ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

RÉGIMES TRANSITOIRES

CIRCUIT RLC SÉRIE - SOLUTIONS

LEÇON 14

Électrotechnique II

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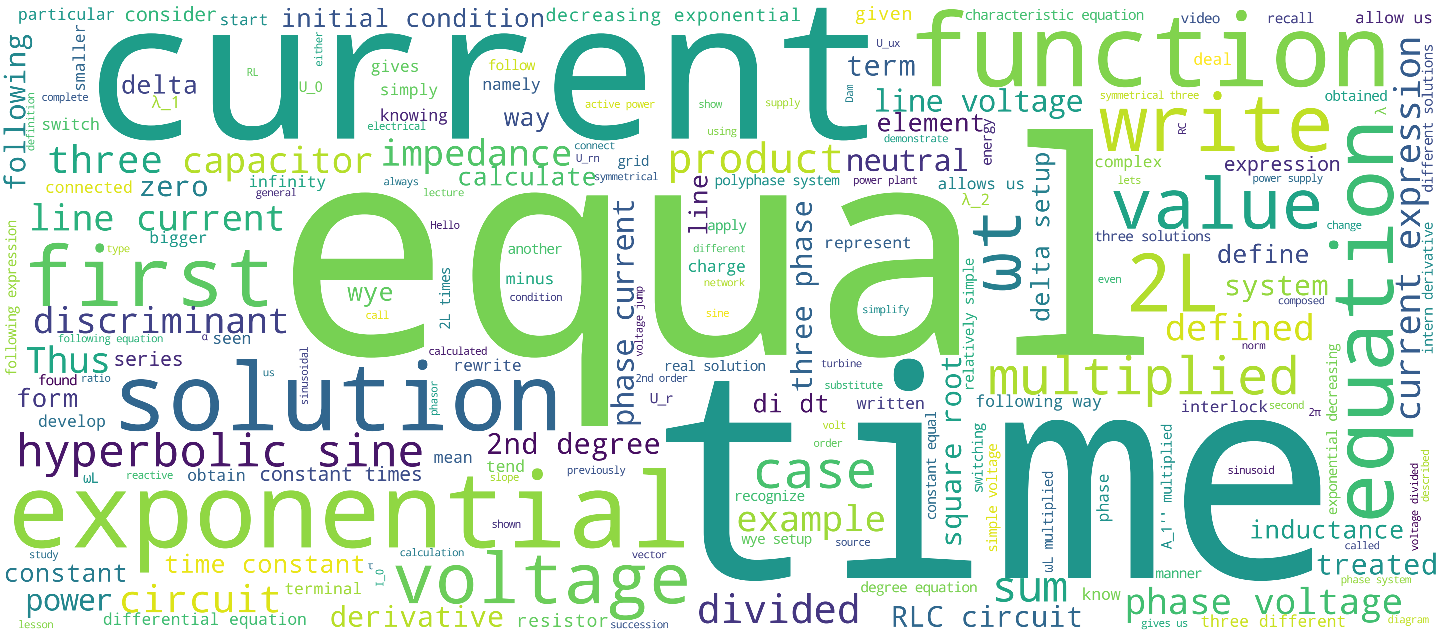
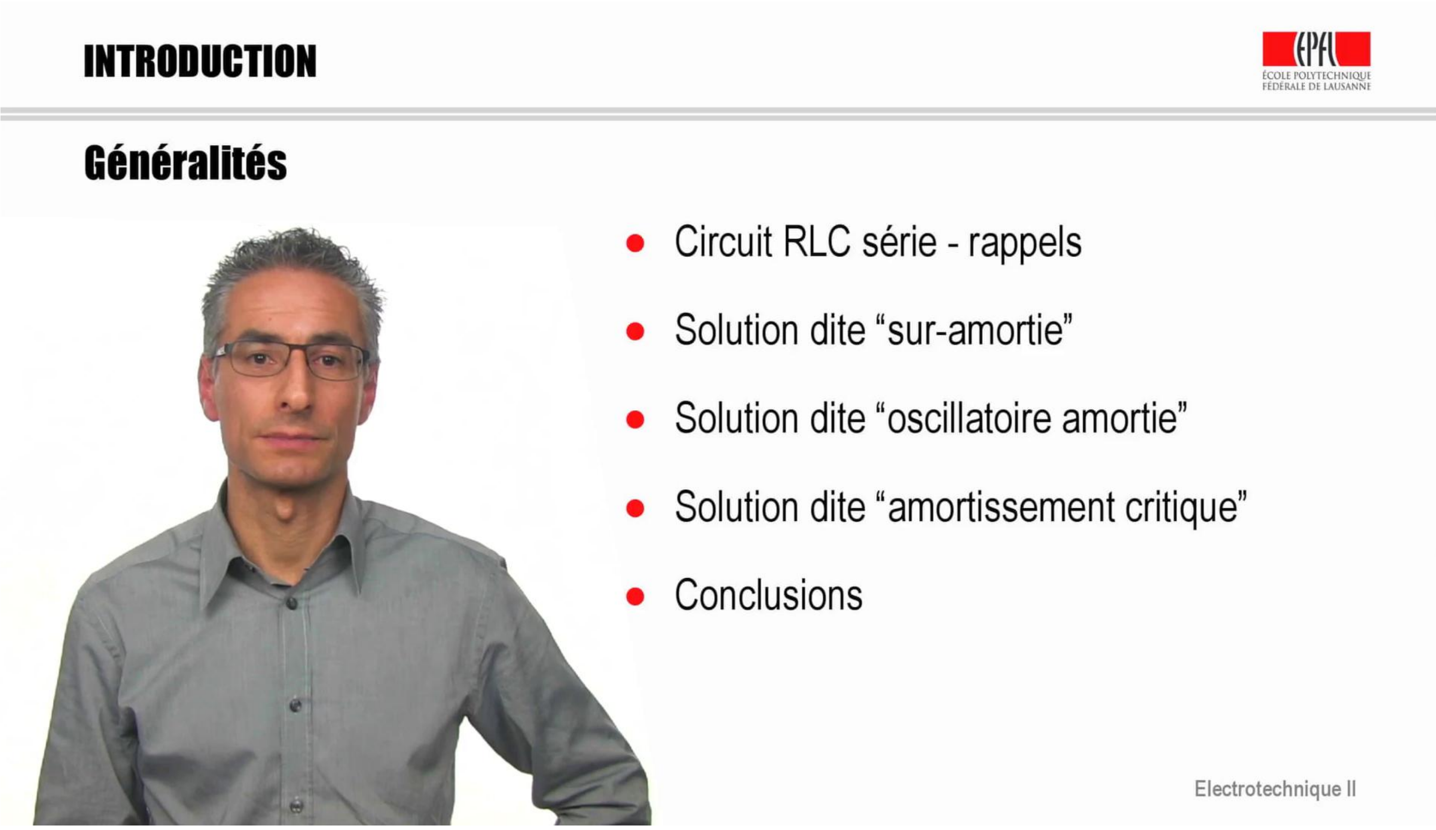


Video

EPFL



Hello, during the last lecture we have treated the most complete case of an RLC circuit in series. We have defined the equations that would allow us to find the solutions for current expression. According to the discriminant sign, the solutions of the characteristic equation, of the 2nd degree are three different solutions During this lecture, we will develop these three solutions We will start with a recall of the essentiel results that we got at the previous lecture and then, we will develop the three solutions for the current expression that are solutions, called, "Over-damped" "damped oscillatory" and "critical damping".

Notes

Summary

0m 04s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RLC SÉRIE

- **Rappel de la solution :**

$$i(t) = A_1 \left(e^{\lambda_1 t} - e^{\lambda_2 t} \right) \qquad \lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = -\frac{R}{2L} \pm \omega$$

$$i(t) = A_1 \, e^{-\frac{R}{2L} t} \cdot \left(e^{\omega t} - e^{-\omega t} \right)$$

- **Rappel des conditions initiales en $t = 0$: $i(0) = 0$ et $u_c(0) = 0$**

Pour $t = 0$: $U = L \cdot \frac{di}{dt}$

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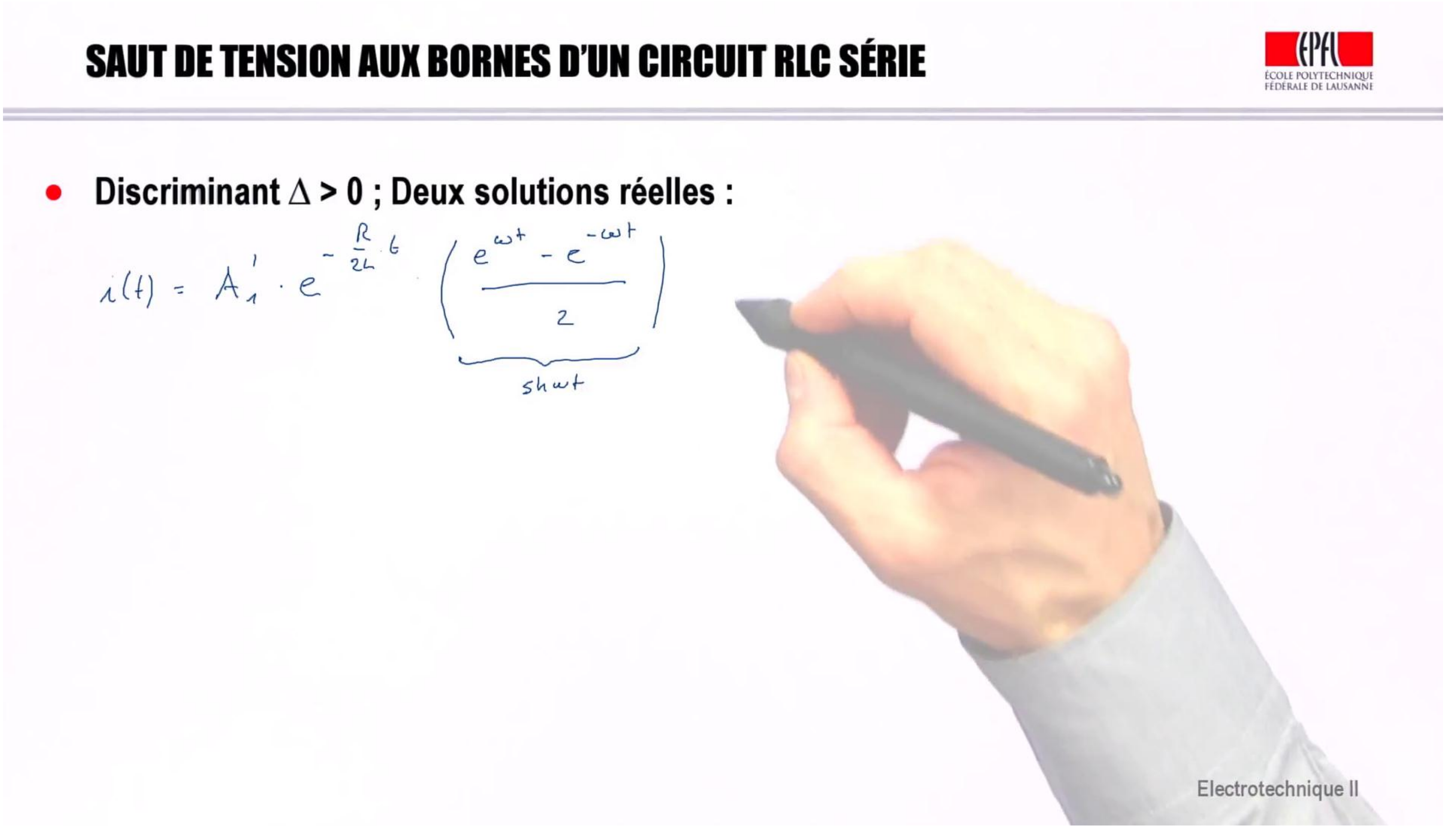
The current expressions that we have obtained were under the form i(t) is equal to a constant times a sum of exponentials. Where λ_1 and λ_2 are the solutions of the characteristic of 2nd degree that we can write, for simplifications -R/2L ± ω. The expression of λ_1 and λ_2 being a sum of two terms that will be here, in the exponential. And knowing that the exponential of a sum is the product of the exponential, we can rewrite the equation of the current, in the following way i(t) equal to the constant times the exponential of (-R/2L) * t times the sum of the two exponentials. We recall again that the initial conditions at time t = 0 were the current at time t = 0 is equal to 0 and the voltage at the capacitor's terminals, at time t = 0, is equal to 0, and the capacitor is initially empty. For t=0, we had U = L * (di/dt).

Notes

Summary

0m 46s



If we consider the first case where the discriminant is bigger than 0 we get two real solutions And in that case, the current equation can be written, slightly modified, as i(t) which is equal to a constant times a first exponential times the sum of the exponentials e^ωt - e^(-ωt) divided by two, and we recognize this expression being the expression of a hyperbolic sine. We can therefore write this equation in a light way and we have that A' multiplied by e^((-R/2L) * t) multiplied by hyperbolic sine of ωt.

Notes

Summary

2m 23s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RLC SÉRIE

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- **Discriminant $\Delta > 0$; Deux solutions réelles :**

$$i(t) = A_1' \cdot e^{-\frac{R}{2L} t} \underbrace{\left(\frac{e^{\omega t} - e^{-\omega t}}{2} \right)}_{sh\,\omega t} = A_1' \cdot \underbrace{e^{-\frac{R}{2L} t}}_{f} \cdot \underbrace{sh\,\omega t}_{g} \qquad (f \cdot g)' = f'g + fg'$$

$$U = L \cdot \frac{di}{dt} = L \cdot A_1' \cdot \left[-\frac{R}{2L} \cdot \underbrace{e^{-\frac{R}{2L} t}}_{1} \cdot \underbrace{sh\,\omega t}_{0} + \omega \cdot \underbrace{e^{-\frac{R}{2L} t}}_{1} \cdot \underbrace{ch\,\omega t}_{1} \right]$$

En $t = 0$

$$A_1' = \frac{U}{\omega L}$$

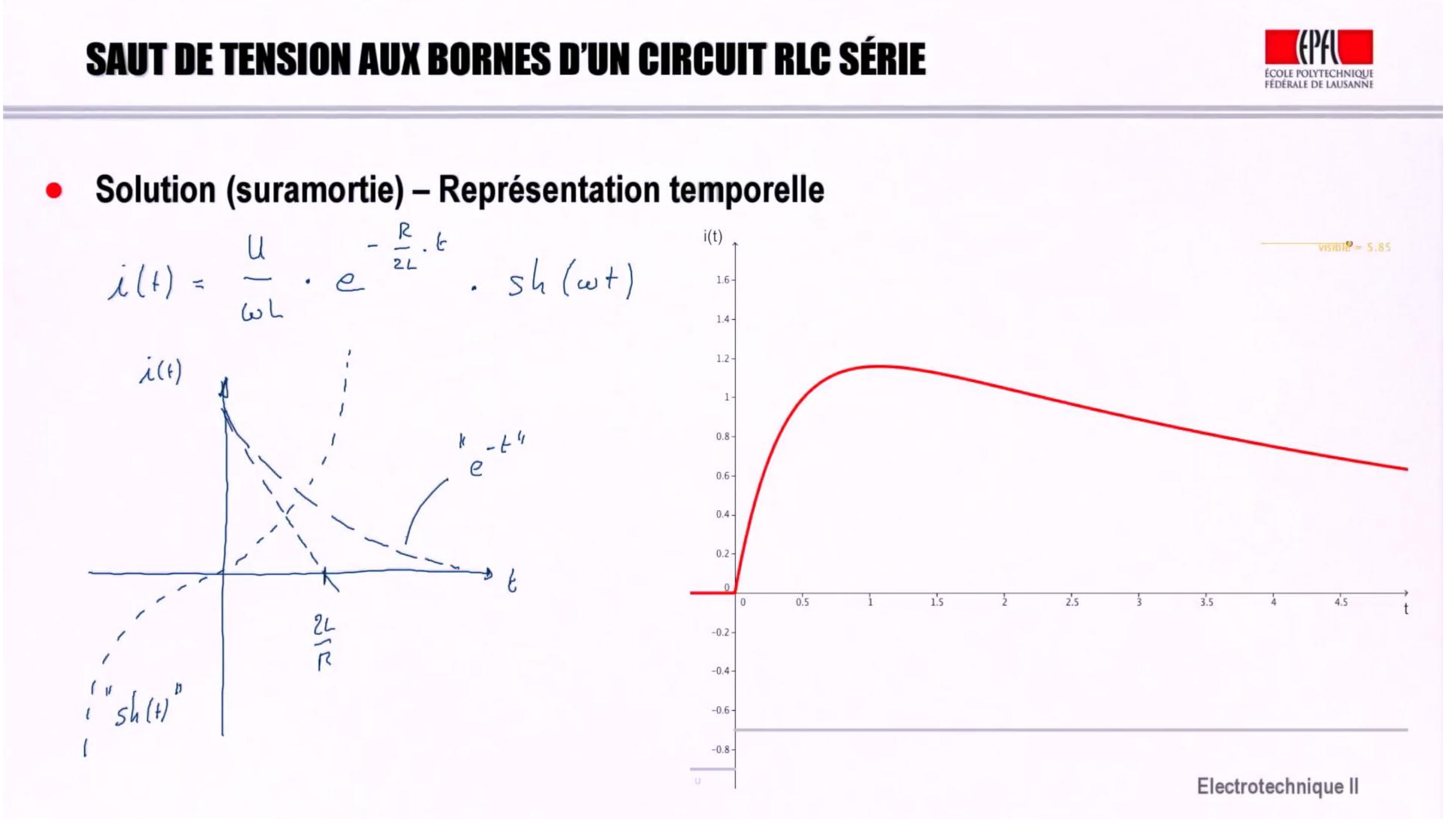
Electrotechnique II

The initial condition that we have described before and that gave us the following equation: U = L*(di/dt) allows us to rewrite with the derivative of the current, the following expression knowing that the current expression is composed of a product of two functions, a first function f and a second fonction g and knowing that the derivative of a product function is equal to the sum f ' * g + f * g' we get an expression a little bit heavier, but that is written as: L*A_1' times the derivative of the function f, it is the function itself times the intern derivative minus R divided by 2L times the exponential -R/2L * t times the function g meaning, the hyperbolic sine of ωt plus the function f exponential of -R/2L * t times the derivative of the function hyperbolic sine that is a hyperbolic cosine of ωt times the intern derivative if we apply this condition at time t = 0 this term turns 1 this term becomes 1 as well this one 0 and this one 1 We therefore get a relatively simple expression that allows us to say that A_1' = U/ωL.

Notes

Summary

3m 19s



We can finally write that the current expression as function of time being the product of a constant, the constant that we just computed U/ωL, times an exponential of exponent -R/2L * t times the hyperbolic sine of ωt. It is then, the product of two functions. If i represent them separately in a graphic, we first have a decreasing exponential two time constant 2L/R therefore, something as a decreasing exponential times a hyperbolic sine of ωt with that appearance. The hyperbolic sine as function of time can be complex in terms of mathematics, we can verify that the solution is still bounded when t tends to infinity. And the product of the two gives the function of the current. Thus, if we have an RLC circuit with R, L and C such as the discriminant under square root of the solution of the equation of second degree is bigger than 0 we then obtain a temporal evolution of the current, which is the product of these two functions that we will see in the video that will follow. We see first an increase of the current, and then a decrease.

Notes

Summary

5m 34s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RLC SÉRIE

- **Discriminant $\Delta < 0$; Deux solutions imaginaires :**

$$\omega = j \cdot \omega'$$

$$i(t) = A_1'' \cdot e^{-\frac{R}{2L} \cdot t} \cdot \underbrace{\left(\frac{e^{j\omega' t} - e^{-j\omega' t}}{2j} \right)}_{\sin \omega' t} = A_1' \cdot e^{-\frac{R}{2L} \cdot t} \cdot \sin \omega' t$$

En $t = 0$

$$A_1'' = \frac{U}{\omega' L}$$

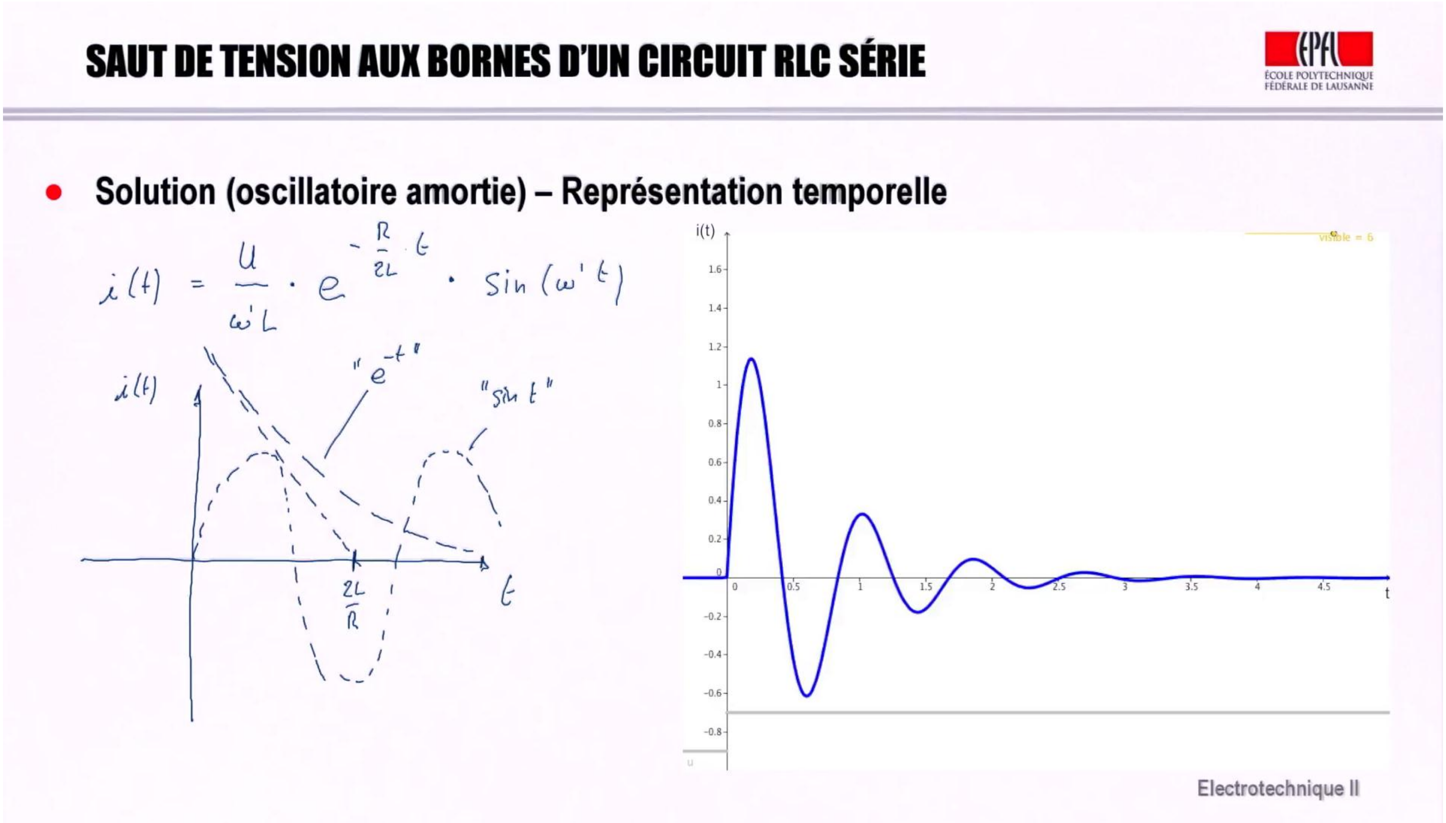
Electrotechnique II

In the case where the discriminate is smaller than 0 we get two imaginary solutions. If we write the following expression ω = j*ω' we can write the equation of the current which is transformed as follows. i(t) is equal to a second constant A_1" multiplied by the exponential of -R/2L times t multiplied by the sum of the two exponentials divided by 2j where we recognize here, the expression of a sine of ω't. Thus, we can write this equation in a lighter manner, such as A_1" multiplied by the exponential of -R/2L* t multiplied by a sinus ω't. Once again, at time t=0 the same initial conditions as the previous case allow us to obtain the integration contant such as A_1" is equal to U divided by ω'L.

Notes

Summary

7m 17s



Therefore, once again, we can write the temporal expression of the current as the product of the constant that we have just determined ω'L multiplied by an exponential function where the exponent is equal to -R/2L * t and, this time, multiplied by a sinusoidal function sin(ω't). If we represent these two functions separately in a temporal graphic we have first of all a sinusoid of a certain amplitude multiplied by an exponential decreasing function, where this is of type sinusoidal this of type exponential decreasing exponent with its time constant equal to 2L/R and, again, if we deal with this RLC circuit where R,L and C are defined such as the discriminant under the square root of the solution of the 2nd degree equation is smaller than 0 then, the current in the circuit evolves in the following way, as shown in the video. We have a sinus that the amplitude decreases exponentially.

Notes

Summary

8m 49s

- **Discriminant $\Delta = 0$; Une solution réelle :**

$$i(t) = \frac{U}{\omega L} \cdot e^{-\frac{R}{2L} \cdot t} \cdot sh\,\omega t$$

pour $\omega t \to 0$: $sh\,\omega t \to \omega t$

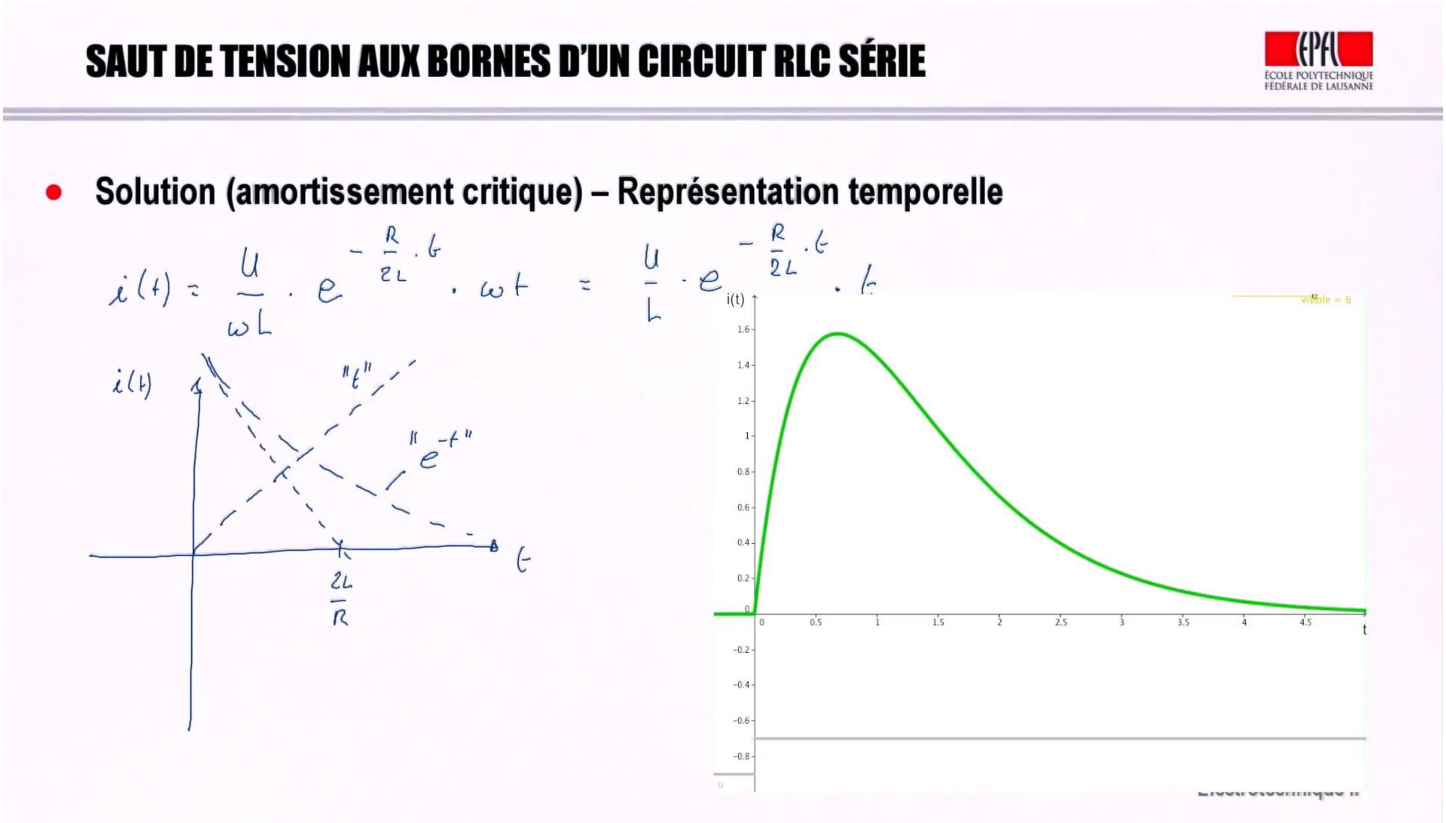
Electrotechnique II

The last case, is the case where the discriminant under the square root is equal to 0. In that case, we have only one real solution for ω. In this critical case, the solution is obtained from either solution. We will consider the 1st one that we found. i(t) is equal to U over ωL, multiplied by the exponential multiplied by the hyperbolic sine of ωt. And when moving to the limit, meaning ω tends to 0, we can substitute the hyperbolic sine function with the first term of the taylor series that we won't demonstrate here, but for ωt that tends to zero we have that the hyperbolic sine of ωt tends to ωt.

Notes

Summary

10m 21s

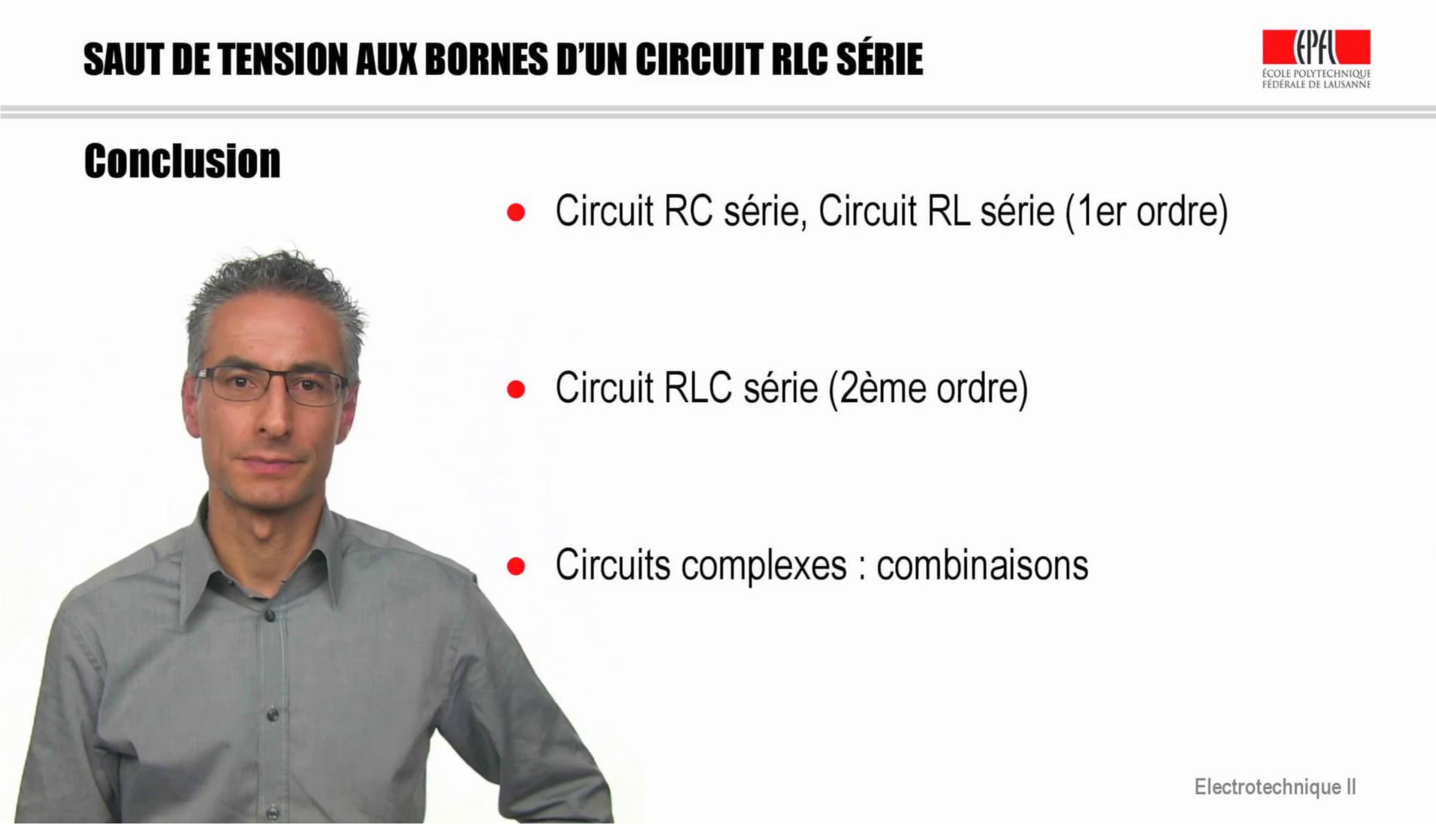


The solution of critical dumping could be written under the form i(t) which is equal to the constant U/ωL multiplied by the exponential of -R/2L * t multiplied by ωt. And this, we can simplify it into U/L multiplied by the exponential of -R/2L * t times t. They are two temporal functions where one is a decreasing exponential again with a time constant equal to 2L/R multiplied by (this is the exponential with a negative exponent) multiplied by a function t which is a simple straight. Once again, if we deal with an RLC circuit in series where R,L and C are defined such as the discriminant under the square root of the solution of the 2nd degree equation is equal to 0, then we get the following current evolution. It looks similar to the first solution, but of different nature.

Notes

Summary

11m 26s



Here we are, we have treated the cases RC and RL, where the differential equation of 1st order, leads to exponential solutions. We have treated the RLC case, the most complete, that gives de differential equations of 2nd order, and therefore three different solutions. For the most complex circuit, you should know that it is the combinations of these cases, and are therefore treated in the same way.

Notes

Summary

12m 51s