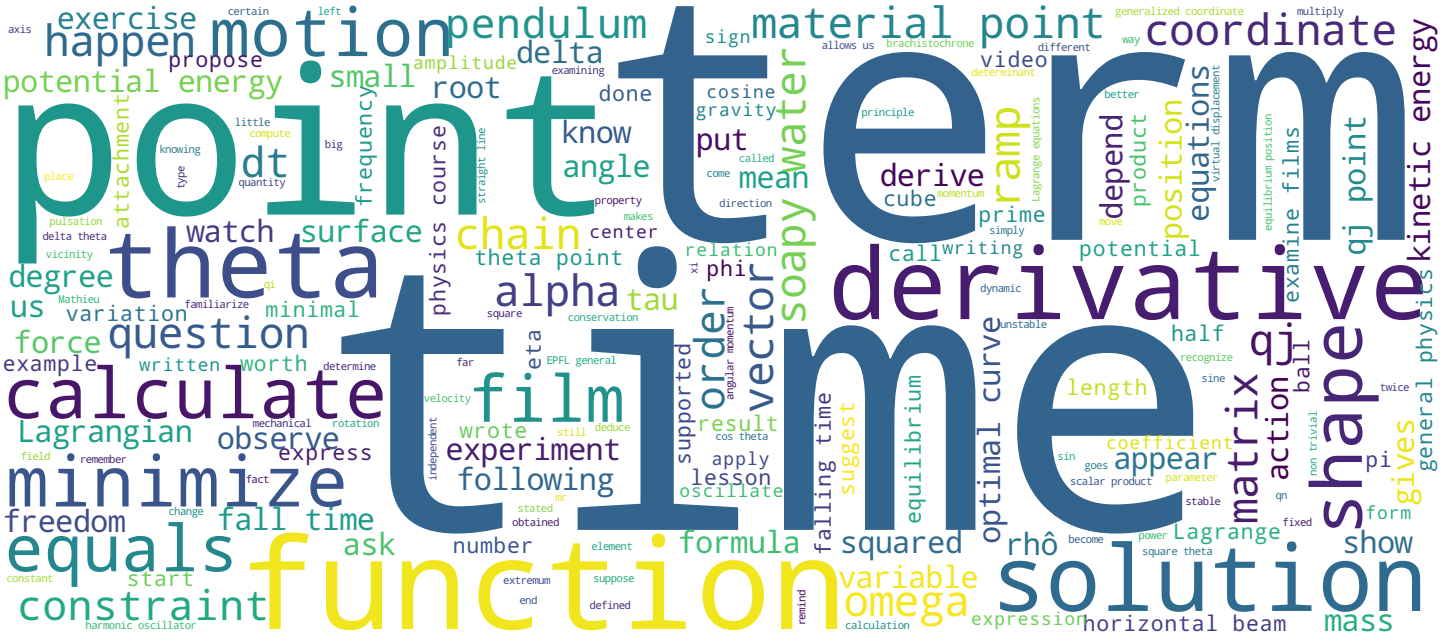


Expériences : minimisation

- Brachistochrone
- Chainette
- Film d'eau savonneuse

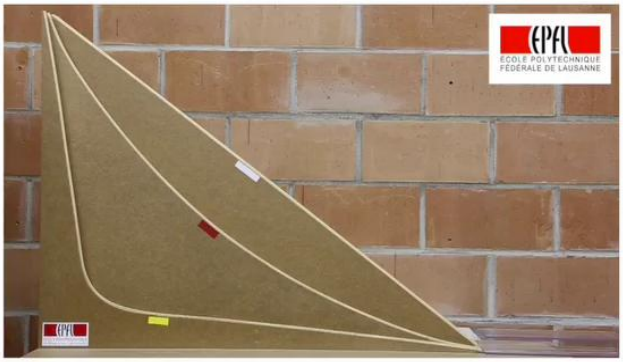
Welcome to the EPFL general physics course. In this lesson, we stated the following principle: the motion of any mechanical system is that which minimizes the action. This is a variational principle. In this experimental module, I propose to familiarize you with this new mathematical notion by examining some experiments. The first one is the "brachistochrone. " It is a question of knowing what is the shape of a ramp on which a material point descends that minimizes the fall time. The second is the question of what is the shape of a "chain," suspended in the field of gravity. Finally, we will examine "films of soapy water," in this case it is the surface of the film that is minimal.

Notes

Summary

0m 04s

Brachistochrone



- Rampes de formes différentes. Sur laquelle le temps de chute est-il minimal ?

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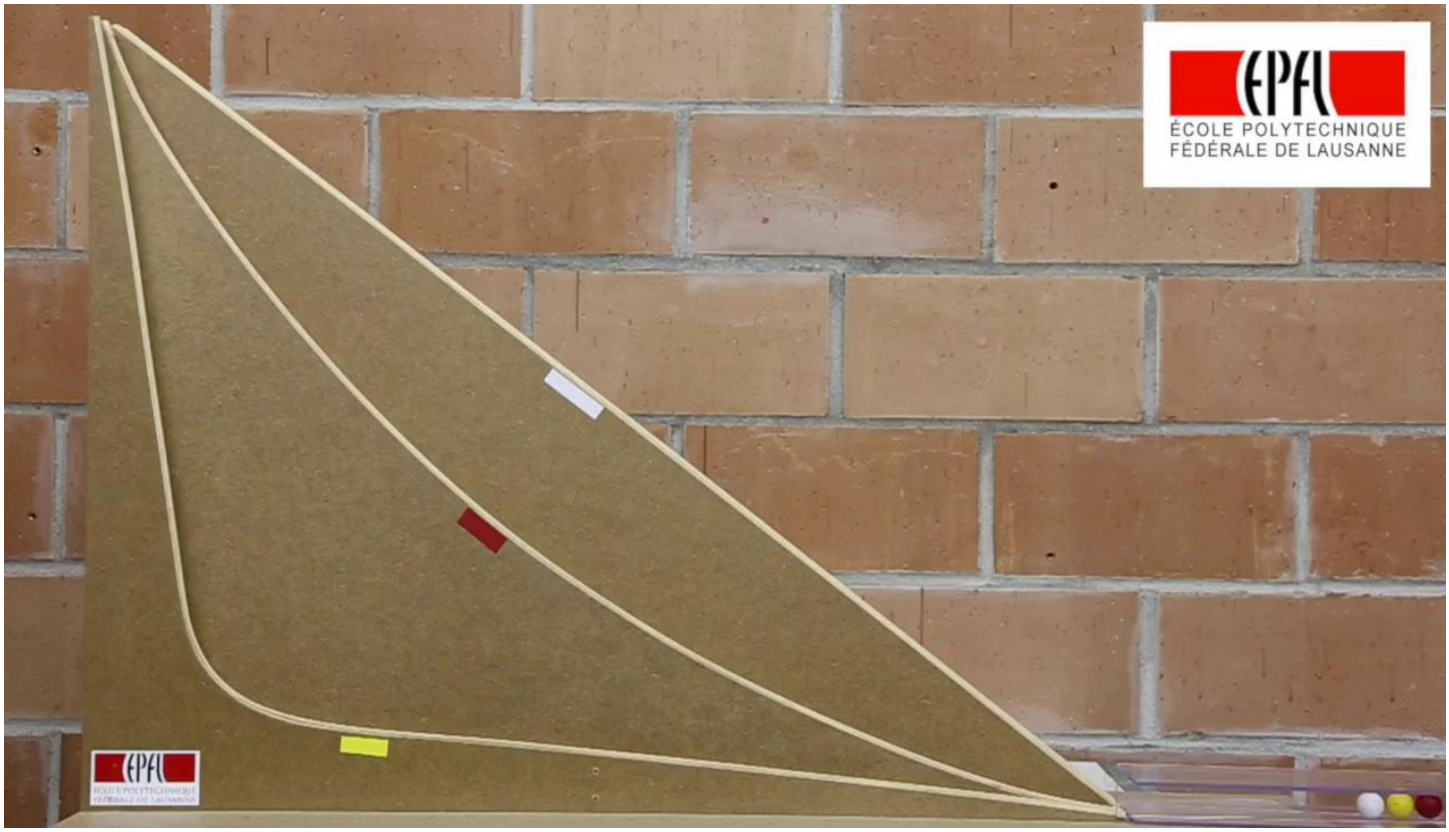
Here is an experiment which is very similar to what we have done so far, but here the question is very different; we would like to know what is the shape of the ramp which minimizes the falling time of the balls which we are going to make go down the ramps.

Notes

Summary

1m 03s



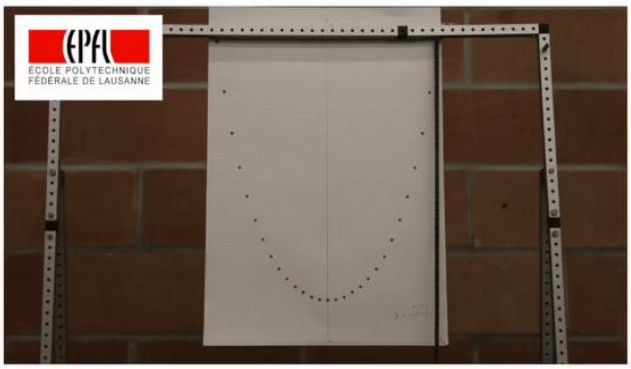
I suggest you watch the video: and now we will see it in slow motion: a small device allows us to see which marble arrived first. We considered several ramps and we see that the straight line does not minimize the fall time. The ramp with a very large initial slope is better, but it is the intermediate curve in our experiment that gives the shortest time. It is a cycloid, it is the mathematical solution of the problem. One can imagine constructing curves in the vicinity of the optimal curve and observe how much the falling time changes.

Notes

Summary

1m 24s

Chainette



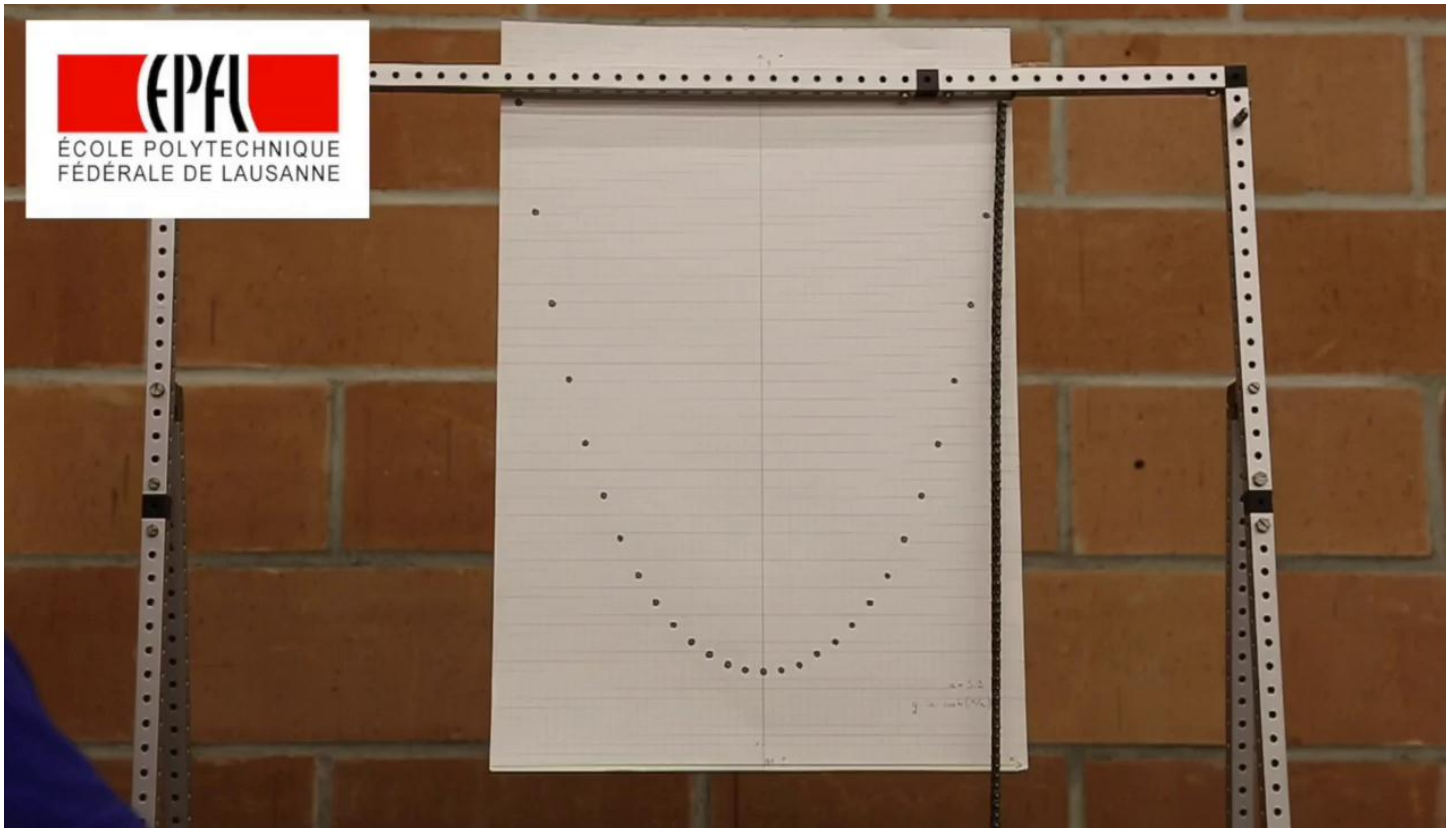
- Chainette dans un plan vertical
- Forme prévue : cosh

By analogy, in order to find this curve analogically one has to consider infinitesimal variations from the optimal curve. And this optimal curve is obtained by the following condition: the variations of the fall time must be zero at first order. Let us now consider the question of the shape of a chain hanging from a horizontal beam.

Notes

Summary

2m 08s

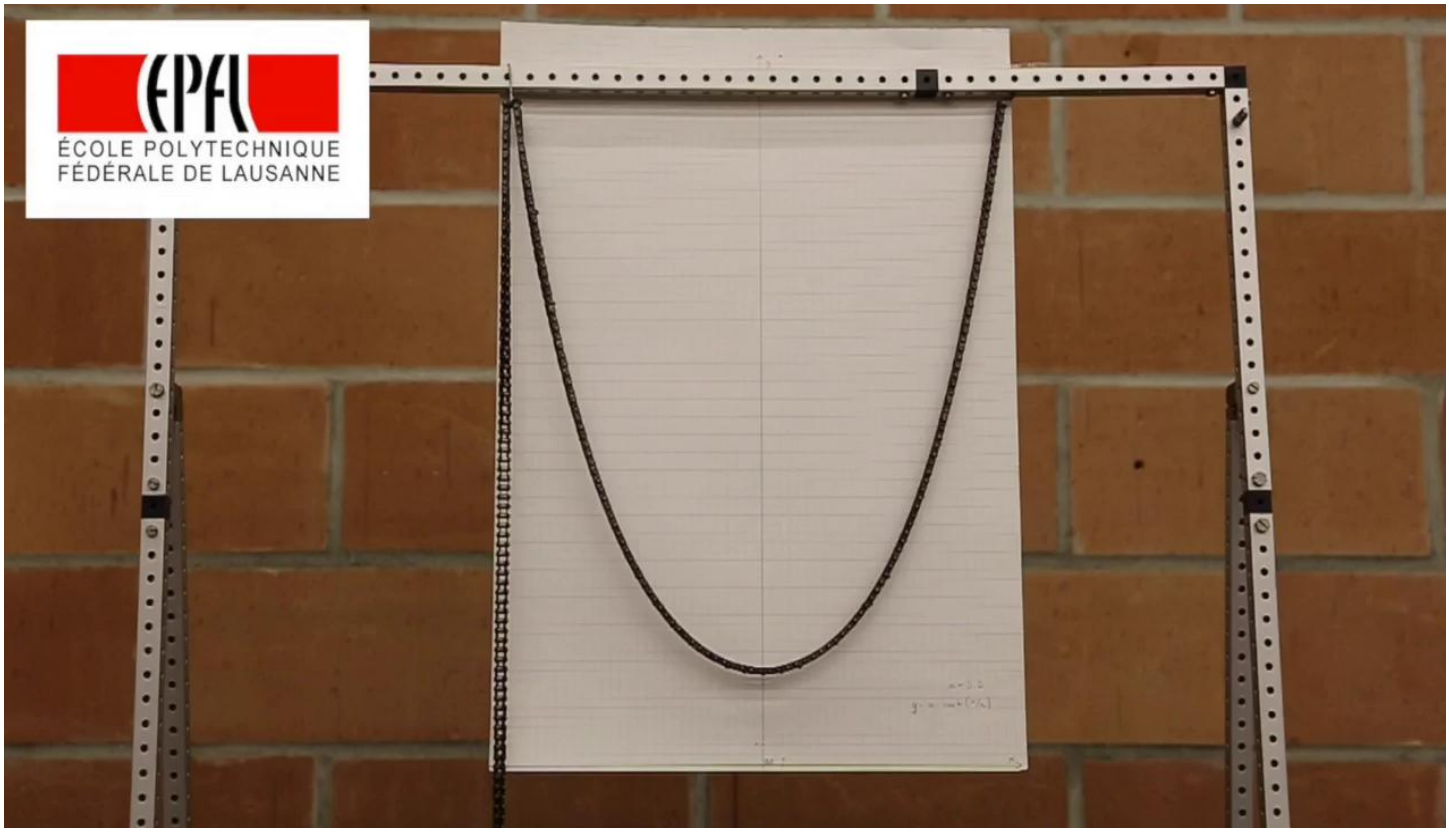


As an exercise, I ask you to determine this shape as the one that minimizes the potential energy of the chain. Here, I give you the solution: it is a hyperbolic cosine. The preparator marked some points of this function on a sheet of paper that he hung on the horizontal beam.

Notes

Summary

2m 37s



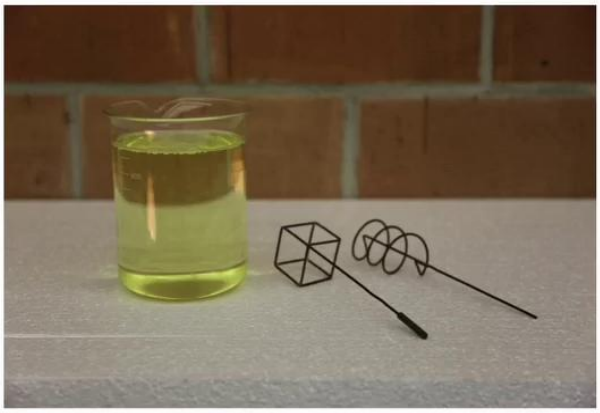
Let's watch the video: We see that the points marked on the sheet, fall quite exactly at the place of the chain. In the exercise, I ask you to deduce this shape as being the one which minimizes the potential energy of the chain, the two points of attachment, and the length of this chain being fixed.

Notes

Summary

2m 58s

Bulle de savon



- Eau savonneuse
- Support de formes variées

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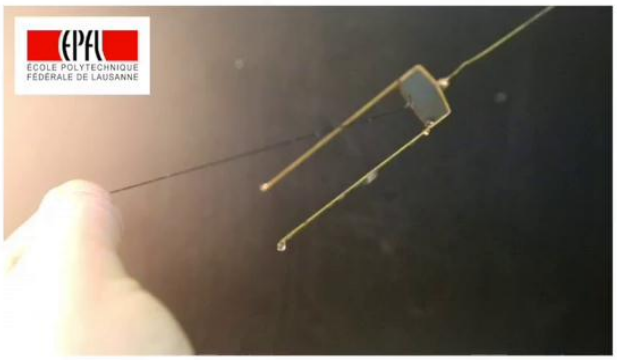
Finally, let's examine films of soapy water.

Notes

Summary

3m 25s

Bulle de savon



- Cadre rectangulaire
- Le côté libre laisse le film se rétracter complètement

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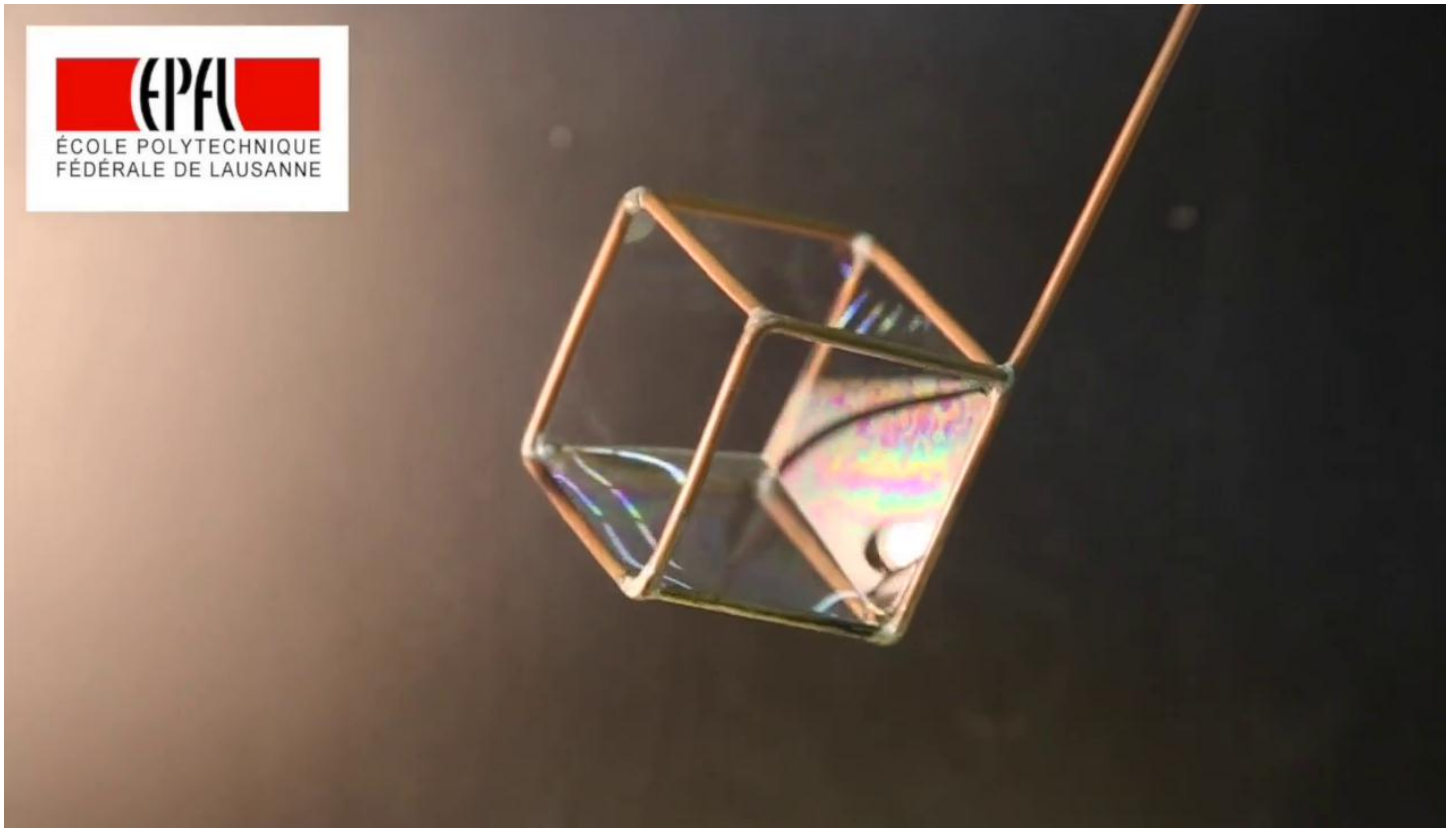
A film of soapy water will arrange itself so that its surface is minimal. To show this, I suggest you look at what happens when you stretch a film of soapy water over a rectangle with one side left free: The film tends to shrink completely.

Notes

Summary

3m 29s



Let's now look at what happens if the film is supported on the edges of a cube: We observe a non-trivial structure in the center of the cube.

Notes

Summary

3m 54s

Bulle de savon

- Cube
- Structure particulière au centre du cube

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Finally, let's look at what happens if the film is supported by a spiral:

Notes

Summary

4m 26s