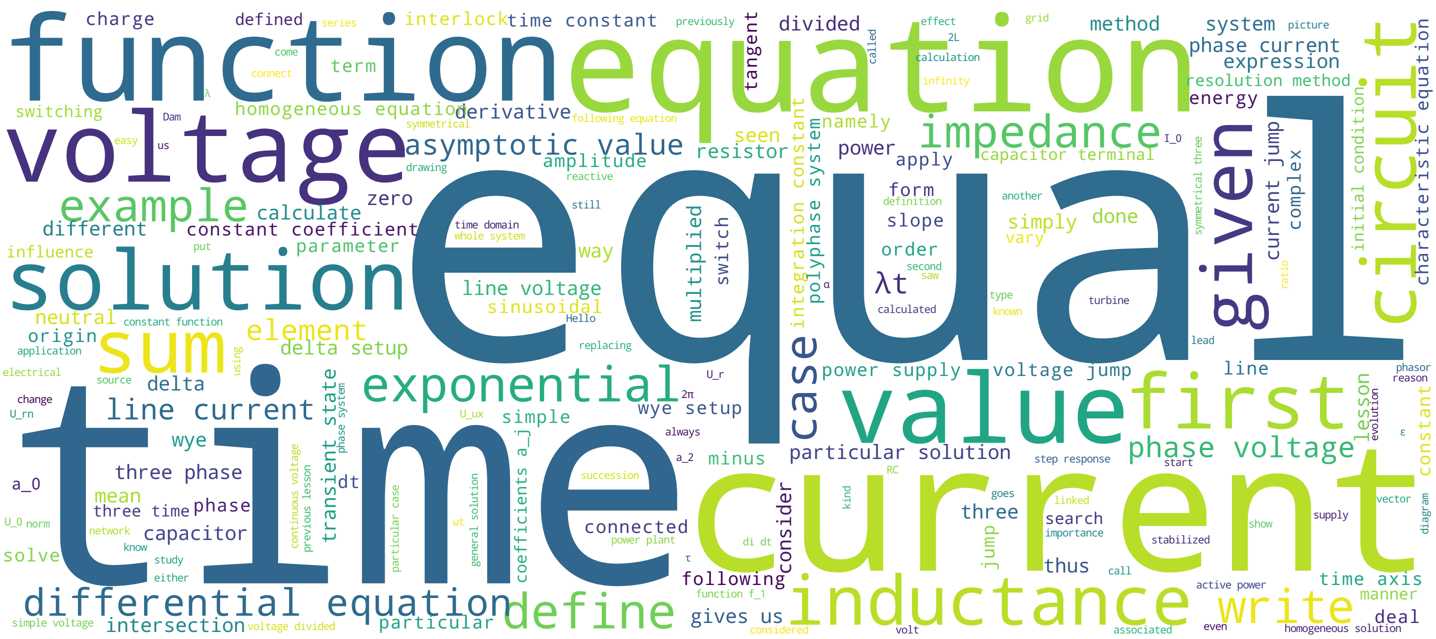
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

RÉGIMES TRANSITOIRES

MÉTHODES DE RÉSOLUTION

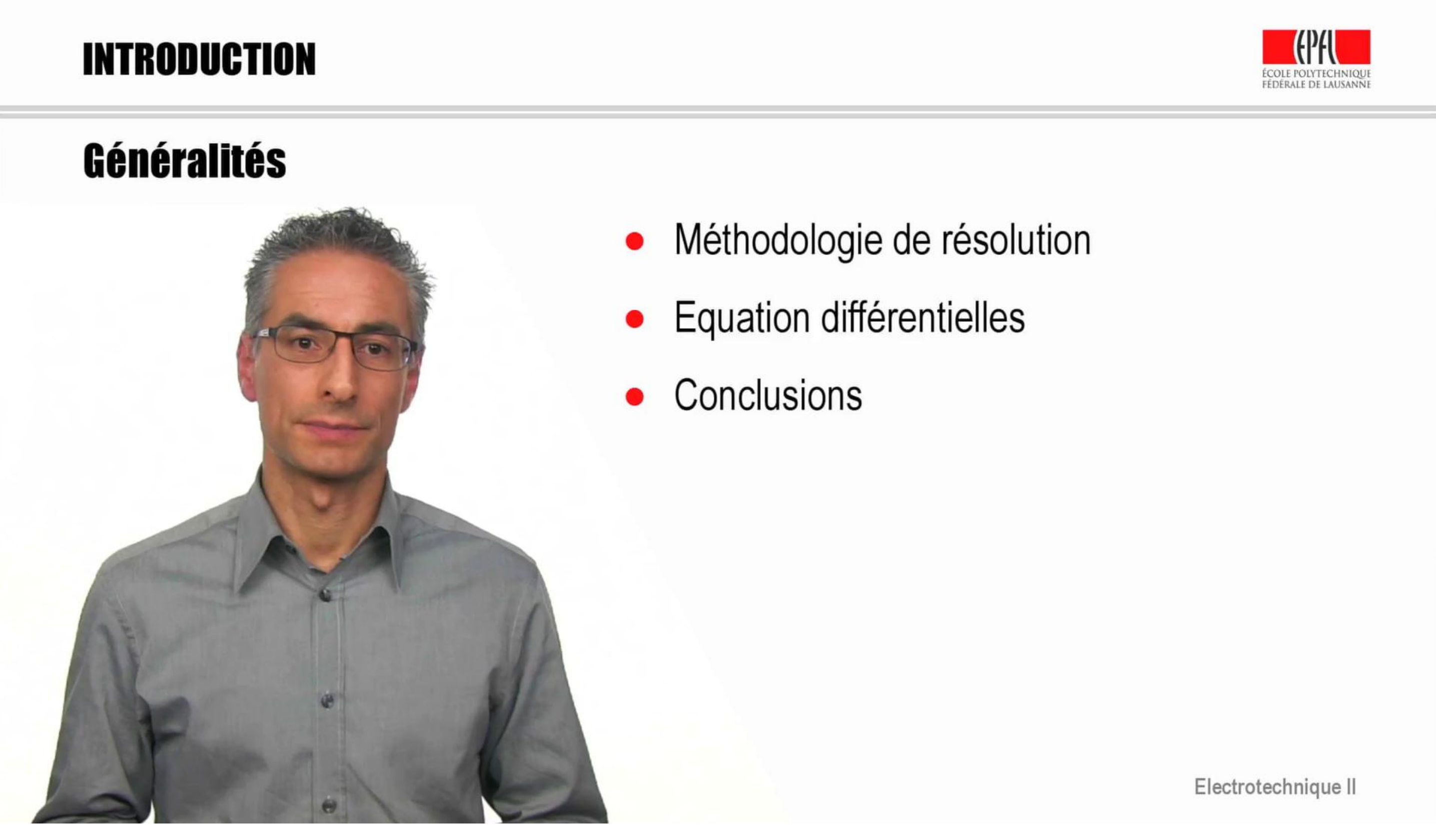
LEÇON 10

Électrotechnique II
Yves PERRIARD & Paolo GERMANO
Laboratoire d'Actionneurs Intégrés



Video

EPFL



Hello, during the previous lessons, we saw what transient states are and how they are mathematically defined We have also seen the fundamental physique principles that governs the ideal components R,L and C. A few applications showed the importance of those transient states. In reality the elements L and C can't exist alone, they are always combined by a parasite resistor a resistor also has a parasite capacitor and inductance. During this lesson, we will elaborate a resolution method to find the step response of a circuit including elements R,L or C subject to a jump. We will see, first of all, the mathematical fundamentals to solve differential equations derived from this method.

Notes

Summary

0m 03s

MÉTHODOLOGIE DE RÉSOLUTION

- **Traitement du schéma du circuit**
  - Dessin du schéma
  - Définition de toutes les grandeurs
  - Réduction du schéma
  - Equations de tension et de courant
  - Application des lois de Kirchhoff
  - Mise en forme des équations sous forme de somme de dérivées
- **Résolution du système d'équations différentielles**
  - Solution de l'équation sans second membre
  - Solution particulière (ou "régime permanent")
  - Solution générale
- **Détermination des constantes d'intégration**
  - Expression des impossibilités (saut de *i* dans L ou saut de *u* aux bornes de C)
  - Résolution du système correspondant
  - Expression finale de la solution

Electrotechnique II

The resolution method that we propose, conist in three main blocks First of all, the processing of the circuit sketch. Lets start by sketching. Then we define all the values that are on the drawing. Then reduce it, if necessary write down the voltage and current equations that is to say, the characteristic equation of each element R, L, C. And finally apply the Kirchhoff laws, that is to say, the laws of the bounds between each element derived from the formatting of the equations as the sum of derivatives. The resolution of the differential equations is done by the search of the solution of the homogeneous equation the search of the particular solution in steady state, a settled state and the sum of both which leads us to the general solution. The third block consists in defining the integration constants that is to say, the expression of the impossibilities either a current jump in an inductance, or a voltage jump of a capacitor terminal. Finaly, we solve the whole system to lead to the final expression of the solution.

Notes

Summary

0m 59s

EQUATIONS DIFFÉRENTIELLES

- LINÉAIRES, - DU 2ÈME ORDRE, - À COEFFICIENTS CONSTANTS

- **Définition**

$$a_0 x + a_1 \frac{dx}{dt} + a_2 \frac{d^2x}{dt^2} = f(t) \quad (1)$$

a_j : coeff. constants

$f(t)$: fonction temporelle, connue

- **Forme de la solution**

  - Solution sans second membre

  $$a_0 x + a_1 \frac{dx}{dt} + a_2 \frac{d^2x}{dt^2} = 0$$

  - solution particulière satisfaisant (1)

Electrotechnique II

The differential equations that we will deal with, are linear differential equations until the second order, and with constant coefficient. The differential equation is chararcterized by the sum of the derivatives of an unknown function x with constant coefficient. We can write a such equation, wher the coefficients a_j are constant coefficients and the function f(t) is as function of time which is known. The solution is the sum of two partial solution. The homogeounous solution that is given by a_0*x plus a_1 dx/dt plus a_2 d^2x divided by dt^2 is equal to 0. And the particular solution that satisfies the equation (1).

Notes

Summary

2m 17s

- LINÉAIRES, - DU 2ÈME ORDRE, - À COEFFICIENTS CONSTANTS

- **Solution de l'équation sans second membre**

$$x_s(t) = A \cdot e^{\lambda t}$$

$$a_0 + a_1 \lambda + a_2 \lambda^2 = 0 \quad \rightarrow \quad x_s(t) = \sum_{j=1}^{n} A_j \cdot e^{\lambda_j t}$$

- **Solution particulière**

① $f(t) = K$: constante $\quad \rightarrow \quad x_p = \frac{K}{a_0}$

② $f(t) = \hat{K} \cdot \cos(\omega t + \alpha) \quad \rightarrow \quad x_p = \hat{K}_p \cdot \cos(\omega t + \alpha_p)$

Electrotechnique II

The solution of the homogeneous equation is a sum of exponential functions of the form x_s(t) is equal to A times the exponential of λt. By replacing this expression, this homogeneous solution whose sum of derivatives of first order is equal to 2, it comes that after simplification by e^(λt) a such equation has n solutions, here, n is equal to 2 n solutions λ_j real or complex. The final solution of the homogeneous equation is therefore given by: For the particular solution, its form is directly linked to the function f(t). We will deal with two particular cases that are specially useful in this application and that are easy to solve. It is first of all a constant function f(t) = K. It is here a constant continuous voltage of a power supply. A second kind of time function. For a constant function the particular solution is constant and given by x_p where the p stands for particular and is equal to K on a_0. For a sinusoidal or cosinusoidal function, the solution is a function of the same kind.

Notes

Summary

3m 44s

EQUATIONS DIFFÉRENTIELLES

- LINÉAIRES, - DU 2ÈME ORDRE, - À COEFFICIENTS CONSTANTS

- **Solution résultante**

$$x(t) = x_s(t) + x_p(t) = \sum_{j=1}^{n} \underline{\underline{A_j}} \cdot e^{\underline{\underline{\lambda_j}}\, t} + x_p(t)$$

• pas de saut de i dans L

• pas de saut de u sur C

- **Propriété de l'exponentielle**

$$f_1(t) = A \cdot e^{-\lambda t} \qquad \xrightarrow{\text{pente à } t=0} \qquad \left.\frac{df_1(t)}{dt}\right|_{t=0} = -A\lambda$$

$$f_2(t) = A\left(1 - e^{-\lambda t}\right)$$

Electrotechnique II

The resulting solution is the sum of the homogeneous solution and the particular solution. Which gives us: The integration constants a_j and λ_j that are given by the initial conditions that are linked to physical impossibilities depending on inductances and capacitors, which mean that we can not do a current jump in an inductance and that we can not perform voltages jumps of the capacitor terminals. Applying these conditions lead to a linear system of equations allowing to define the coefficients a_j. We will now see a few properties of the exponential. An exponential with a negative exponent has one of the two following forms: f_1(t) is equal to A times e^(-λt) or f_2(t) is equal to A times 1 minus e^(-λt). If we consider the first function f_1(t) and that we define its initial slope it is equal to th derivative of f_1(t) as function of time at the time t=0 and this is equal to -Aλ.

Notes

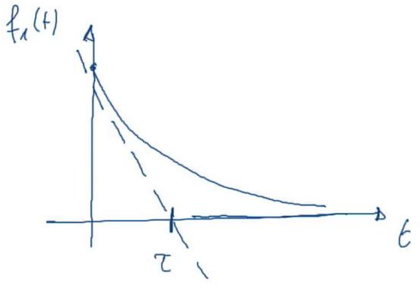
Summary

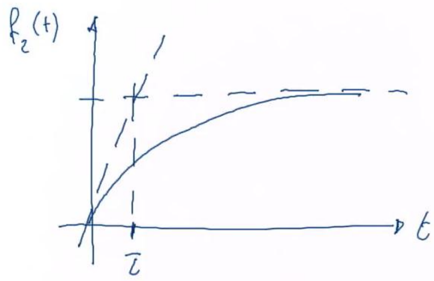
5m 44s

- LINÉAIRES, - DU 2ÈME ORDRE, - À COEFFICIENTS CONSTANTS

- **Propriété de l'exponentielle** (suite)

$0 = 1 - \lambda t_{asy}$ ou $t_{asy} = \tau = \frac{1}{\lambda}$: constante de temps τ





Electrotechnique II

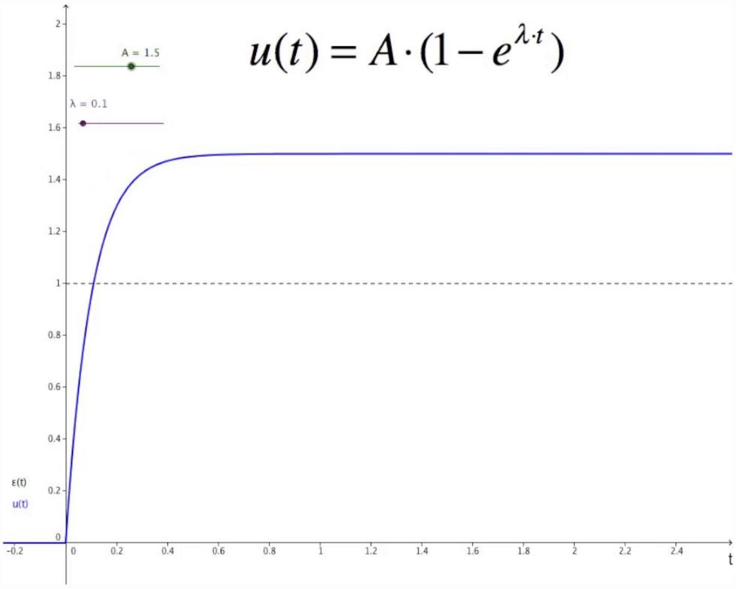
The intersection point between the asymptotic value, which mean the intersection of the time axis and its tangents, is given by the following equation. 0 = 1 - λ*t_asy or t_asy = τ = 1/λ. And this, is the time constant, that we call τ (tau). The graphical representation, gives us the following curve as function of time the function f_1(t), which is an exponantial but if we consider the slope origin its intersection with the asymptotic value which is the time axis is done at the time τ. For the second function that we have considered in the time domain here, the equation is very simple, the equation of the tangent is very simple because it goes through the origin. It's simply the slope the intersection with the asymptotic value, which is, the stabilized and permanent value of the system is done at the time t = τ.

Notes

Summary

7m 25s

Analyse de la fonction exponentielle - Temps



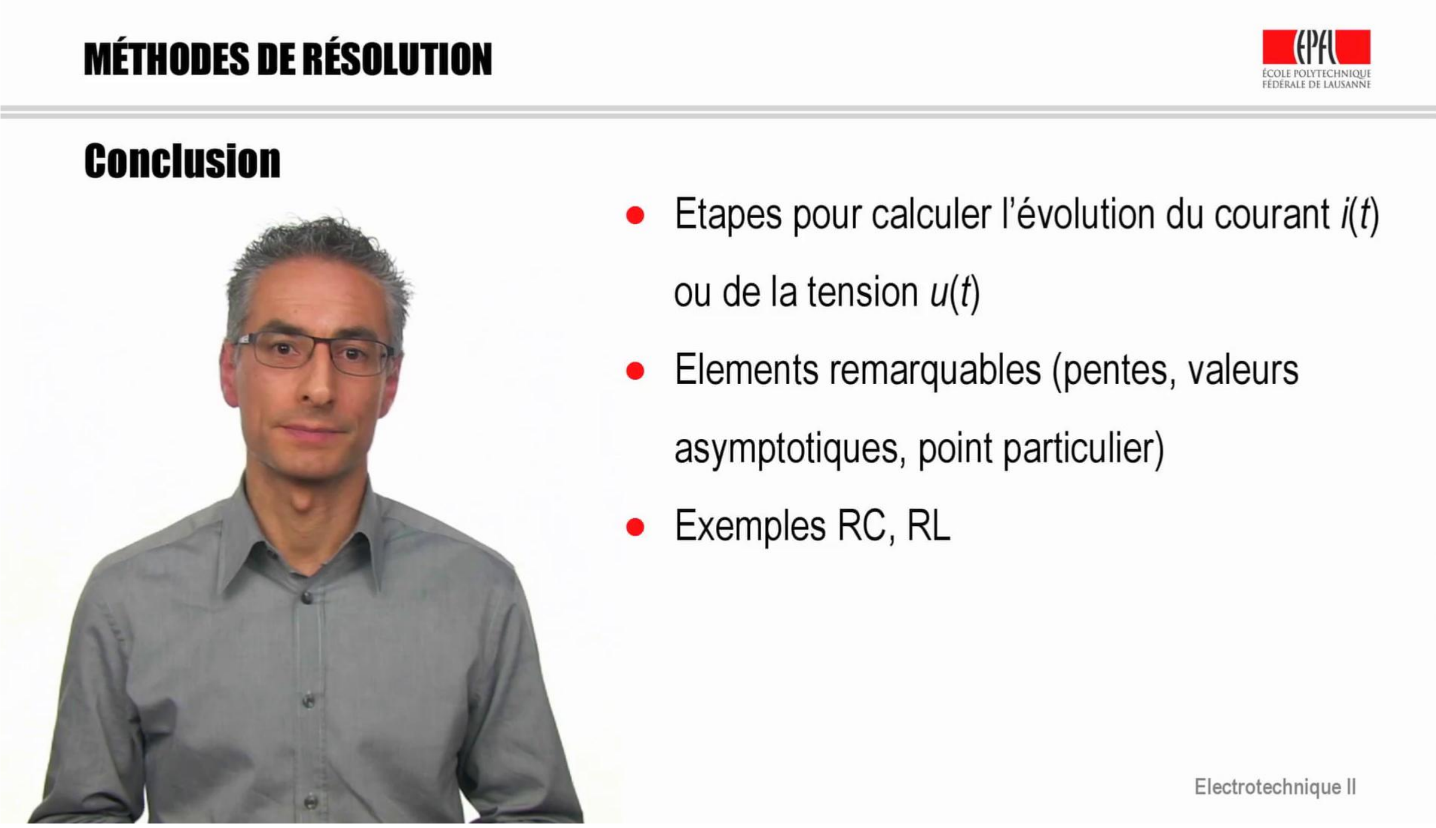
Electrotechnique II

If we picture a such equation like, for example, u(t) = A*(1-e^(λt)) and we vary the first parameter A we can see that the parameter A influences the amplitude of the signal. We vary A, we see that the amplitude of the signal Using the same equation and this time by varying the parameter A we see that the influence of this λ parameter doesn't effect the amplitude anymore but the speed of the rising of the curve till the asymptotic value. And it's for this reason that the λ is associated with the time constant.

Notes

Summary

9m 01s



During this lesson, we have seen the method to describe mathematically a circuit, and how to put it under an equation form, in this calculation purpose and the evolution of the current or the voltage at a particular spot of the circuit. Once the solution found, we have analyzed this solution and brought out some noteworthy elements as the slopes at the origin the asymptotic values, or particular points. During the following lessons, we will apply those methods on circuits of the type RL or RC.

Notes

Summary

9m 58s