RÉGIMES TRANSITOIRES

INTRODUCTION - SAUTS ET RÉPONSES INDICIELLES

LEÇON 8

Électrotechnique II
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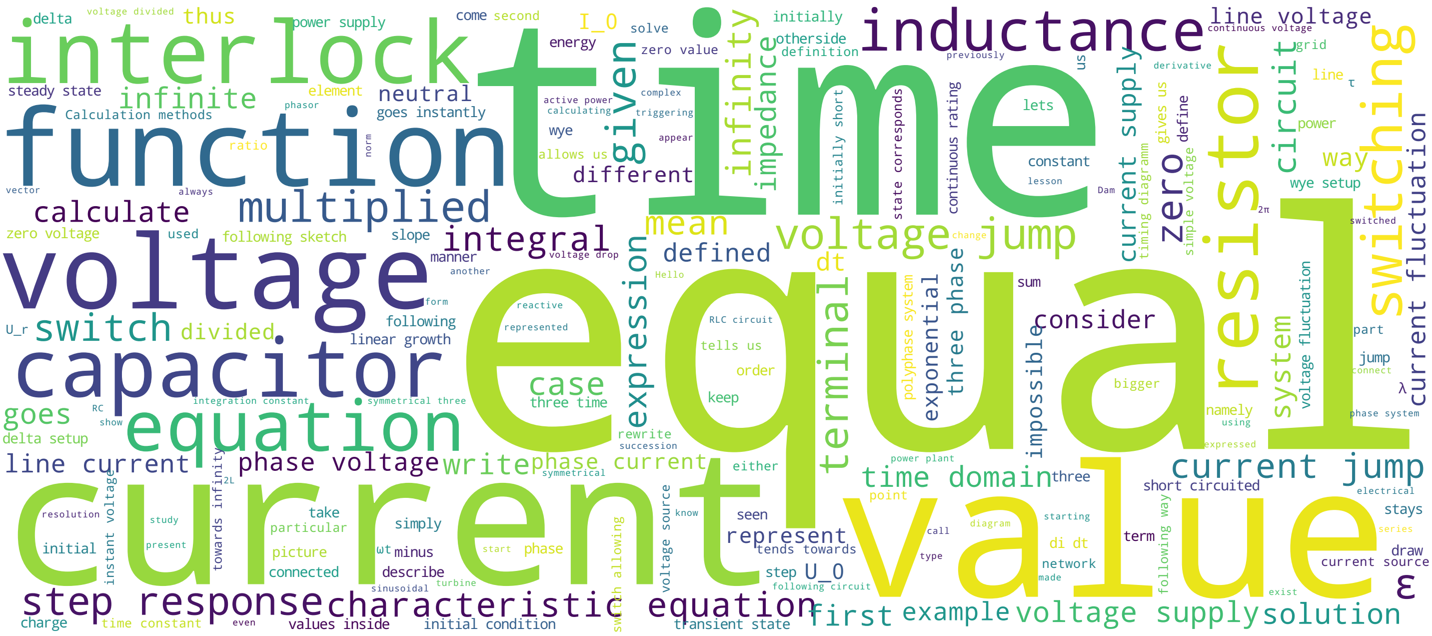
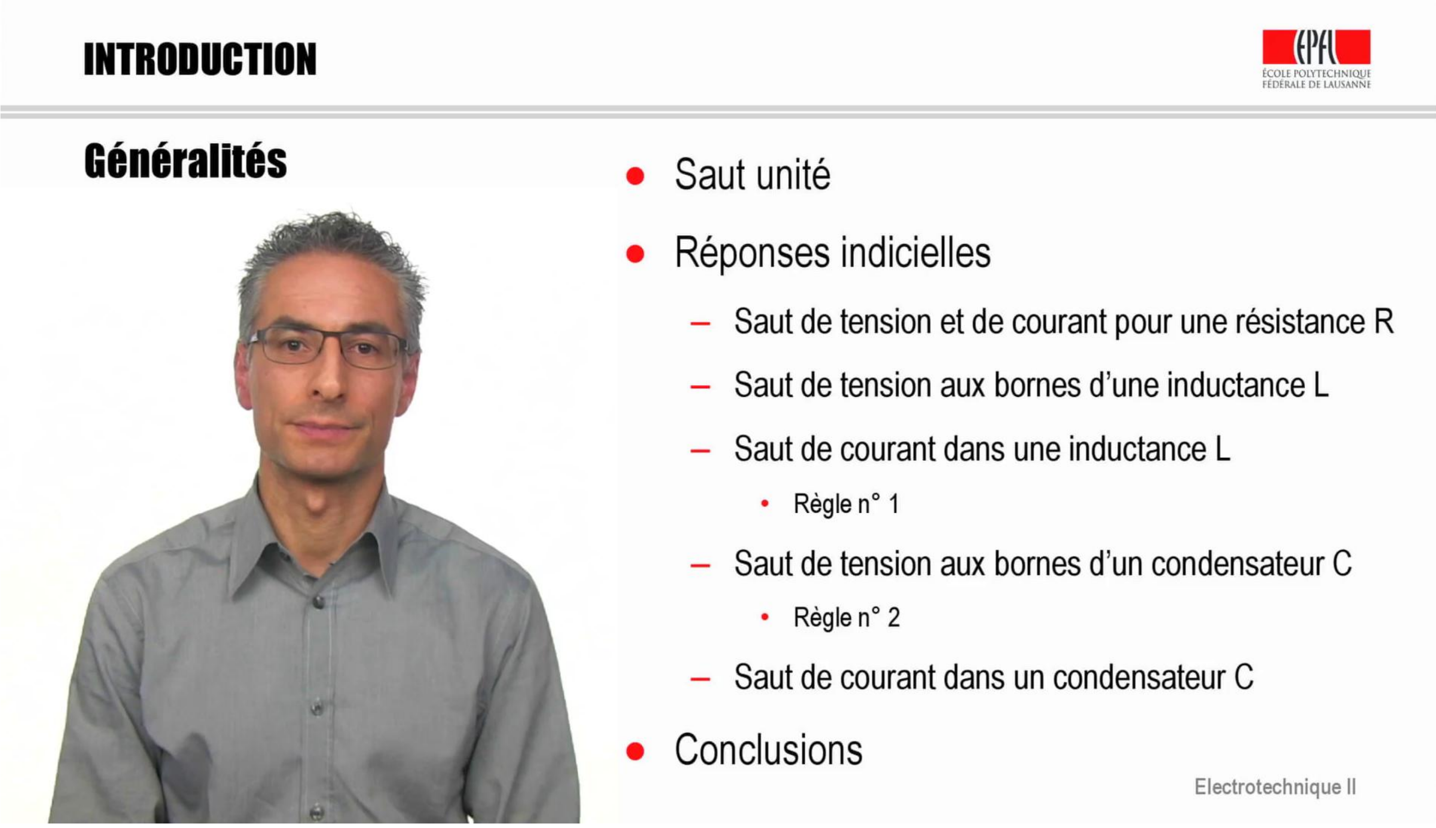


Video

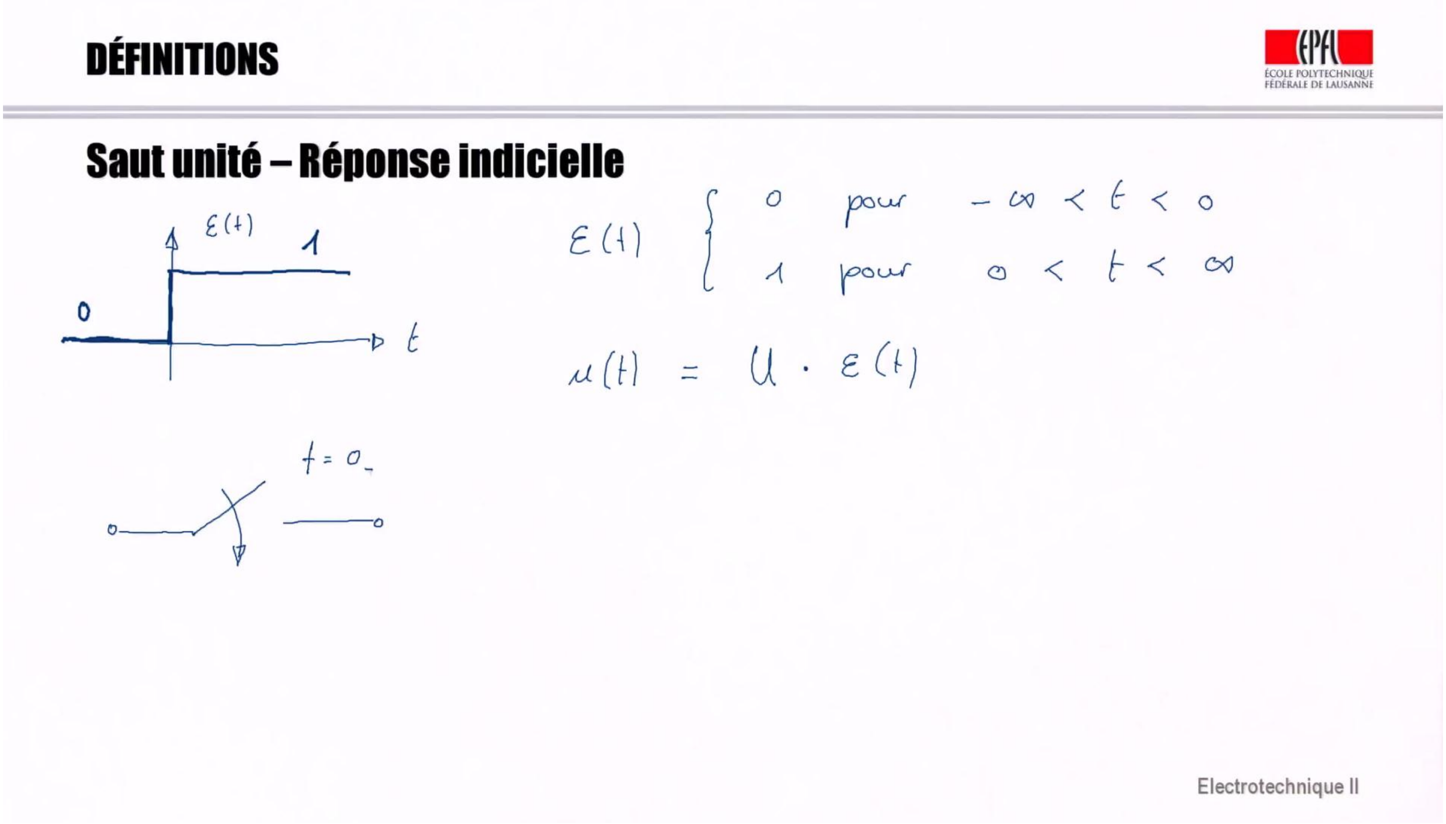


Hello, to this point, we have only dealt with steady state power supply in this electrotechnology class, which means that the excitations of the circuit, whether currents or voltage, are supposed to be established since an infinite time whether it is continuous rating which generates continuous currents and voltages, or sinusoidal steady state, which generates sinusoidal currents and voltages. Is defined as transient state every state change in a system which the corresponding disturbance is a length similar or inferior to the biggest time constant of the system. The most frequent case matches with the interlock or the triggering of the RLC circuit on its power supply. But it could also be a breakdown, a dysfunction or a particular functionning as those used in many common electronic devices. Calculation methods developed until today are not valid anymore because it is not a continuous rating nor an alternatif rating with only one frequency. New calculation methods will be established. Mathematically speaking, the interlock is transcribed by a function called "Out-of-Step". that we will name ε(t) which can take for value 1 or 0. We will see how does a resistor, a capacitor or an inductance behaves, if we send in a jump. We will now calculate their step response. We will set 2 rules to govern those phenomenons. Finally, we will present you a method to solve a given problem.

Notes

Summary

0m 04s

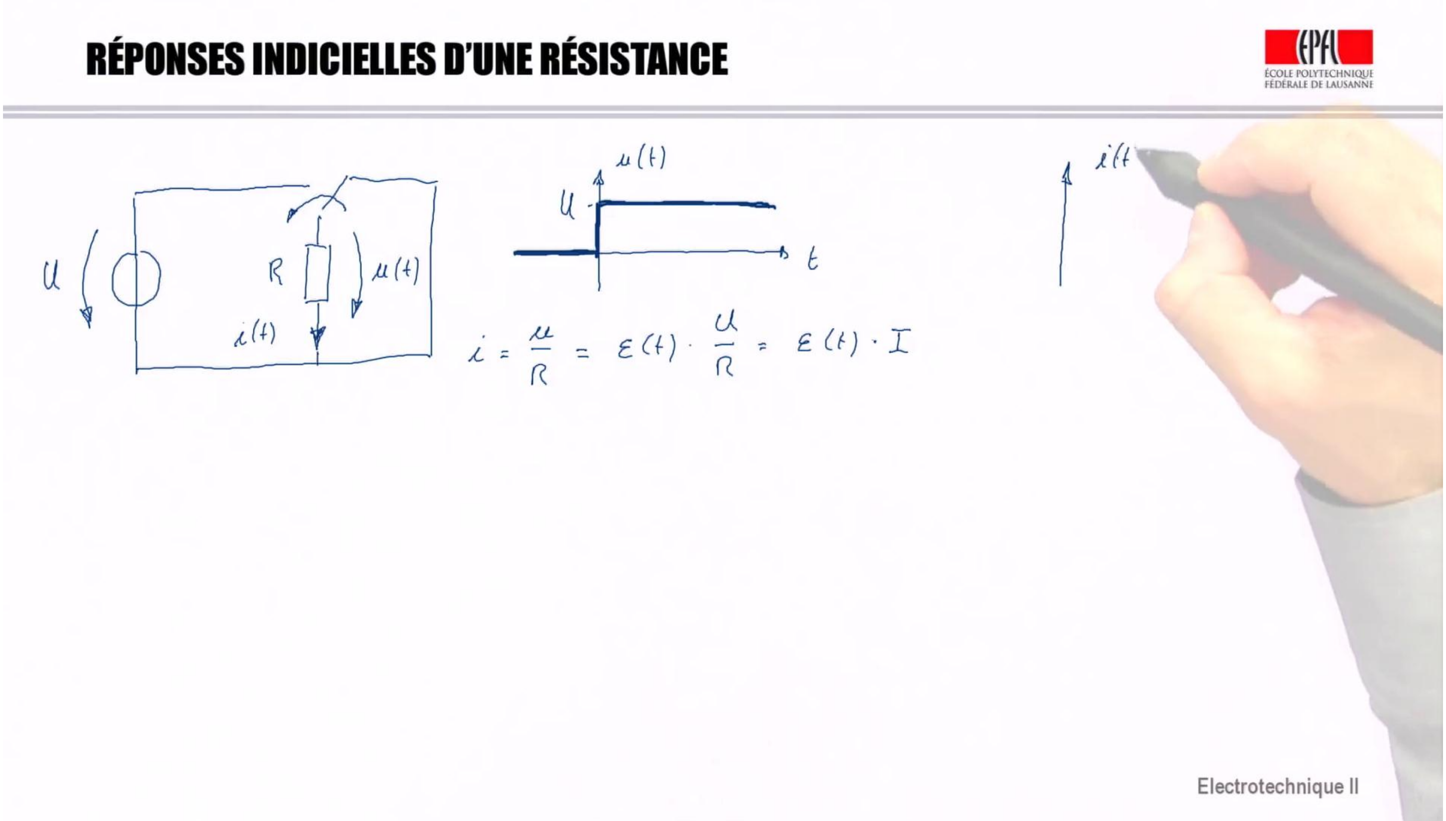


The out of step is defined as a time function. This function is called ε(t). Its value is zero for a time between minus infinity and 0. It goes instantly to the value 1 at the time t=0, and is equal to 1 for a time between 0 and infinity. The state 0 corresponds to a circuit triggered, and the 1 state corresponds to an interlocked cicuit. We can write the mathmatical relation of ε(t). It is equal to 0 for a time between minus infinite to 0. and it is equal to 1 for a time between 0 and infinite. A voltage jump. will be defined by the following relation the voltage as function of time is equal to the continuous voltage that we switch to the circuit multiplied by this function ε(t). We call step response, the response of a voltage jump or a current of the circuit Practically this jump will be made by a two position switch that we will represented this way. A switch is open, before the switching moment where t is equal to zero just before the switching. Lets look at the step response of linear elements known as R,L and C.

Notes

Summary

1m 43s



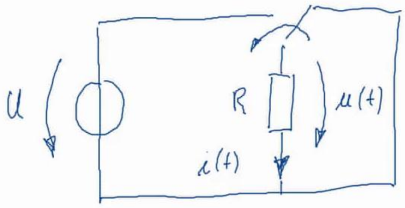
Now, let's calculate the step response of a resistor that is switched on a voltage supply. Let the following sketch be a voltage supply whith a value U A resistor R that is initially short-circuited on itself at a time t=0, we will switch it to the voltage supply U. Those are the following values inside the circuit the current that goes through the resistor i(t), and the voltage measured on the terminals of the resistor called u(t). In the time domain this representation of u(t) is given by the following graph. u(t) as function of time before the switching, the voltage is 0 it goes immediately to the value U, at t=0 and keeps this value U infinitely. The characteristic equation that describes the behavior of the resistor is given by i = u/R. We have determinated with the out-of-step the expression of u, that is equal to ε(t) mutliplied by U/R and this is simply equal to, ε(t)*I. This is the expression of the step response, i caused by the voltage jump of the resistor that we can also represent in the time domain the following way. i(t) as a function of t is equal to 0 before the interlock goes instantely to the value I and stays to this value after the interlock.

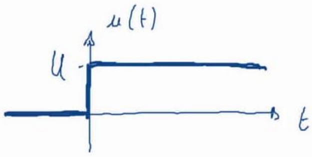
Notes

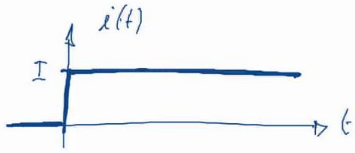
Summary

3m 48s

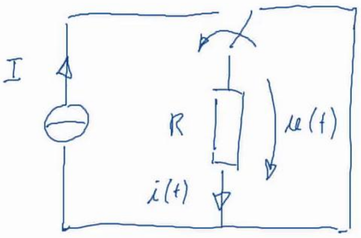
RÉPONSES INDICIELLES D'UNE RÉSISTANCE

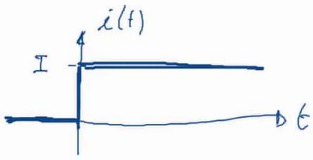


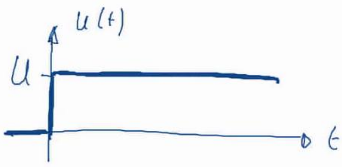




$$i = \frac{u}{R} = \varepsilon(t)\cdot\frac{U}{R} = \varepsilon(t)\cdot I$$







$$u = R\cdot i = \varepsilon(t)\cdot I\cdot R = \varepsilon(t)\cdot U$$

Le courant est l'image de la tension et reciproquement.

Electrotechnique II

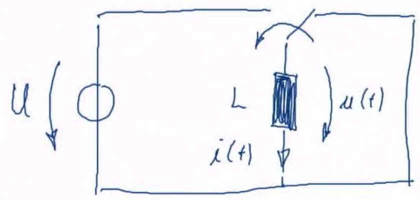
So we can see that the step response of a resistor to a current jump is also a current jump. We can use the following reasoning for a current jump we can represent the following sketch, a current source for value I that we switch to a resistor R. Initially the resistor is short-circuited on itself, so there is no current at t=0 we switch the resistor to the current source. The values inside the circuit, are again the current as function of time that goes through the resistor and the voltage drop doesn't appear on the resistor, u(t) Again, if we express the current as function of time, so in a time domain we get the following sketch as function of time i(t). We have no current before the interlock and a current that goes instantly to the value i when t=0 and stays at the value i. The characteristic equation is given by u what we are looking for is equal to R * i i is expressed with this concept of out-of-step ε(t) multiplied by i multiplied by R, and this is also ε(t) multiplied by U that we can picture again in the time domain the time axis of t is equal to 0 before the interlock and instantly takes the value u at the time t=0 and keeps this value. What we see now, is that the current is the image of the voltage, and vice-versa.

Notes

Summary

6m 11s

RÉPONSE INDICIELLE D'UNE INDUCTANCE À UN SAUT DE TENSION



$$u = L \cdot \frac{di}{dt}$$

$$i = \frac{1}{L} \int_{-\infty}^{t} u \cdot dt$$

Saut de tension : $u(t) = \varepsilon(t) \cdot U$

Condition initiale : $i(0) = I_0$

$$i = \frac{1}{L} \int_{0}^{t} U \cdot \varepsilon(t) \cdot dt + I_0$$

Résol. pour $t > 0$: $i = I_0 + \frac{U \cdot t}{L}$

Electrotechnique II

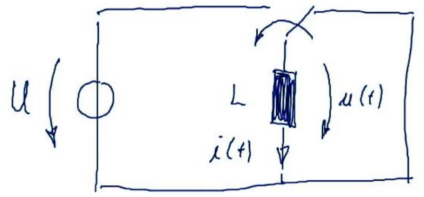
Now let's calculate the step response of an inductance to a voltage jump. We consider the following sketch: a voltage source for value U an inductance L that is short-circuited on itself initially a switch allowing the switching of the inductance on the voltage source at the time t=0. The characteristic equation of the voltage u(l) as function of the current i that goes through it is given by u = L *(di/dt). By integrating this equation we get that the current is equal to 1/L multiplied by the integral, from -infinity to t, of u(t)dt. In this case a voltage jump the voltage of the inductance as function of time, is equal to ε(t) * u. We consider the initial condition, which means, that the current that goes through the inductance at the time t=0 i(t=0) = I_0 This is a general case. So the characteristic equation becomes i and is equal to 1/L, multiplied by the integral taken from 0 to t. Which means that starting to the interlock of U multiplied by ε(t) multiplied by dt plus the initial current in the inductance, I_0. The resolution of this equation for t bigger than 0 gives us that i(t) is equal to I_0 plus (U*t) / L. We can see a linear growth of the current as function of time.

Notes

Summary

8m 36s

RÉPONSE INDICIELLE D'UNE INDUCTANCE À UN SAUT DE TENSION



$$u = L \cdot \frac{di}{dt}$$

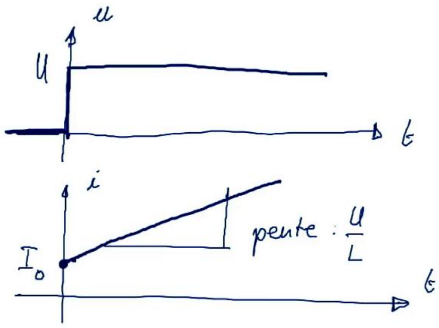
$$i = \frac{1}{L}\int_{-\infty}^{t} u \cdot dt$$

Saut de tension : $u(t) = \varepsilon(t) \cdot U$

Condition initiale : $i(0) = I_0$

$$i = \frac{1}{L}\int_{0}^{t} U \cdot \varepsilon(t) \cdot dt + I_0$$

Résol. pour $t > 0$: $i = I_0 + \frac{U \cdot t}{L}$



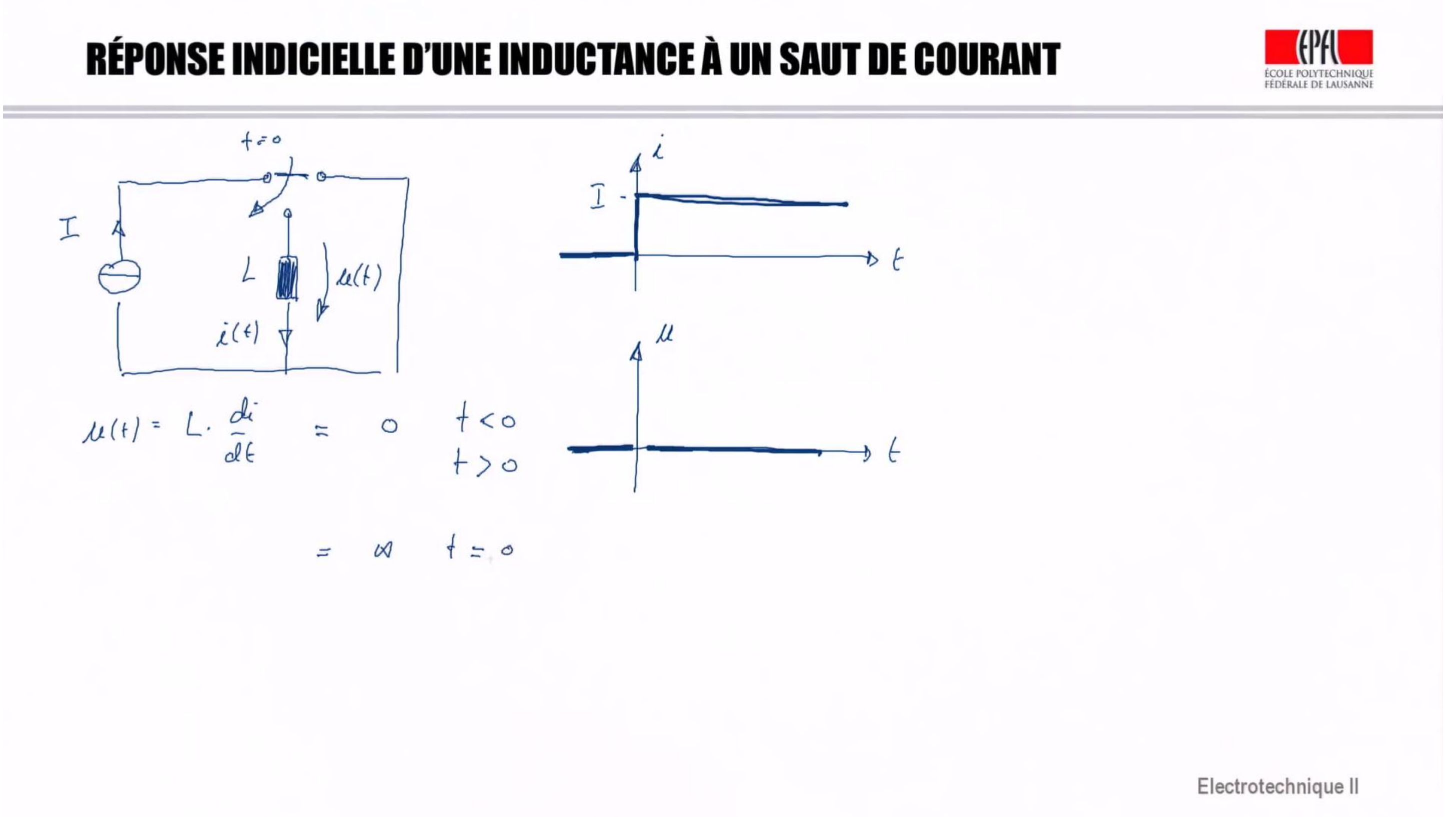
Electrotechnique II

Represented in the time domain we get for the voltage as function of time a zero voltage before the interlock an instant voltage jump at the time t=0, and a value that is equal to the value u(t) which is constant after the interlock. For the current the temporal representation starts at a value I_0, which is the initial condition, then a linear growth of the current according to the time whose slope is equal to U / L.

Notes

Summary

11m 03s

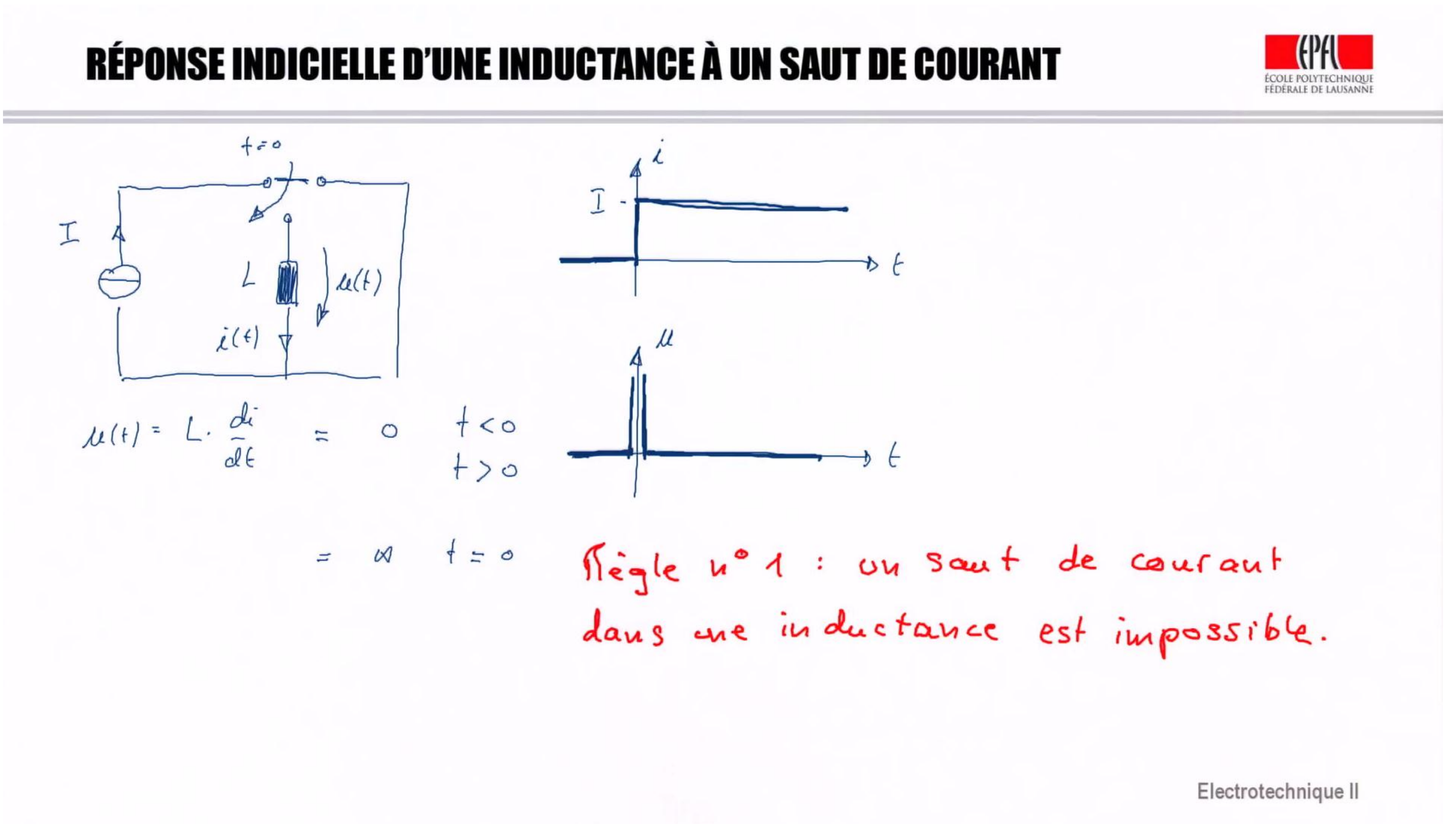


Now lets perform a current jump through an inductance. We consider the following electric circuit a current supply for value I an inductance for value L and a switch allowing the switching at a time t=0 of the current supply on the inductance The characteristic equation is given by u(t) = L*(di/dt). Warning u(t) is the current that goes through the inductance i(t). We can represent in this time domain the variation of t as function of time zero before the switching and instantly the current will get to the value i at the time t=0. We can see that the expression of u is equal to 0 when there is no current fluctuation. Which means that before the interlock, there is no current fluctuation, and after the interlock, there is no current fluctuation Which means that this expression is equal to zero for t < 0 or t > 0 that we can represent as function of time like this of t. Zero voltage before the interlock and a zero voltage after the interlock, because there is no current fluctuations. On the otherside this equation this voltage is infinite a the time t = 0 Why? Well, because the current fluctuation is infinite and so the voltage given by the current supply should be infinite.

Notes

Summary

11m 52s

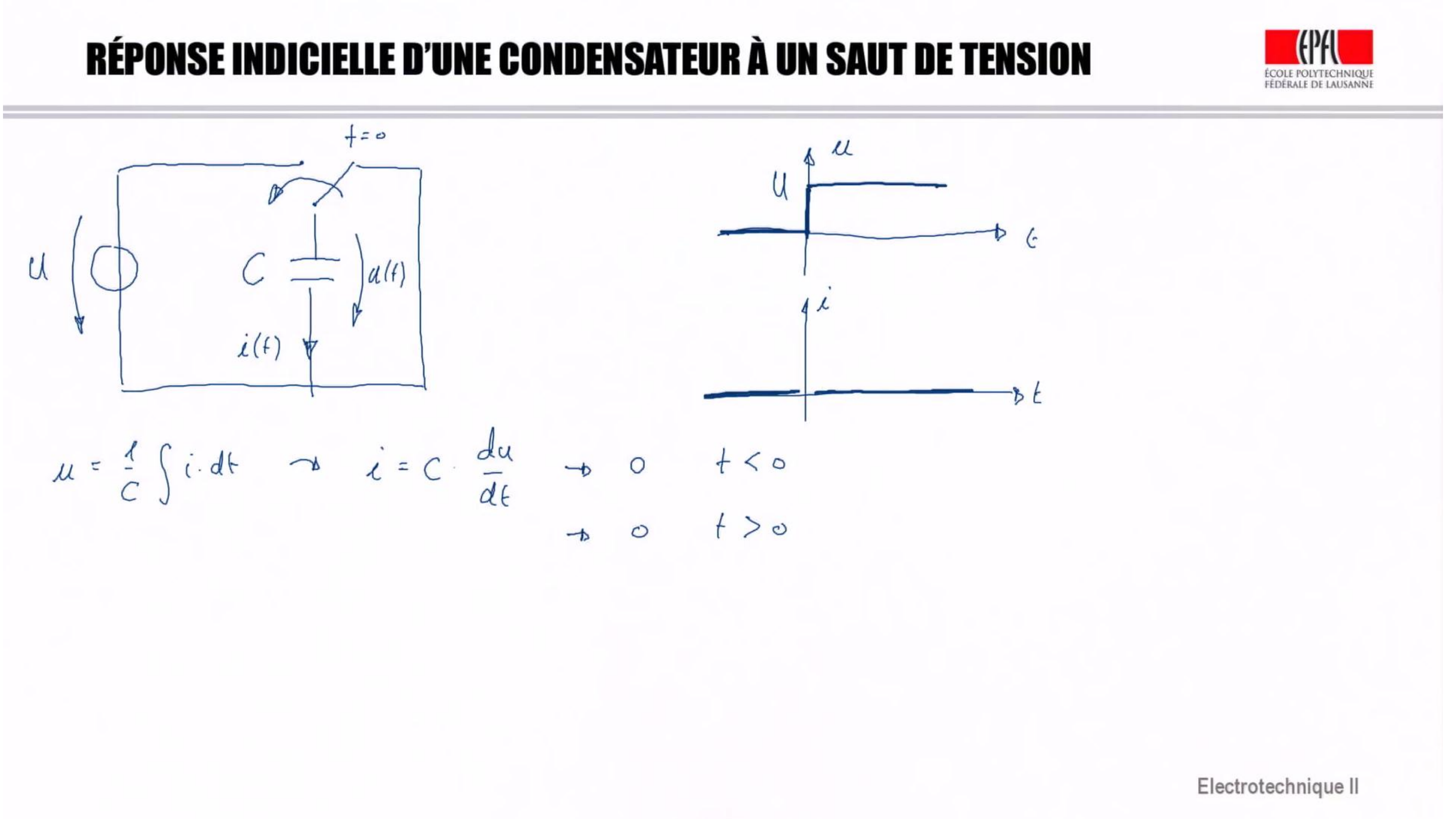


This function that we call "Dirac function" is not physically reachable in physics, the notion of infinity doesn't make sense, because infinity can never be reached. In math, there is no way to describe it easily. we come to a first rule which tells us that a current jump in an inductance is impossible Now, lets do a voltage jump u(t) = ε(t)*U on the terminals of a capacitor.

Notes

Summary

14m 04s

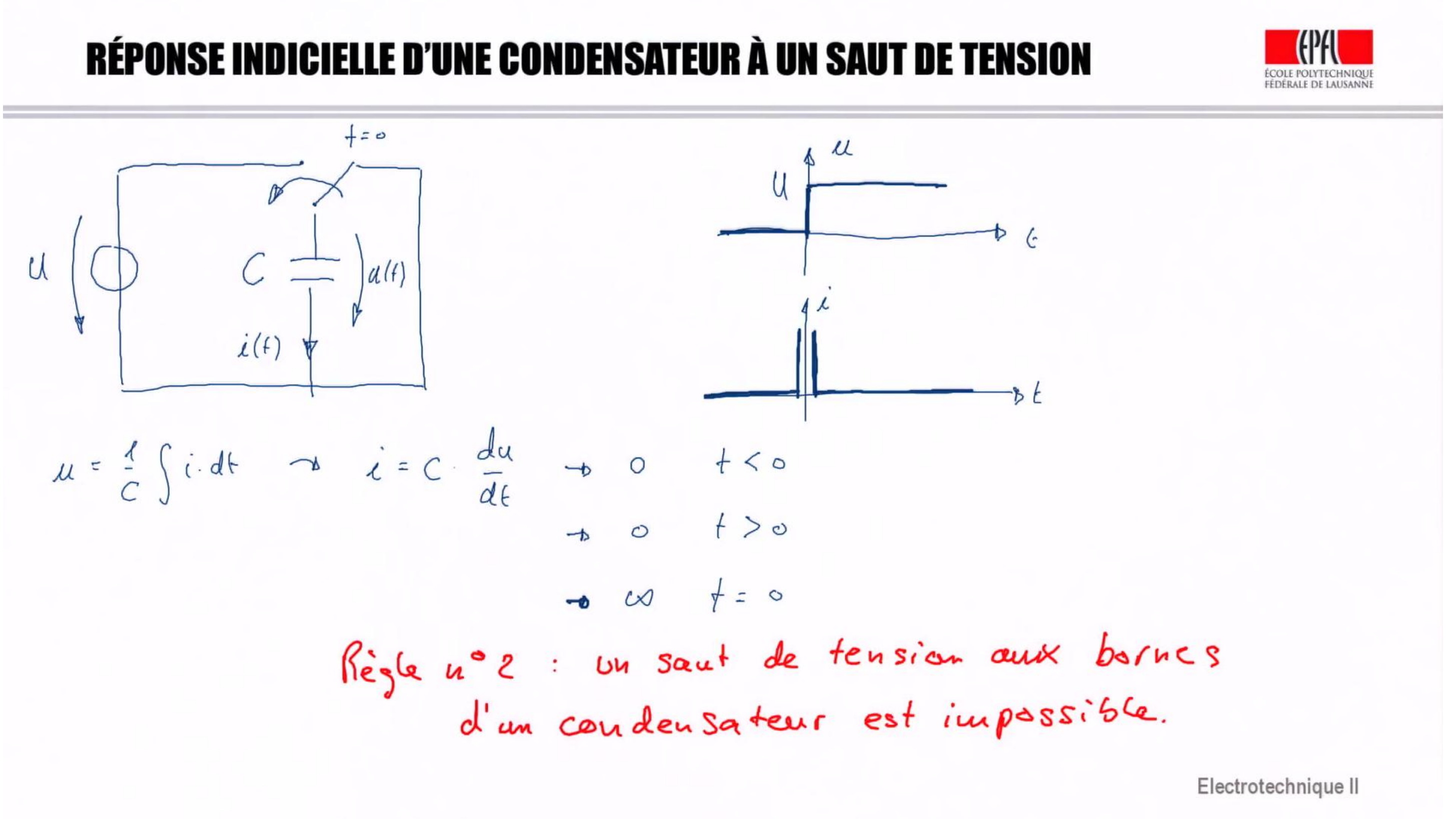


We considerate the following circuit, made of a voltage supply for value U a capacitor that is initially short-circuited on itself, and that we will switch thanks to a switch on the voltage supply U at a time t = 0. The values inside the circuit are the current that goes through the capacitor i(t) and the voltage at the terminals of the capacitor u(t). The characteristic equation of the capacitor is given by u = 1/C times the integral of i*dt. If we integrate this equation we get that i = C*(du/dt). This current takes the value 0 for t < 0 and also 0 for t > 0 because the voltage fluctuation is zero. The voltage is zero before the interlock and goes instantly to the value u at the time t = 0. If we draw a step response i, as function of time we have values that are zero before the interlock because there are no voltage fluctuation and also a zero value after the interlock because there is no voltage fluctuation. So the current is equal to 0 before the interlock and equal to 0 after the interlock. On the otherside during the transition, when we do a voltage jump on the capacitor this expression tends towards infinity and so the current tends towards infinity during the switching This means again that the voltage supply should give us an infinite current to do an instant voltage jump on the terminals of a capacitor.

Notes

Summary

14m 38s

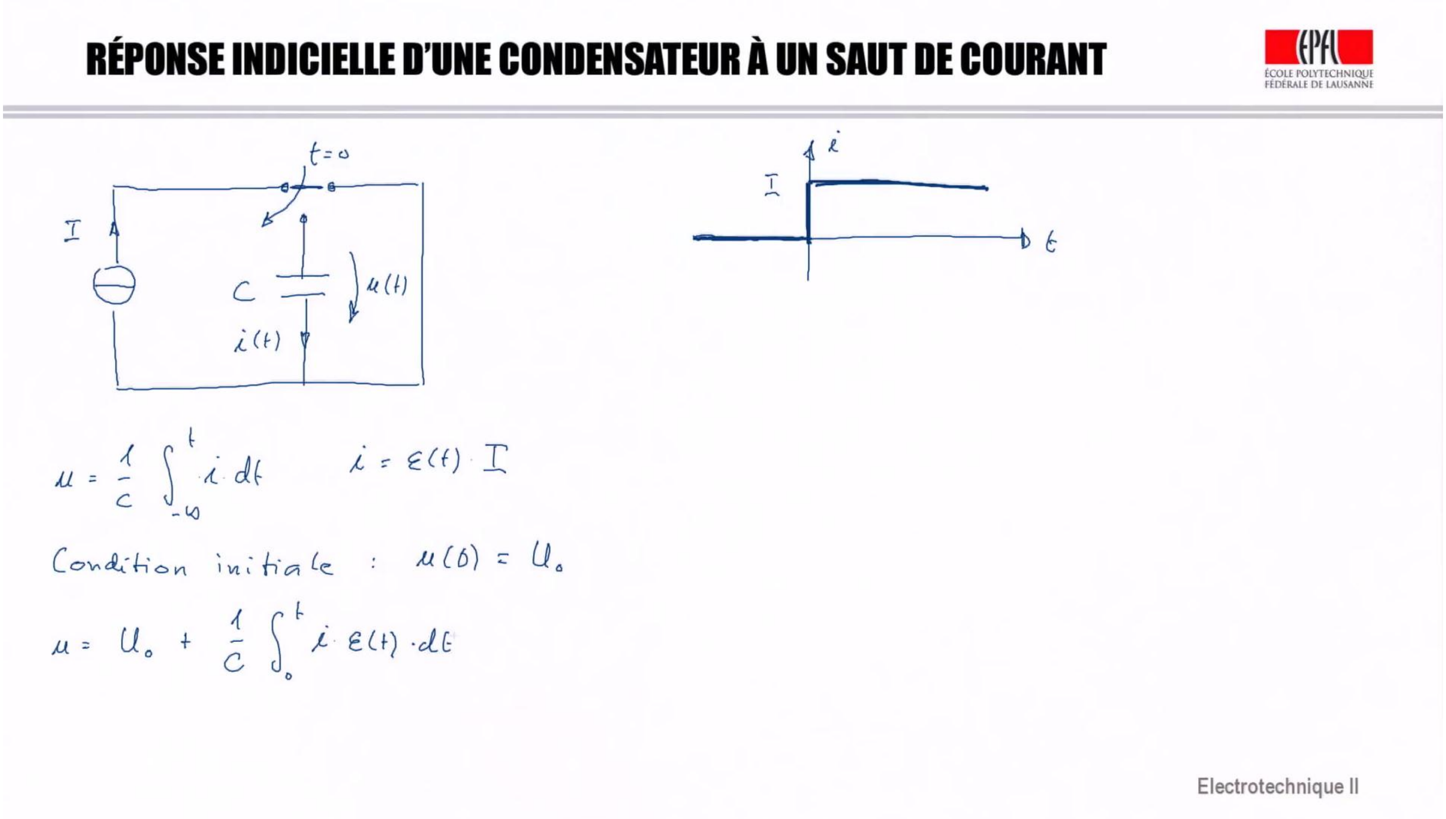


We get to a second rule which tells us that a voltage jump on the terminals of a capacitor is impossible.

Notes

Summary

17m 01s

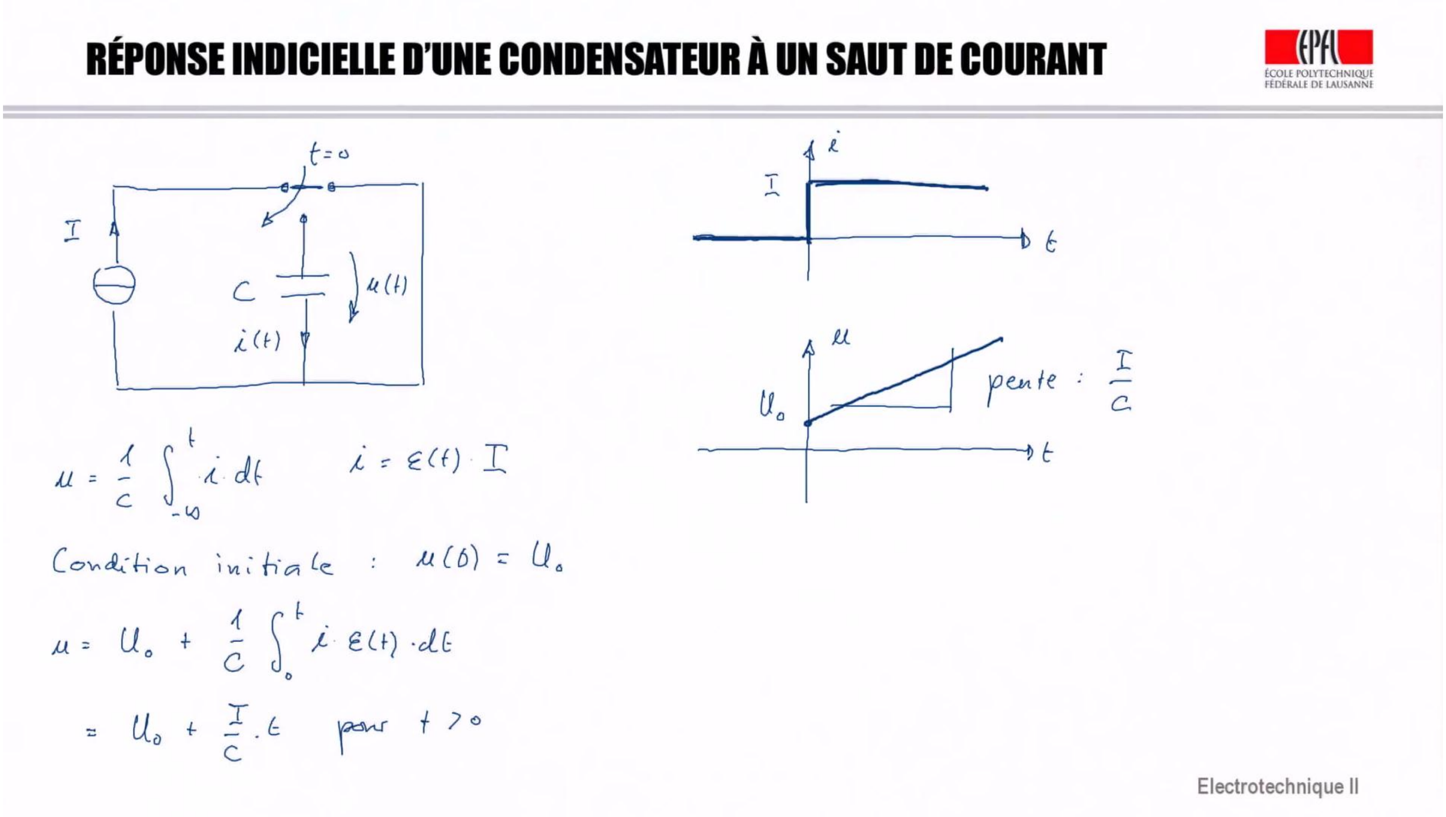


Now lets considerate a forth case which consists of calculating the step response of a capacitor subjected to a current jump. We consider again the following electronic sketch, made of a current supply equal to I a capacitor C and a switch that allows the switching of the capacitor on the current supply at a time t = 0. The characteristic equation of the voltage on the terminals of the capacitor as function of the current given by the expression U = 1/C times the integral to -infinity to t of i*dt. Knowing that i = ε(t) * I that we picture here, in a timing diagramm. A zero value before the switching, and a value i after the switching. Again, the initial condition is the volatge which exist on the terminals of the capacitor before the switching of the switch at the time t=0, and is equal to U_0. So we can now rewrite the equation with the integral by splitting this integral from -infinity to 0 and from 0 to infinity. So it comes that u is equal to U_0 which is the part of the integral before the switching plus 1/C times the integral from 0 to t of i*ε(t)dt. By solving this equation we get that the voltage is equal to U_0 plus (I/C)*t and this for t>0.

Notes

Summary

17m 10s

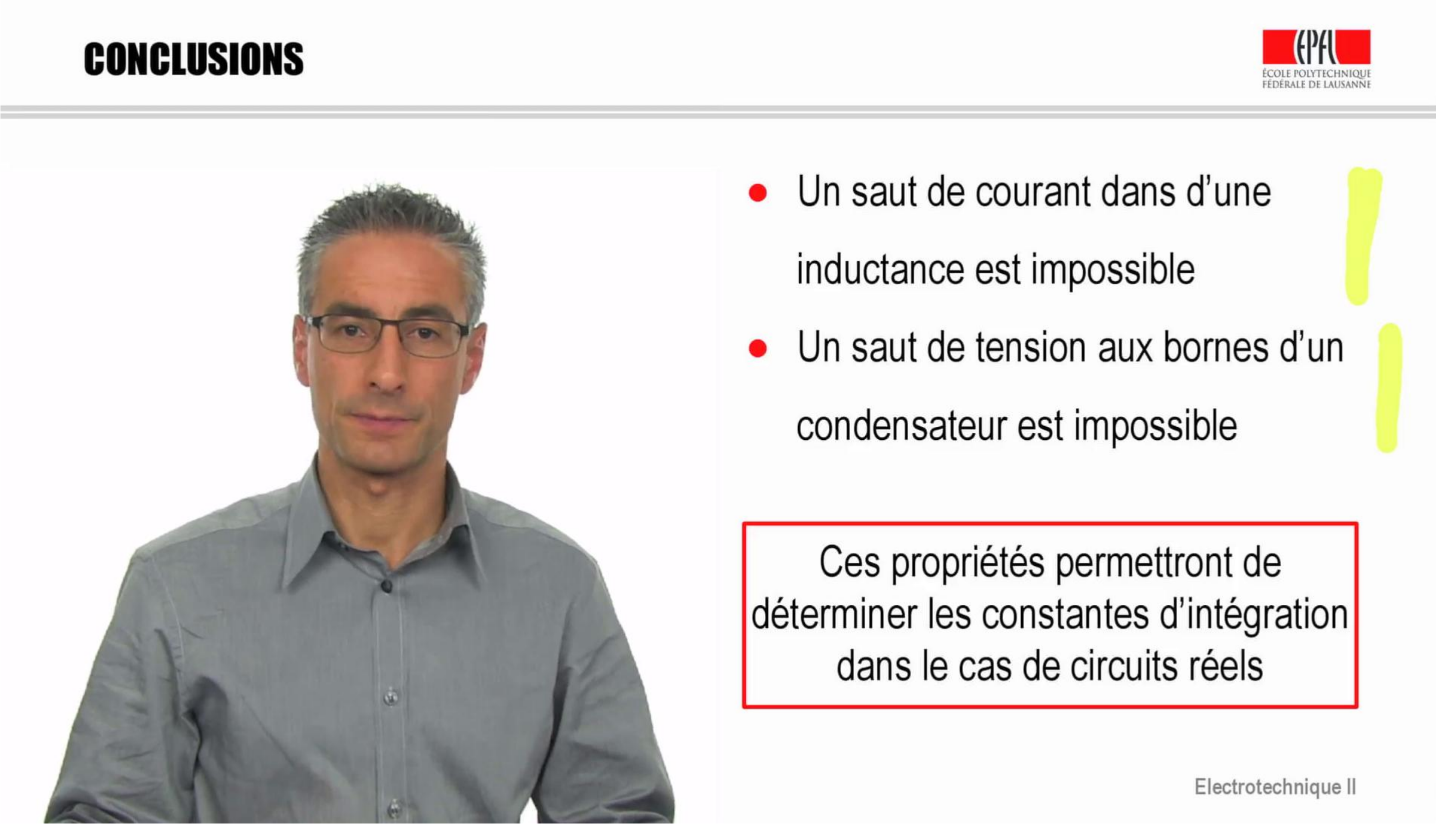


If we picture this equation in the timing diagramm we get the following curve, as function of time at an initial value before the switching, and after the switching we see that we have a straight line a linear growth of the voltage with the current which the slope is equal to I/C.

Notes

Summary

19m 18s



After studying all theses cases of step response we can draw the following conclusions which tell us that, a current jump in an inductance is impossible, and that a voltage jump at the terminals of a capacitor is also impossible. These propreties allows us to establish the integration constants in the case of a real circuit.

Notes

Summary

19m 47s