# RÉGIMES TRANSITOIRES

## CIRCUIT RL SÉRIE

LEÇON 12


**Électrotechnique II**

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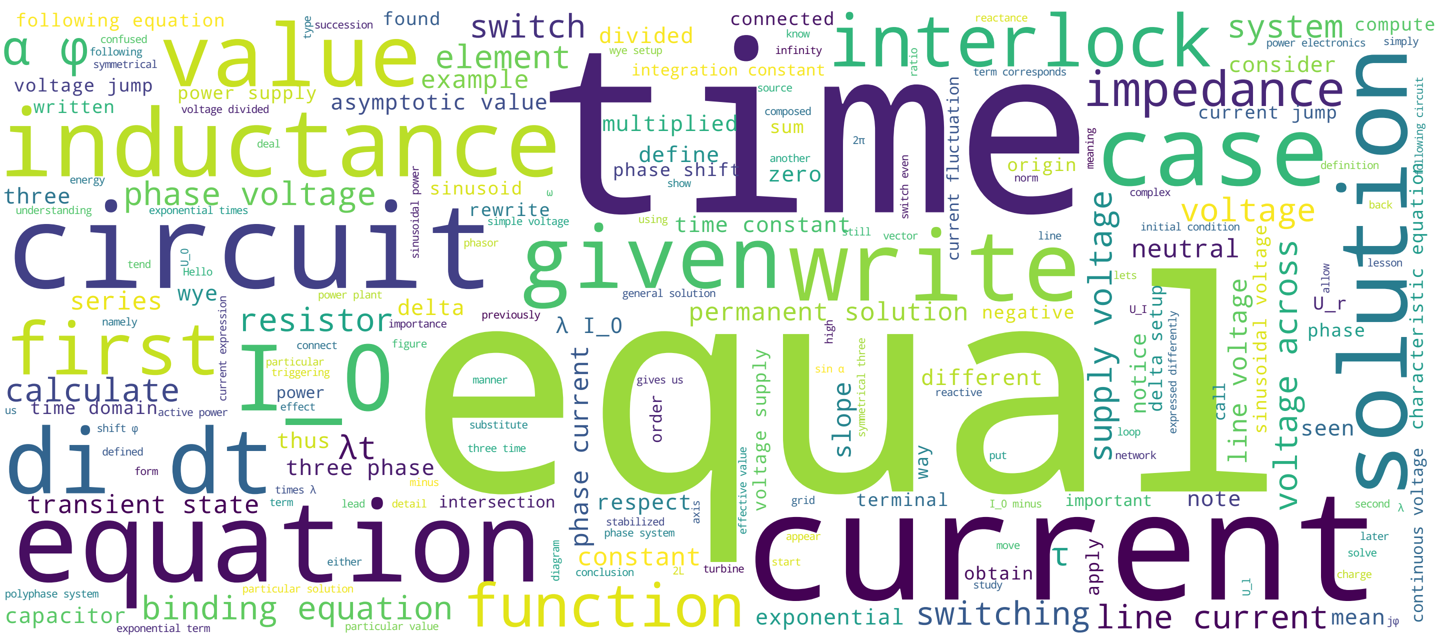



Video

## Généralités

- Exemple : Circuit RL série
- Saut sur une source de tension continue
- Saut sur une source de tension sinusoïdale
- Conclusions

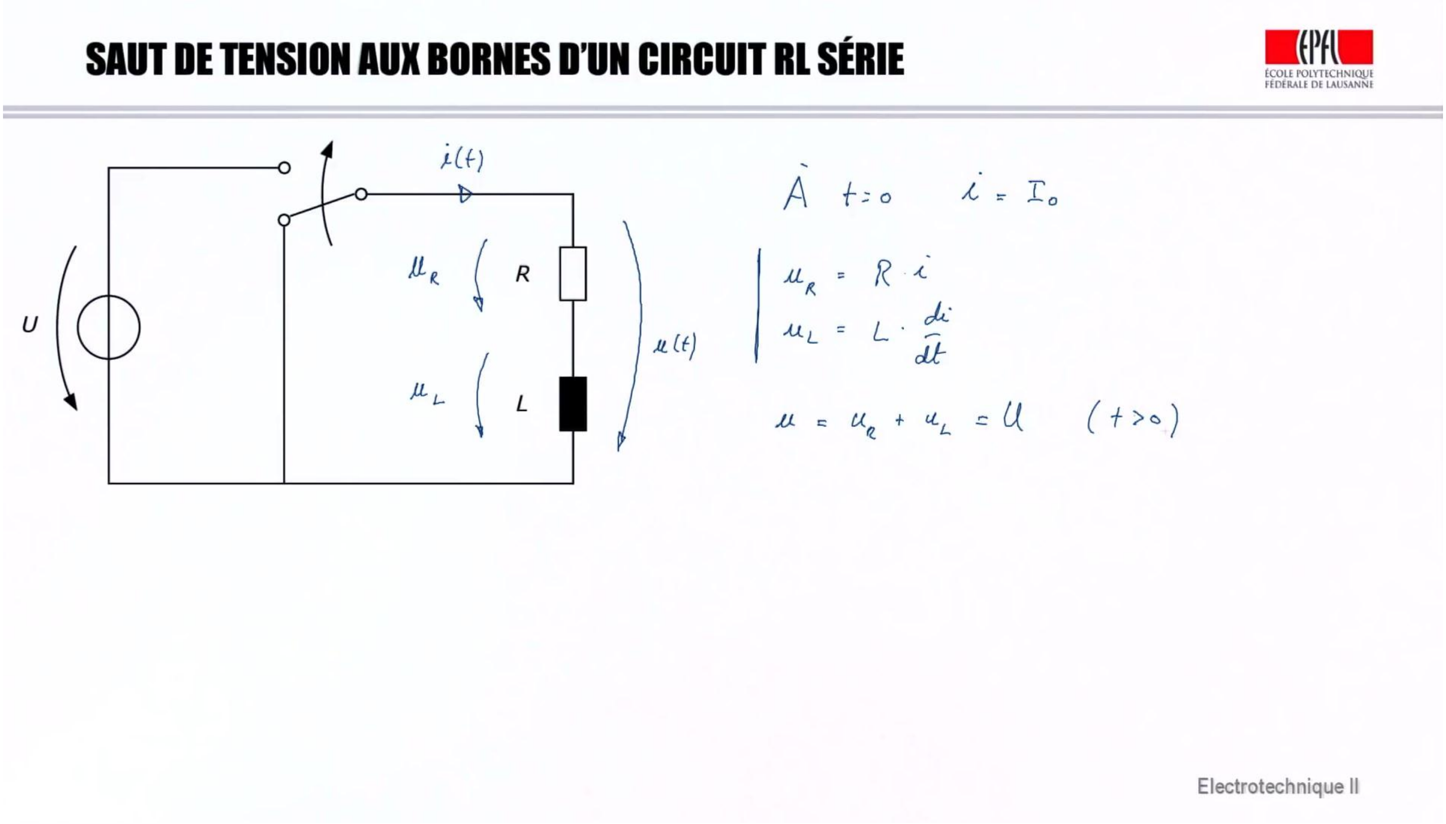
Electrotechnique II

Hello, in today's lecture, we will study the RL circuit in series. This is an important subject as the circuit is omnipresent in the field of motors, transformers or power electronics. As for RC circuit, we will first deal with the case where we apply a continuous voltage jump to a circuit then, we will apply to a circuit a voltage jump sinusoidal Let's start with a continuous source We consider the following circuit, composed of a resistor in series and an inductance, in series, switched at time t=0 under a supply voltage U.

Notes

Summary

0m 04s



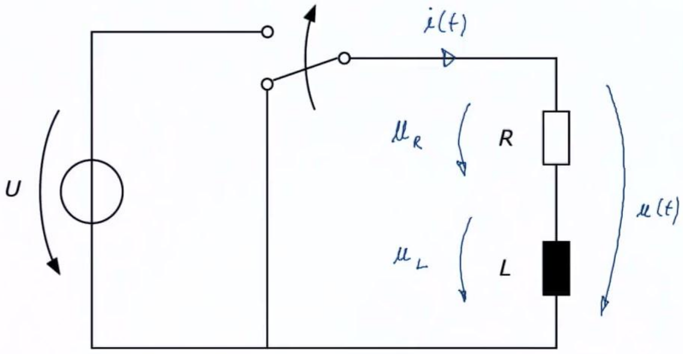

We write all the values according to the circuit First of all the current in the circuit unique loop i(t) the voltage across the resistor that is equal to U_r the voltage across the inductance equal to U_I and the total voltage across both elements which is equal to u(t) To deal with a general case, at time t=0 We will consider that the current in the inductance is already equal to I_0 at t=0 i is equal to I_0 We then write all the characteristic equations of each element, which means that for U_r this is equal to R*i and for U_I the voltage across the U_l is equal to L * (di/dt). These are the characteristic equations. Then we consider the loop and write the binding equations Which means that u is equal to U_r + U_l is equal to U At time t = 0 and after (t>0). Therefore, this binding equation is written here as R*i plus L di/dt is equal to U We've seen that such equation has an homogenous solution, of exponential type and permanent solution I_p, equal to for the settled state U/R, the inductance doesn't see any current fluctuation, the voltage across the inductance is equal 0, and therefore, the current in the loop, is equal to U/R.

Notes

Summary

0m 47s

# SAUT DE TENSION AUX BORNES D'UN CIRCUIT RL SÉRIE




À $t=0 \quad i = I_0$

$$\begin{cases} u_R = R \cdot i \\ u_L = L \cdot \frac{di}{dt} \end{cases}$$

$$u = u_R + u_L = U \quad (t>0)$$

$$R i + L \frac{di}{dt} = U$$

$$i_s = A \cdot e^{\lambda t} \; ; \; i_p = U/R \rightarrow i = A \cdot e^{\lambda t} + \frac{U}{R} \rightarrow \frac{di}{dt} = \lambda \cdot e^{\lambda t} \cdot A$$

$$R i + L \cdot \frac{di}{dt} = U \rightarrow R \cdot \cancel{A \cdot e^{\lambda t}} + \cancel{\frac{U}{R} R} + L \cdot \lambda \cdot \cancel{A e^{\lambda t}} = \cancel{U}$$

$$R = -L \cdot \lambda \rightarrow \lambda = -\frac{R}{L} = -\frac{1}{\tau} \qquad \underline{\tau = \frac{L}{R}}$$

Electrotechnique II

This leads to the general solution that is equal to I_s + I_p, meaning that i is equal to A times e^(λt) plus U/R that is, after derivation, equal to λ, the internal derivative of the exponential, times the same exponential times the constant A. If we substitute these equations of i and di/dt, we obtain in the binding equation to find the constant λ, that R_i + L * (di/dt) is equal to U substituting i(t) and di/dt in this binding equation, we have R times i, meaning A e^(λt) plus U/R times R plus L times di/dt, meaning that, L times λ times A e^(λt) is equal to U. We can simplify these two termes, as well as these ones divided in both sides of the equation by this term and therefore, R is equal to -L times λ, or expressed differently λ is equal to -R/L, which is equal to -1/τ. τ being time constant which is equal to L/R for a circuit RL in series.

Notes

Summary

2m 57s

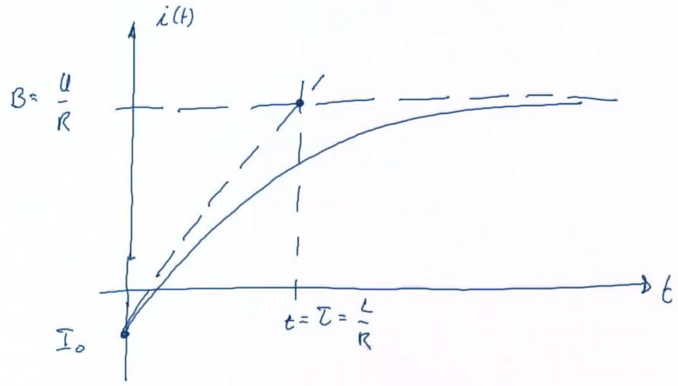
# SAUT DE TENSION AUX BORNES D'UN CIRCUIT RL SÉRIE

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Constante A :

À $t=0$ : pas de saut de courant :

$$I_0 = \frac{U}{R} + A \rightarrow A = I_0 - \frac{U}{R} \quad : \quad i(t) = \frac{U}{R} + \left(I_0 - \frac{U}{R}\right)\cdot e^{-\frac{R}{L}t}$$




Electrotechnique II

We should now define the integration constant A and to do so, we move to t=0 at the interlock where there is no current fluctuation then, at t=τ, the exponential is equal to 1, and we can write to following equation : I_0 = U/R + A, therefore A is equal to I_0 moins U/R. we finaly rewrite the equation of the current which is equal to i(t) which is equal at the permanente solution U/R, plus the current expression already determined. Graphically represented in the time domain, we get the following curve i(t) as function of time the curve start from I_0 an unknown value, that we can choose to be negative and will tend to the asymptotic value and reach U/R, which is the permanente value that we call B for the computations that will follow. The intersection between the slope at the origin and the asymptotic value is at time t=τ, which is equal to L/R.

Notes

Summary

5m 09s

# SAUT DE TENSION AUX BORNES D'UN CIRCUIT RL SÉRIE

Calcul de la pente à l'origine :

$i(t) = B + (I_0 - B)\cdot e^{-\lambda t}$ ; $\frac{di}{dt} = -\lambda (I_0 - B)\cdot e^{-\lambda t}$

en $t=0$ : $\frac{di}{dt} = -\lambda (I_0 - B)$ $\xrightarrow{\text{droite}}$ $i' = -\lambda (I_0 - B)\, t + I_0$

intersection entre la pente à l'origine et la valeur asymptotique :

$i' = B \rightarrow B = -\lambda (I_0 - B)\cdot t + I_0$

$B - I_0 = -\lambda (I_0 - B)\, t \rightarrow t = \frac{1}{\lambda} = \tau = \frac{L}{R}$ constante de temps

Que vaut $i$ pour $t = \tau$ : $i = B + (I_0 - B)\, e^{-1}$

si $I_0 = 0$ $\quad i = B(1 - e^{-1})$

$= B\left(1 - \frac{1}{e}\right) = 0{,}632 \cdot \frac{U}{R}$

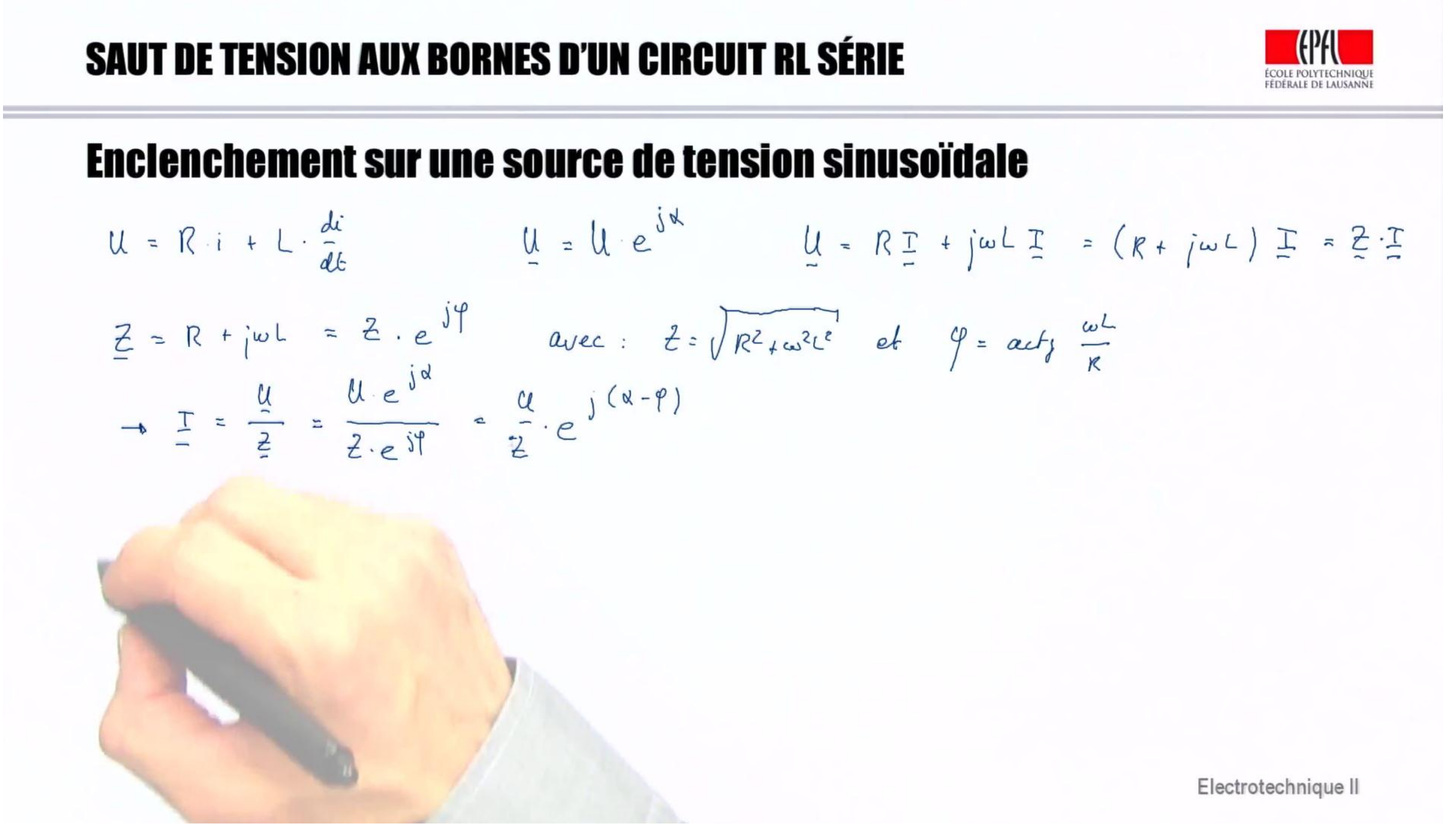
Electrotechnique II

Now let's compute the slope at the origin. We repeat current equation i(t)=U/R that we have substituted by B, I_0 minus B,times e^(-λt) and its derivative di/dt, that is equal to -λ I_0 minus B times e^(-λt). At t=0, this slope is equal to -λ(I_0 - B) the right hand side is therefore given by the expression -λ plus the absciss Now, we can compute the intersection between this slope at the origin and the asymptotic value. Meaning that we should solve the following equation : i' = B B is therefore equal to -λ(I_0 - B) plus I_0. Then, B - I_0 is equal to -λ(I_0 - B)*t. The intersection occurs at time t = 1/λ = τ. L/R being the time constant. Finally, we will calculate the value of i for t = τ which is the time constant if we substitute in the current equation, the time t by τ, we obtain the following equation i = B + (I_0 - B)e^(-1). and this is equal to t/τ. If I_0 is equal to 0, then the current i is equal to B times (1 - e^(-1)) or written under this form, B(1 - 1/e) is equal to 0.632 times the asymptotic value.

Notes

Summary

6m 54s




Now, let's compute the case of an interlock, not under a continuous voltage supply, but under a sinusoidal voltage supply. For such interlock we will consider initial conditions. Only the permanent component changes and is given by the complex calculation associated to the circuit. We rewrite the binding equation the supply voltage written under a complex form is U and is equal to the effective value times e^(jα). If we rewrite the binding equation considering the supply voltage, we have U = R * I complex + jωLI. It is also equal to (R + jwL)*I this is equal to Z*I. expressed differently The impedance Z, which is equal to R + JωL Z times e^(jφ) with Z equal to √(R^2 +ω^2*L^2) and the angle phi, given by the arctan of the reactance over the resistor. And finally, the current is given by U/Z can be expressed as U*e^(jα) divided by Z*e^(jφ) which is equal to U/Z times e^j(α - φ). If we go back to time domain, we can rewrite this equation which is the inhomogeneous solution of the current I_p = √2*(U/Z)*sin(ωt + α - φ).

Notes

Summary

9m 45s

## Enclenchement sur une source de tension sinusoïdale

$u = R\cdot i + L\cdot\frac{di}{dt}$ $\qquad \underline{U} = U\cdot e^{j\alpha}$ $\qquad \underline{U} = R\underline{I} + j\omega L\underline{I} = (R + j\omega L)\underline{I} = \underline{Z}\cdot\underline{I}$

$\underline{Z} = R + j\omega L = Z\cdot e^{j\varphi}$ avec : $Z = \sqrt{R^2+\omega^2L^2}$ et $\varphi = \arctan\frac{\omega L}{R}$

$$\rightarrow \underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{U\cdot e^{j\alpha}}{Z\cdot e^{j\varphi}} = \frac{U}{Z}\cdot e^{j(\alpha-\varphi)}$$

Domaine temporel : $i_p = \sqrt{2}\cdot\frac{U}{Z}\cdot\sin(\omega t+\alpha-\varphi)$

$$i = \sqrt{2}\,\frac{U}{Z}\sin(\omega t+\alpha-\varphi) + \underline{\underline{A}}\cdot e^{-\frac{R}{L}\cdot t}$$

$$A = -\sqrt{2}\,\frac{U}{Z}\sin(\alpha-\varphi)$$

$$\Rightarrow \underline{i = \sqrt{2}\,\frac{U}{Z}\left[\sin(\omega t+\alpha-\varphi) - \sin(\alpha-\varphi)\cdot e^{-\frac{R}{L}\cdot t}\right]}$$

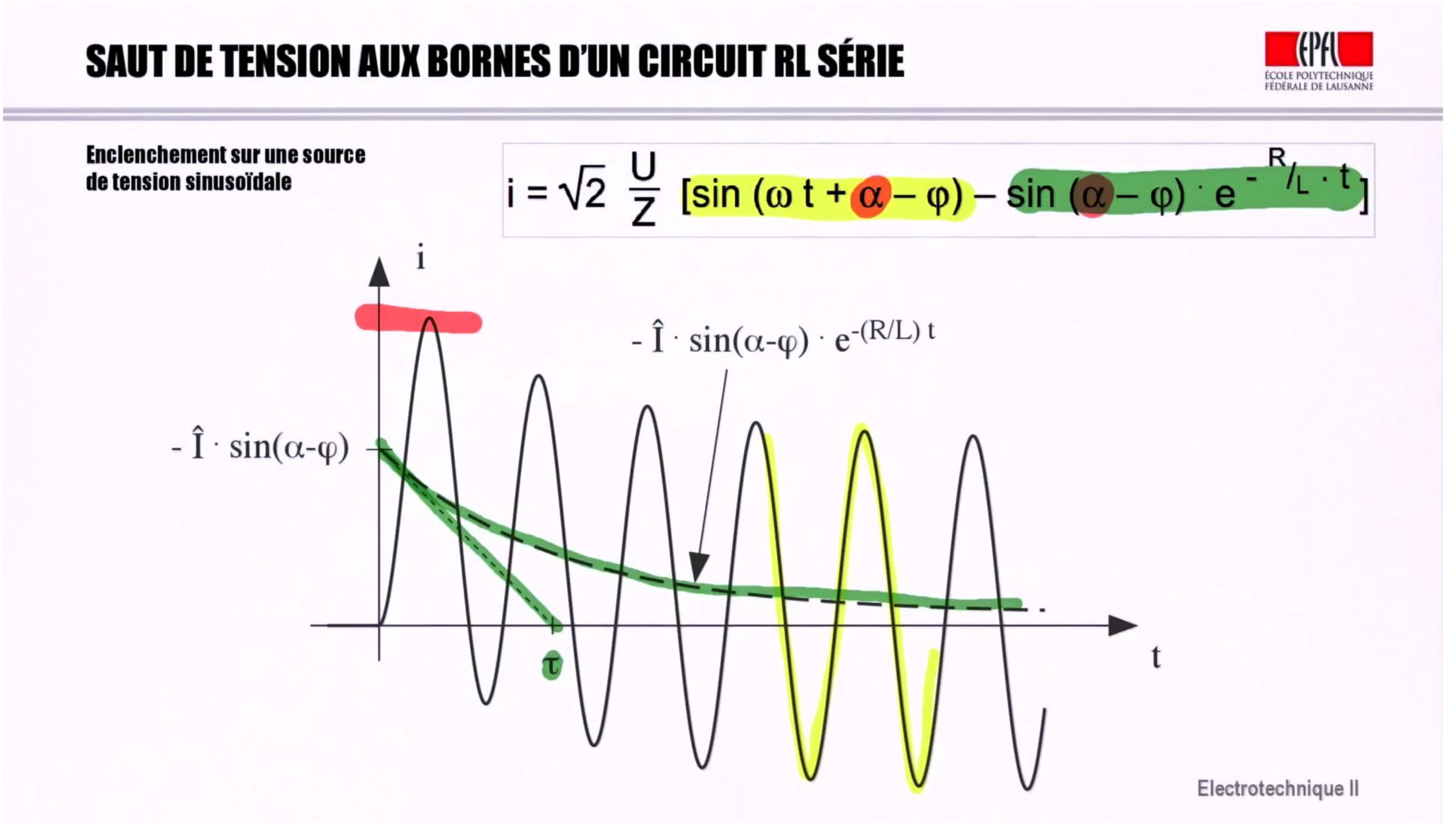
Electrotechnique II

As I have already mentioned before only the particular value changes Therefore, the solution of the second membre stays, we can then write the equation of the current which is the particular solution plus the permanent solution Again, we should calculate the integration constant A and to do so, we should move to time t=0, at the interlock, assuming i=0 We have A equal to -√2*(U/Z)*sin(α - φ) And finally, the solution is given by the current which is equal to This is the final expression, we notice that the moment of the switching with respect to the sinusoidal voltage is very high we will see that in the graph of the following page.

Notes

Summary

12m 22s

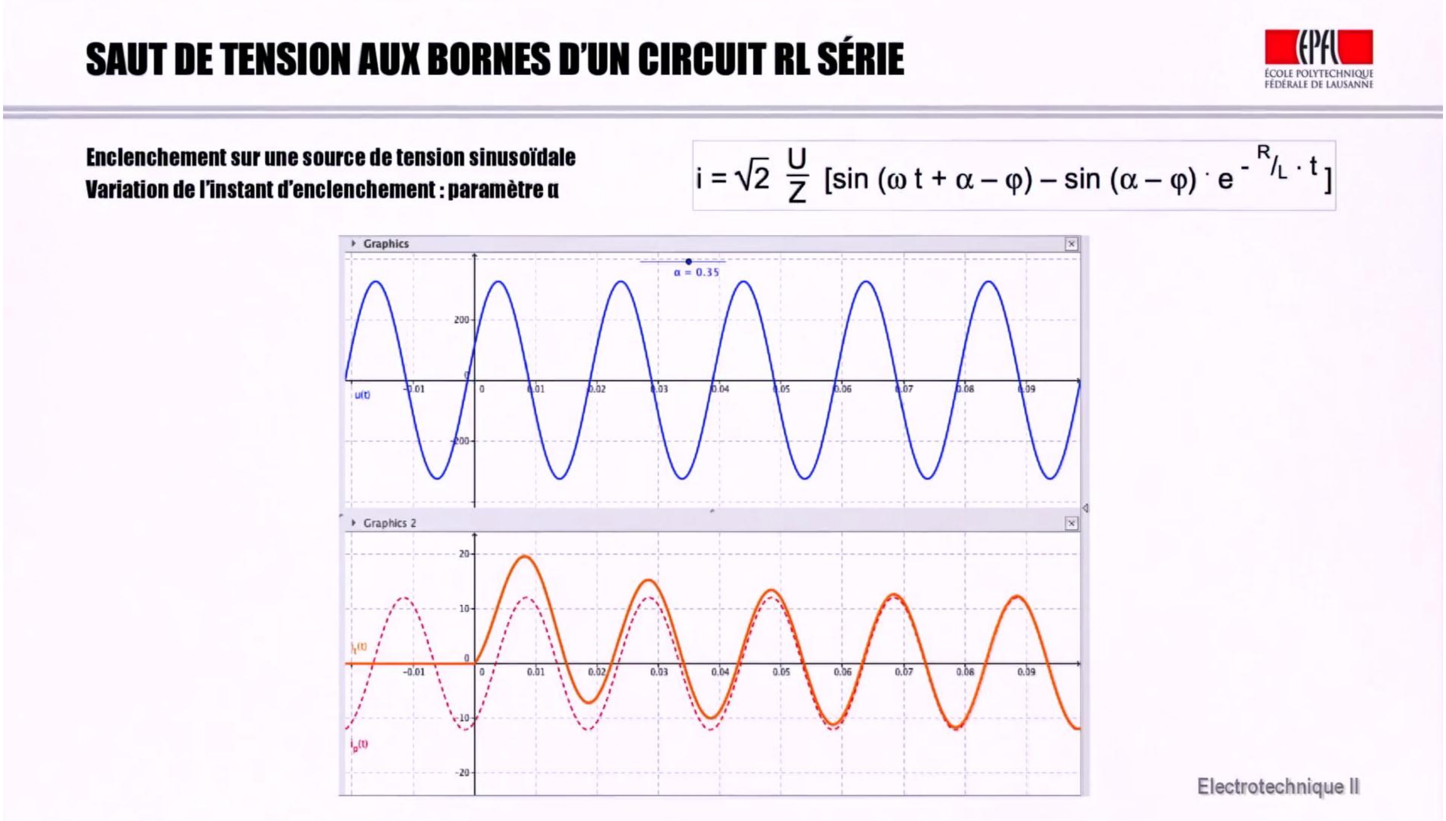



The temporal graphical representation of the solution, for the current, is given by that figure. We see a sinusoid this term which is added to an exponential function. The sinusoid corresponds to the sinusoidal power supply, and the exponential corresponds to the interlock. It is the transient state with its time constant. We notice the overcurrent due to the interlock, here that we also notice in case of triggering, and that depend on the switching moment α, here and here with respect to the supply voltage. We will look in details at the effect of the parameter α.

Notes

Summary

13m 33s

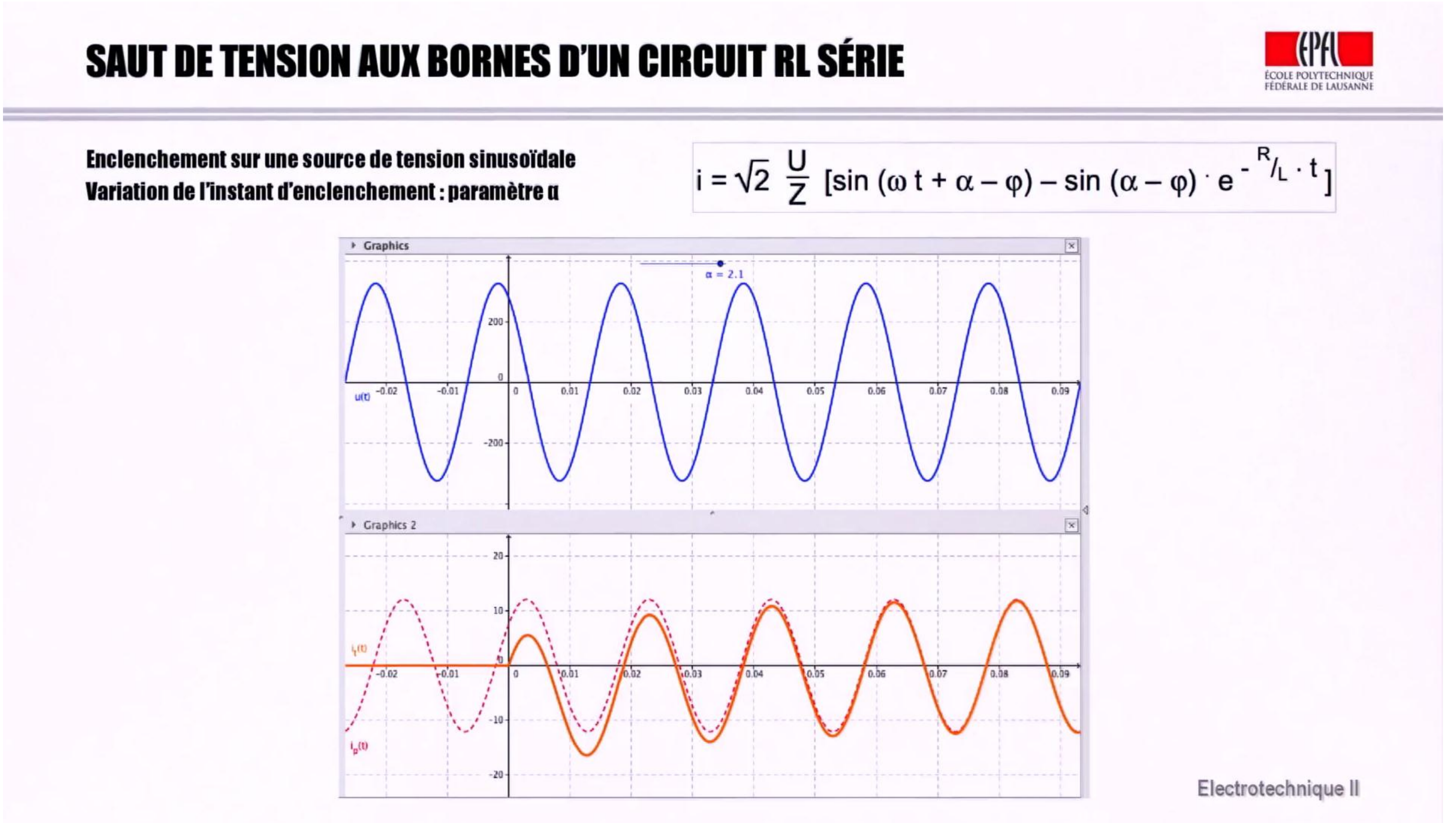



We see in this figure, in blue, the sinusoidal supply voltage, with an angle α = 0. Below, in red is plotted the permanent current in the circuit if the system was stabilized. we put it as reference for the understanding of these explanations. We note that the phase shift φ the phase shift φ which is introduced by the impedance RL, φ is a circuit constant. If we switch the circuit at the represented instant here, at the axis t=0 we obtain, in orange, the previously described curve, meaning that no current here, before the switching, then an equation, which have a sinusoidal term, and an exponential term. Once the transient state is finished, the system is stabilized we still only have the permanent solution and therefore the solution, here that we found, is confused with the permanent solution we note that there is no current jump at the switching, here because the circuit is composed of an inductance in series. We note, here again, the over-current due to the exponential. If we switch, not at a=0, but at a bigger value, therefore later on the voltage sinusoid, here we see, we note that the over-current decrease slightly, which is normal because the term α - φ, in absolute value, becomes smaller.

Notes

Summary

14m 31s




If we switch the circuit further in time, here we see that the exponential term decrease again. An overcurrent decreases. If we switch even further, until we finally arrive at a particular value, $\alpha = \varphi$ in this case, the term $\sin(\alpha - \varphi) = 0$ we notice that the term next to the exponential becomes equal to zero et therefore the transient state doesn't appear. We see in the curve, here that the solution that we have found, is confused with the particular solution, there is no transient state. This is obvious, because we actually switch at the passage of the current by 0, and then, there is no current jump. This is called cross switching 0. This approach is very important in the power electronics where the switchings occur at many kilohertz, even dozens of kilohertz, because they allow to handle the components, avoiding the overload, either of the current, or the voltage. If we switch even further, with respect to the sinusoid, the transient always reappears, but this time in negative. And, again a little bit latter the overload here, is more important.

Notes

Summary

16m 16s

## SAUT DE TENSION AUX BORNES D'UN CIRCUIT RL SÉRIE

### Conclusion

- Un terme correspond à l'alimentation sinusoïdale de la source
- Un terme correspond au fait de faire un enclenchement (exponentielle)
- La solution générale correspond au cumul de ces deux effets
- Importance de l'instant auquel est fait l'enclenchement par rapport à l'alimentation sinusoïdale

Electrotechnique II

In conclusion, we see that a term corresponds to the sinusoidal power supply of the circuit, it is the permanent solution A second term corresponds to the switching, it is the exponential part of the solution. The general solution corresponds to the cumulation of these two effects. Moreover, we have seen the importance of the switching instant with respect to the sinusoidal voltage, that could, more or less, stress out the electronic components, because of an overload.

Notes

Summary

17m 43s