ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

RÉGIMES TRANSITOIRES

CIRCUIT RLC SÉRIE – MISE EN ÉQUATIONS

LEÇON 13

Électrotechnique II
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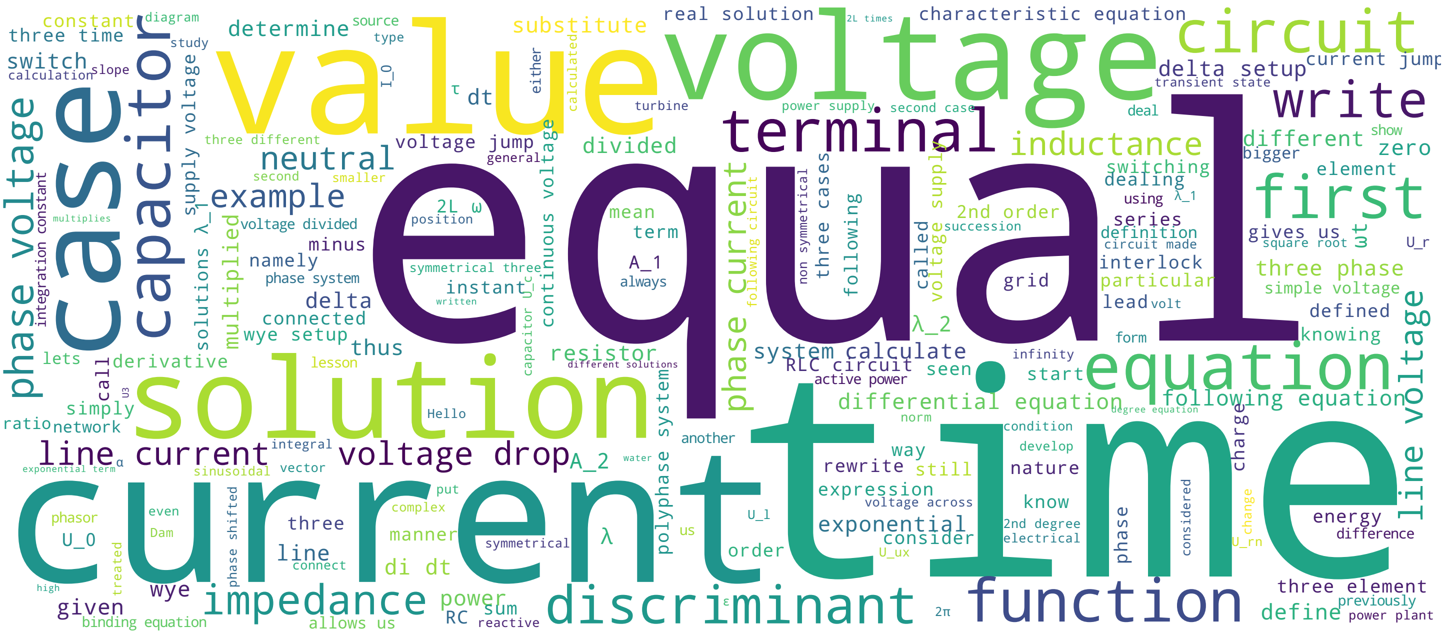

Video

EPFL

Hello, in the previous lectures, we have treated the case of RC or RL circuits that lead to some differential equations of the 1st order, and then to homogenous solutions of exponential type. During today's lecture we will see the case of a voltage jump, continuous voltage, in an RLC circuit in series. For the equation layout of the circuit, the methodology stays the same, but the solutions that we will find will be very different because the differential equation resulting from the circuit proposed, is of the 2nd order.

Notes

Summary

0m 04s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RLC SÉRIE

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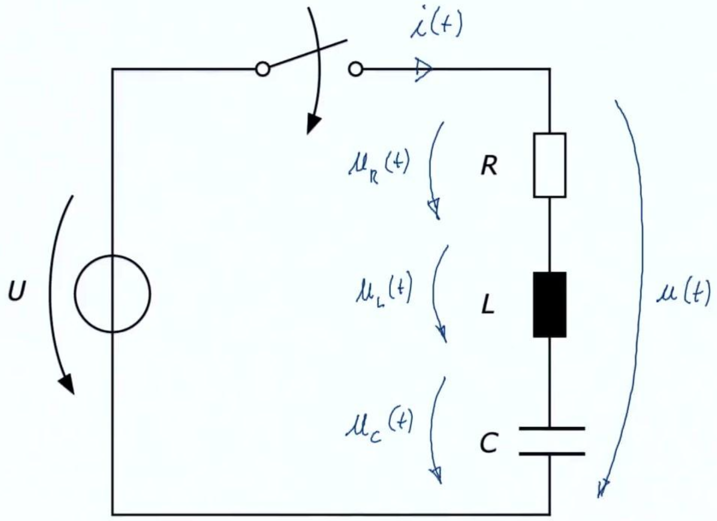

Pour $t \geq 0$:

$$U = R \cdot i + L \cdot \frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i \cdot dt$$

Dérivation →

$$L \cdot \frac{d^2 i}{dt} + R \cdot \frac{di}{dt} + \frac{1}{C} \cdot i = 0$$

Eq. caractéristique associée :

$$\lambda^2 \cdot L + \lambda R + \frac{1}{C} = 0$$

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Let's consider the following circuit, made of a continuous voltage supply, of a switch and of three elements R, L, and C in series. We start with the values of the circuit. i(t) the voltage drop across the resistor U_r(t) the voltage drop at the inductance terminals U_l(t) and, the voltage drop at the terminals of the capacitor U_c(t) the voltage at the terminals of the three elements, that we call U(t). For a time t >= 0, we could write the following equation U is equal to R*i + L*(di/dt), the voltage drop at the terminals of the inductance, plus the voltage drop at the terminals of the capacitor : 1/C * ∫[-∞, t] i(t)*dt. By deriving this equation, we get the following equation L * (d^2*i/dt) + R*(di/dt) + 1/C * i = 0, the derivative of U. At this 2nd order differential equation we can associate the following characteristic equation λ^2 * L + λ*R + 1/C = 0. It is a 2nd degree equation we will then determine the solutions λ_1 and λ_2.

Notes

Summary

0m 36s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RLC SÉRIE

- **Solutions de l'équation caractéristique**

$$\begin{cases} \lambda_1 = -\dfrac{R}{2L} + \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}} = -\dfrac{R}{2L} + \omega \\ \lambda_2 = -\dfrac{R}{2L} - \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}} = -\dfrac{R}{2L} - \omega \end{cases}$$

- **Equation du courant**

$$i(t) = A_1 \cdot e^{\lambda_1 t} + A_2 \cdot e^{\lambda_2 t}$$

A_1 et A_2 à déterminer

Electrotechnique II

This equation of second degree, present two solutions λ_1 et λ_2 that are given by the constant terms of the characteristic equation with a difference a + for the first solution, and a - for the second solution. For visibility matters, we will substitute the square root with the term ω, and we then get λ is equal to -R/2L ± ω. Supposing these two equations we can rewrite the current equation i(t) is equal to A_1*e^(λ_1*t) + A_2*e^(λ_2*t) The constant A_1 and A_2 have to be defined.

Notes

Summary

2m 45s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RLC SÉRIE

- **Détermination des constantes d'intégration**

En $t=0$: pas de saut de courant (L série)

$$i(0) = A_1 \cdot e^{0} + A_2 \cdot e^{0} = 0 \quad \rightarrow \quad A_2 = -A_1 \quad \rightarrow \quad i(t)$$

$$i(t) = A_1 \left(e^{\lambda_1 t} - e^{\lambda_2 t} \right) = A_1 \cdot e^{-\frac{R}{2L} t} \cdot \left(e^{\omega t} - e^{-\omega t} \right)$$

$$\lambda_1 = -\frac{R}{2L} + \omega$$

- **Conditions en $t = 0$: $i(0) = 0$ et $u_c(0) = 0$**

$$u_R(0) = R \cdot i(0) = 0 \quad ; \quad u_C(0) = 0$$

Donc pour $t=0$: $U = L \cdot \frac{di}{dt}$

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In order to determine the integration constants A_1 and A_2, we position at time t = 0 knowing that at that instant there is no current jump in the circuit, because of the inductance in series. Writing the equation of the current at time t=0, we get the following equation this equation, allows us to determine a first relation between A_1 and A_2, that we substitute in the temporal equation of the current, that leads us to the following relation: We can develop this equation knowing that the constant λ_1 is equal to -R/2L + ω. the same for λ_2, if we substitute in the previous equation, we get the exponential term -R/2L times (e^(ωt) - e^(-ωt)). For this case, we put the conditions at time t=0 but we know that the current at time t=0 is equal to 0. The voltage at the terminals of the capacitor at time t=0, is equal to 0 the capacitor is initially empty. This gives us, the voltage across the resistor at time t = 0 is equal to R times i(0) is equal to 0 and, the voltage at the terminals of the capacitor at time t=0, is equal to 0. we therefore still have for the binding equation at time t=0 U, the supply voltage which is equal to L*(di/dt), the two other terms disappear.

Notes

Summary

4m 04s

- **Trois cas à considérer :**

Discriminant sous la racine carrée : $\Delta = \frac{R^2}{4L^2} - \frac{1}{LC}$

1) $\Delta > 0$ → deux solutions réelles (sol. "suramortie")

2) $\Delta < 0$ → deux solutions complexe (sol. "oscillatoire amortie")

3) $\Delta = 0$ → une solution réelle (sol. "amortissement critique")

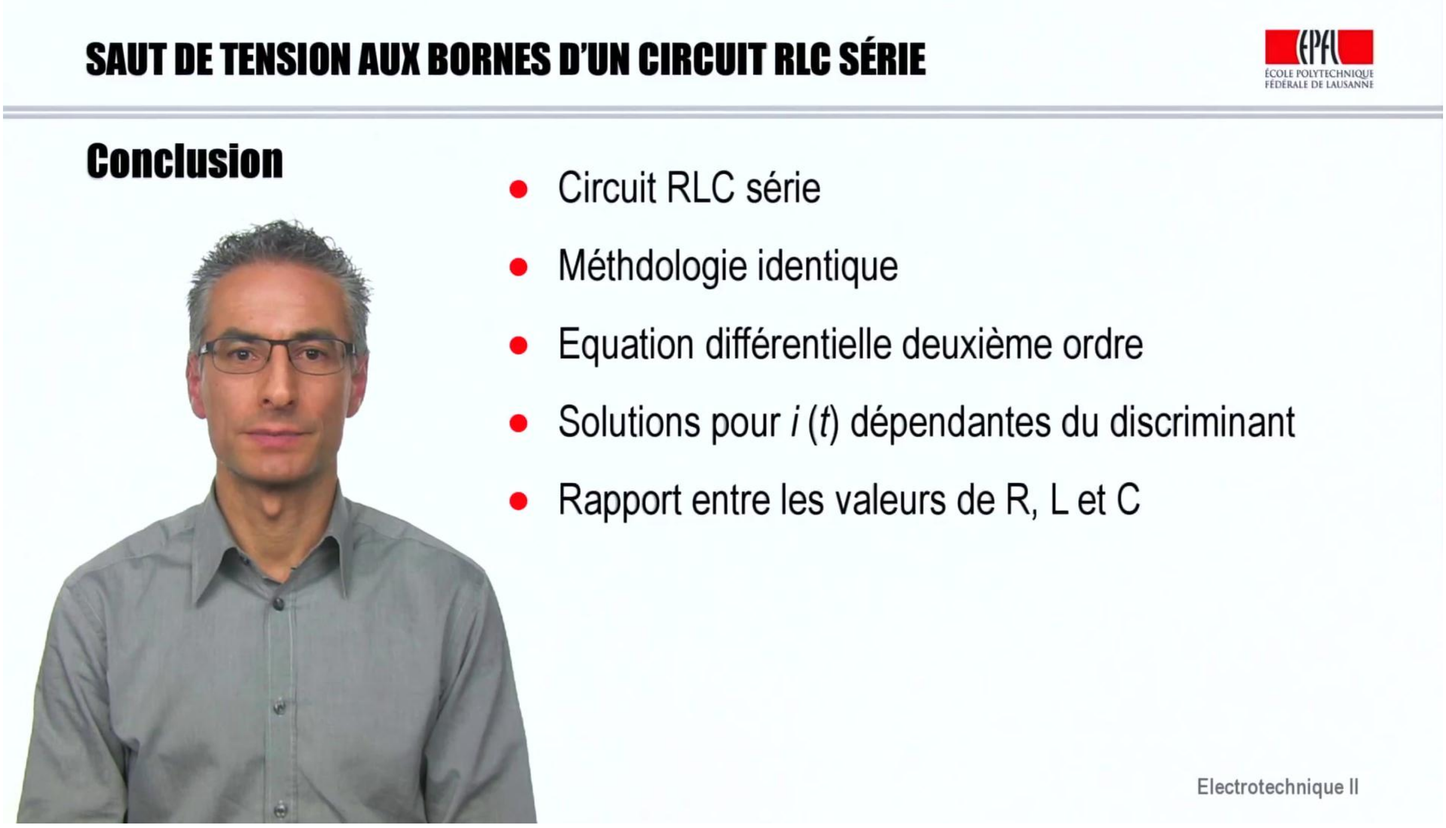
Electrotechnique II

At this instant of the development three cases have to be considered as function of the value of the discriminant under the scare root this discriminant Δ is equal to (R^2/4L^2) - 1/LC. the three cases are the followings: Either the discriminant is bigger than 0 in that case we get two real solutions. That solution is called 'over-damped'. In the second case the discriminant is smaller than 0 we will find two complex solutions, we are dealing with a "damped oscillation" solution And in the third case, when the discriminant is equal to 0 we have an unique and real solution, we are dealing in this cas with a "critical damping" solution.

Notes

Summary

6m 01s



We started dealing the case of an RLC circuit in series switched under a continuous supply voltage The used methodology remains the same The nature of the current leads to a differential equation of the 2nd order. The solutions for the current i(t) will be dependent of the discriminant. The link between the values R, L and C will determine the nature of the solution, and of the current. During next lecture, we will deal with the three cases of the discriminant will lead us to three different solutions.

Notes

Summary

7m 23s