ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

RÉGIMES TRANSITOIRES

CIRCUIT RC SÉRIE

LEÇON 11

Électrotechnique II
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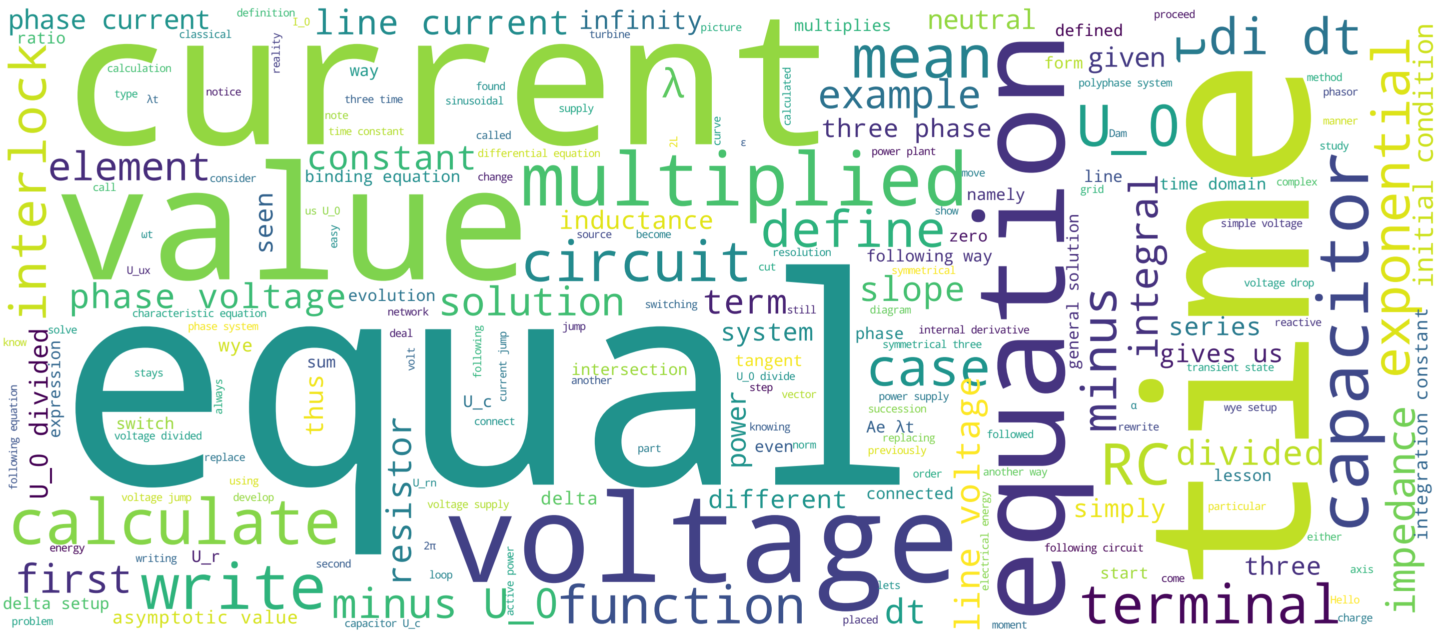
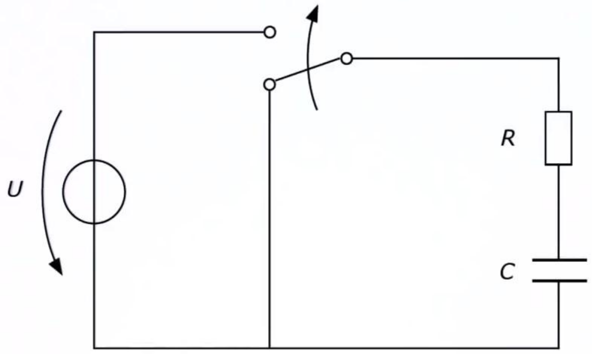


Video

EPFL

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RC SÉRIE



Electrotechnique II

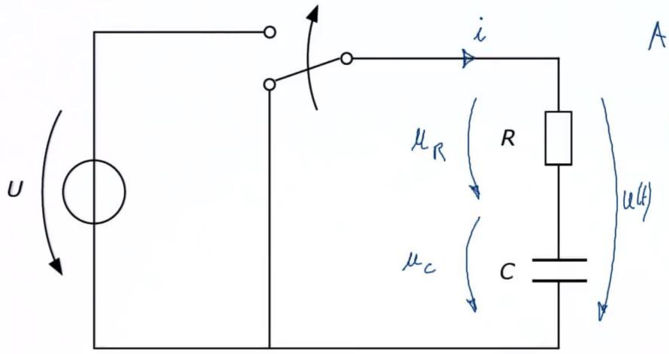
Hello, during the first lesson, we have laid the theoretical bases of the resolution of a problem in transient state we will move to a concrete example with several components. We have seen how a resistor, capacitor or an inductance behaved alone, subject to a voltage or a current jump. In reality the elements L and C can't exist alone, they are always combined with a parasite resistor and even a resistor always has a parasite capacity or inductance. We will see a method that deals with these combined elements. The example that we will see, during this lesson, is a circuit with a resistor in series with a capacitor. It's the classical and widespread charge or discharge of a capacitor. In other words, stocking electrical energy in a capacitor, or an accumulator or even a supercapacitor. Let the following circuit, made of a resistor in series with a capacitor, that we will switch on a constant voltage supply U. We begin by writing down all the values in the circuit, which are the voltage at the terminals of the resistor U_r, the voltage at the terminals of the capacitor U_c knowing that the loop is traveled by a current I in this direction.

Notes

Summary

0m 04s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RC SÉRIE



A $t=0$ $\quad u_c = U_0$

$$u_R = R \cdot i$$

$$u_c = \frac{1}{C}\int_{-\infty}^{t} i\,dt = U_0 + \frac{1}{C}\int_0^t i \cdot dt$$

$$u(t) = u_R + u_c = U \quad \text{pour } t > 0$$

$$u = Ri + \frac{1}{C}\int i\,dt$$

dérivant l'équation : $\quad 0 = R \cdot \frac{di}{dt} + \frac{1}{C} \cdot i$

Solution : $\quad i = A \cdot e^{\lambda t}$ avec A et λ constantes d'intégration

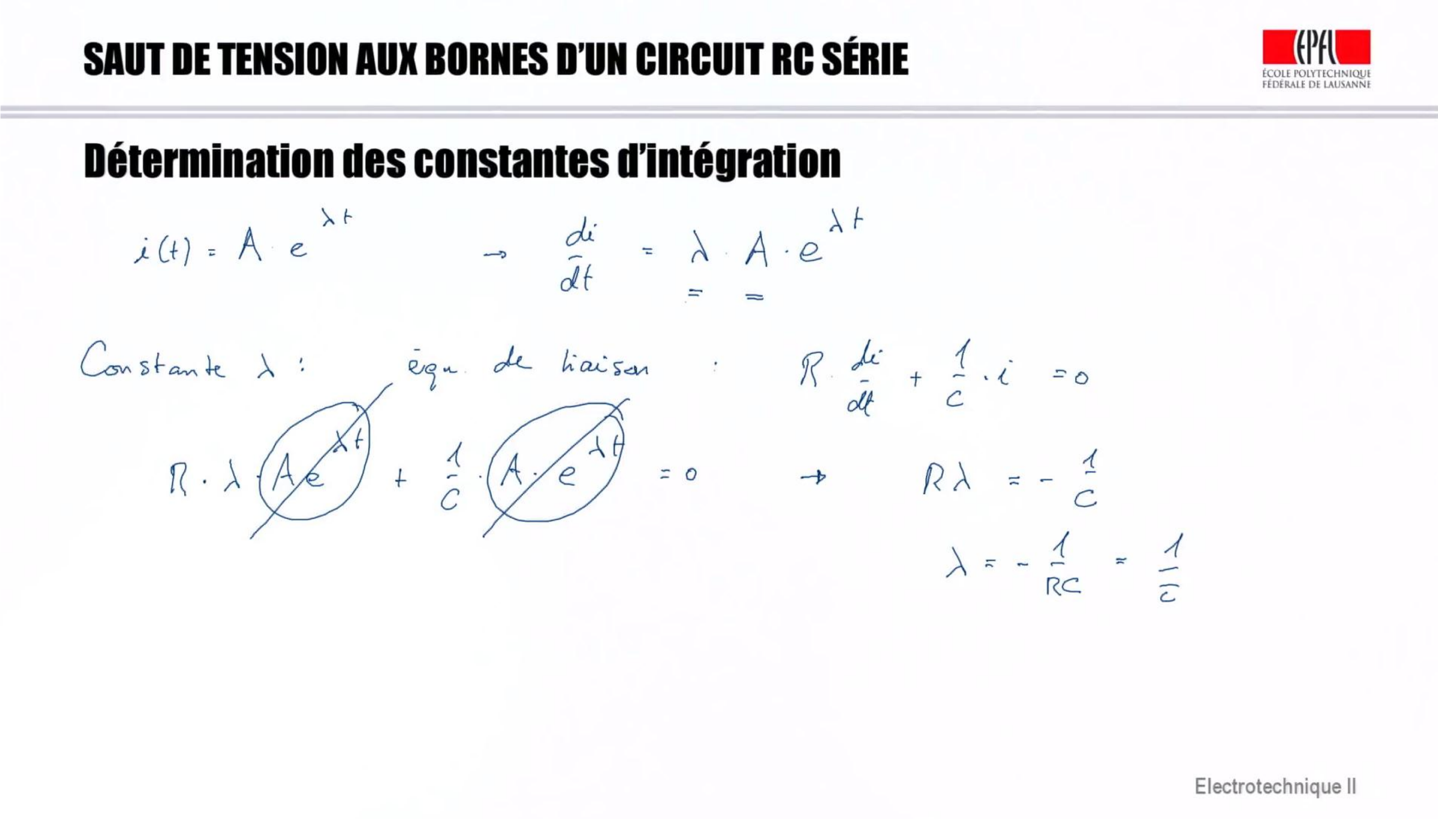
Electrotechnique II

First a condition is placed, at the time t = 0 there is a residual voltage in the capacitor U_c, that is equal to U_0, it is the initial condition of the circuit. We then write the characteristic equations of each element which means that U_r is equal to R*I and U_c is equal to 1/C times the integrale to -infinity to t of i(t)dt that we can express by seperating the integral from -infinity to 0 and from 0 to t. Knowing that the part from -infinity to 0 is in reality, the initial condition of the capacitor, that we can write U_0. So this is equal to U_0 plus 1/C, multiplied by the integral from 0 to t of i(t)dt. We can now write the binding equation, which means that, on the single loop of the circuit, we apply Kirchoff's theorem, and we have therfore that u(t) which is here u(t) is equal to U_r + U_c. And, this is equal to U for t > 0, which means, after the interlock. We can develop this, in the form U is equal to R*i plus 1/C multiplied by the integral of i*dt. We derive in this equation and we get that 0 is equal to R multiplied by di/dt + 1/C times i. We see that this is a first order differential equation whose solution is given by i(t) is equal to A*e^(λt). With A and λ being integration constants.

Notes

Summary

1m 35s



Once this general solution is found, we have to define the integration constants. We rewrite the equation which is the general solution, i(t) is equal to Ae^(λt) and its derivative that stays an exponential and that is equal to λ, the internal derivative, multiplied by the equation itself, the exponential itslef. A, λ, remaining to define. We proceed the following way, we replace i and (di/dt) in the binding equation which gives us R times (di/dt) plus 1/C times i is equal to 0 and this becomes R multiplied by λ times Ae^(λt) plus 1/C that multiplies i(t), it is Ae^(λt). This is equal to 0. By dividing by this term, we get Rλ is equal to -1 divided by C or, express differently λ is equal to (-1/RC) and this is equal to 1/τ the time constant.

Notes

Summary

4m 07s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RC SÉRIE

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Détermination des constantes d'intégration

Constante A : au temps t=0 : pas de saut de tension aux bornes de C

$u_c = U_0$ pour $t=0$

$$u_c = u - Ri = U - R\cdot A\cdot \underbrace{e^{-t/RC}}_{=1} = U - R\cdot A = U_0 \rightarrow A = \frac{U-U_0}{R}$$

$$i = \frac{U-U_0}{R}\cdot e^{-t/RC}$$

Rem: sol. permanente = 0 i_C en DC = 0

Electrotechnique II

To define the constant A we proceed the following way, we solve the equation at the time t=0. U_c is equal to U_0. The initial condition for t=0. By rewriting the equation at the time t=0, we get U_c is equal to U minus R_i, and this is equal to the interlock, at U minus R*A, exponential of -t/RC this term is equal to 1 at the time t=0 so it comes that U minus R*A is equal to U_0 or, the term A that is equal to U minus U_0 divided by R. We have defined the two integration constants and can therefore, now, write the equation for the current that is equal to i which is equal to A, which means U-U_0 divided by R multiplied by the exponential of -t/λ, which means -t/RC. We will notice that the permenant solution is equal to 0 simply because the current in the capacitor in DC is equal to 0.

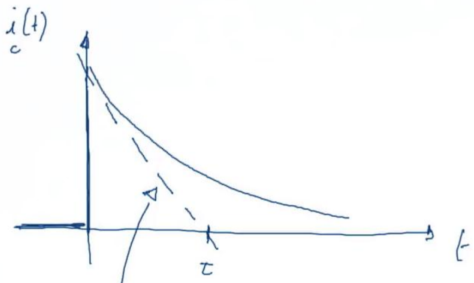
Notes

Summary

5m 51s

Analyse

$i = \frac{U - U_0}{R} \cdot e^{-t/RC}$ pour $t = \tau$ $\rightarrow \frac{1}{e}$ $\quad i\big|_{t=\tau} = \frac{U \cdot U_0}{R} \cdot e^{-1}$



$\frac{di}{dt} = \frac{U - U_0}{R}\left(-\frac{1}{RC}\right) \cdot e^{-t/RC}$ en $t=0$: $\frac{di}{dt}\Big|_{t=0} = -\frac{U - U_0}{R^2 C}$

équation : $i' = \frac{U - U_0}{R} - \frac{U - U_0}{R^2 C} \cdot t$

i is equal to U minus U_0, divided by R, multiplied by e^(-t/RC). We see that, for t=τ well, there is a a current depreciation in a ratio 1/e Why? Because i at the time t, is equal to τ this is equal to (U-U_0)/R multiplied by e^(-1). This current equation is pictured in the time domain by this slope here a zero current before the interlock and a current that does an instant jump like this this is i_c(t). We can calculate the tangent at the origine and calculate what is its its intersection with the stabilized value which means, in our case, the axis t. This intersection is done at the time t=τ indeed, we can write that the derivative of the current at the time t=0 di/dt is equal to (U-U_0)/R that multiplies (1/-RC) multiplied by e^(-t/τ). At t = 0, What do we have? We have that the derivative, di/dt is equal to -(U - U_0)/R^2*C The equation of this tangent is given by i' which is equal to U minus U_0 divide by R it's the value at the time t=0 minus the slope, time t.

Notes

Summary

7m 47s

SAUT DE TENSION AUX BORNES D'UN CIRCUIT RC SÉRIE

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Calcul de l'évolution de la tension aux bornes du condensateur

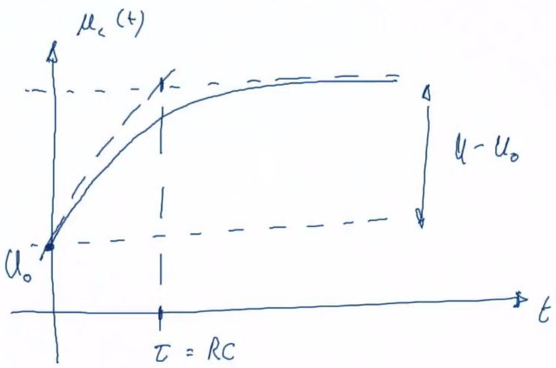
$$u_C = U_0 + \frac{1}{C}\int_0^t i \cdot dt$$

$$u_C = U_0 + \frac{1}{C} \cdot \frac{U - U_0}{R} \int_0^t e^{-t/RC} \cdot dt$$

$$= U_0 + \frac{U - U_0}{RC}(-RC)\, e^{-t/RC}\Big|_0^t$$

$$= U_0 - (U - U_0)\left(e^{-t/RC} - 1\right)$$

$$= U_0 + (U - U_0)\left(1 - e^{-t/RC}\right)$$



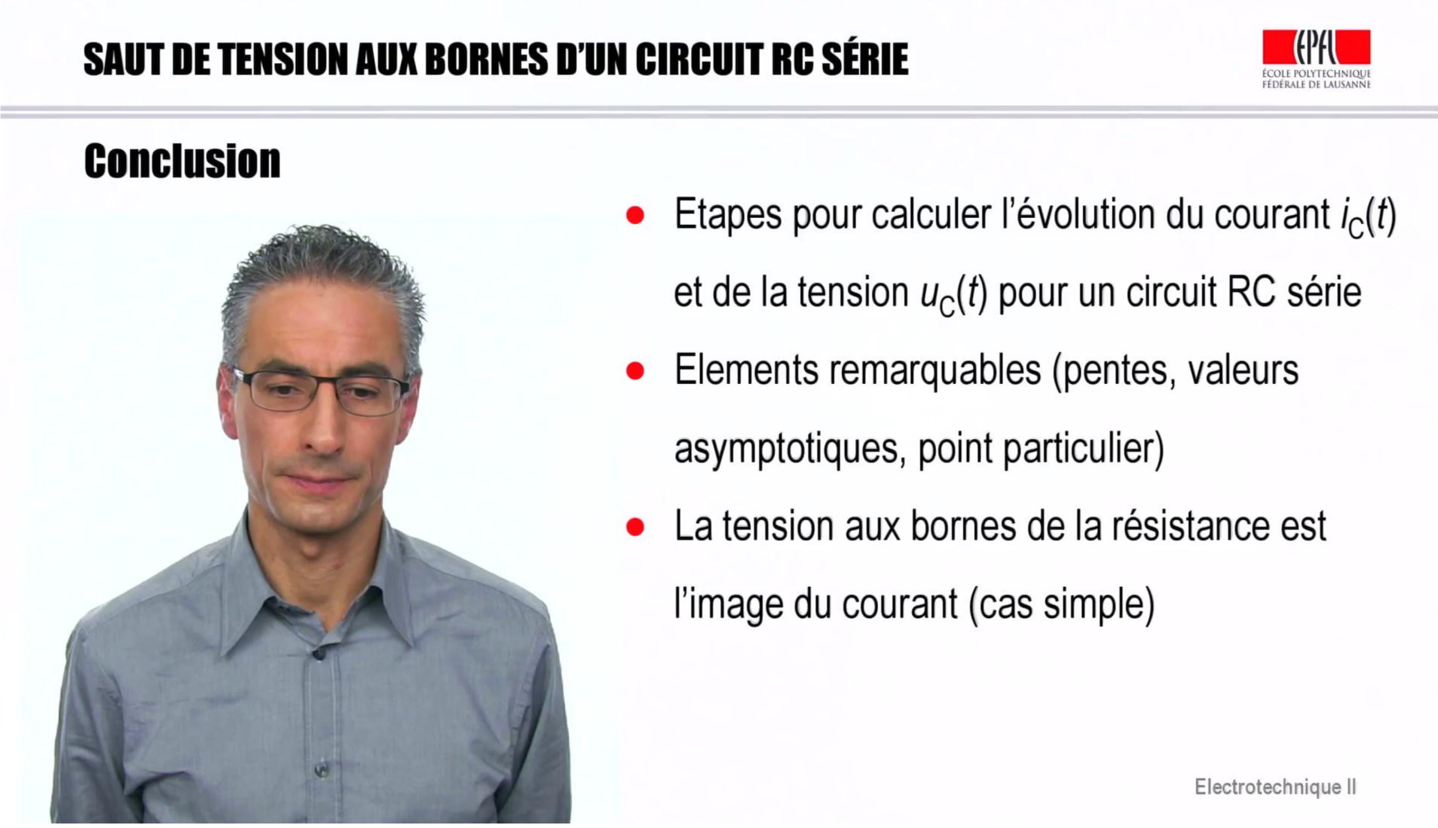
Electrotechnique II

Let's calculate, now, the evolution of the voltage on the terminals of the capacitor. We have the equation at the terminals of the capacitor that is given U_C that is equal to the initial condition U_0 plus 1/C multiplied by the integral from 0 to t of i*dt. By replacing the value of i, by the found solution we get the following equation, U minus U_0 divided by R multiplied by the integral from 0 to t of e^(-t/RC)*dt By solving this we get U_0 plus U - U_0 divide by RC. We calculate its antiderivative that is the same value with the internal derivative taken from 0 to t. And this all calculations done, gives us, U_0 minus plus U minus U_0 that multiplies e^(-t/τ) which means that RC, minus 1. Written in another way, which gives us U_0 plus U minus U_0 that multiplies 1 minus e^(-t/RC). If we picture this slope in the time domain we get the following thing U_C(t), as function of time we start with an initial value U_0 no matter what happened before, the moment of the interlock the value of the capacitor is at U_0, and this curve this exponantial, has this appearance. A jump that is equal to an asymptotic value U-U_0, and a tangent that cuts the asymptotic value at the time t=τ is equal to RC.

Notes

Summary

10m 09s



That's it, we have followed all the steps to calculate the evolution of the current I_c(t) and the voltage U_c(t) for a RC circuit in series. We have seen the noticeable elements as the slope, the asymptotic values and particular ponts. We note that we didn't calculate the voltage drop at the terminals of the resistor because it is very easy to calculate, it is simply R multiplied by the current I that we have calculated.

Notes

Summary

12m 50s