ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

RÉGIMES TRIPHASÉS

TRANSFORMATION TRIANGLE-ÉTOILE

LEÇON 6

Électrotechnique II
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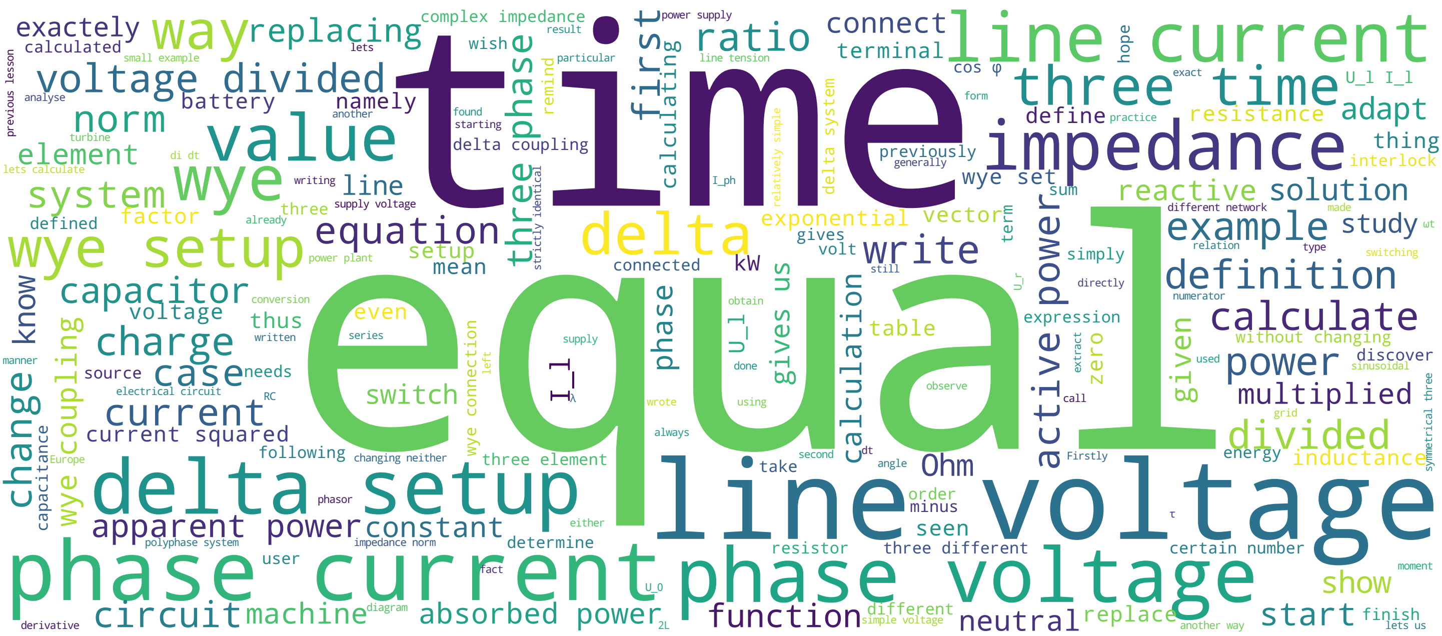
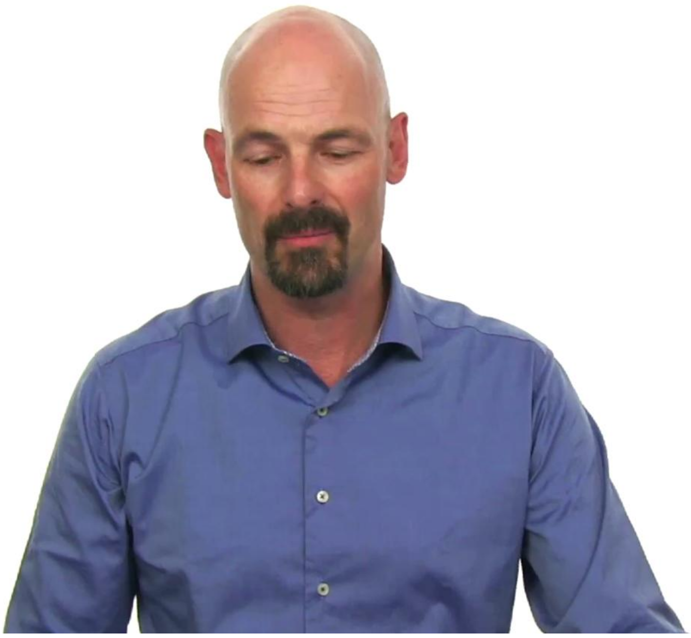


Video

INTRODUCTION



- Introduction
- Réduction de la puissance absorbée
- Adaptation à une tension plus élevée
- Transformation d'un montage triangle en un montage étoile équivalent
- Conclusion

Electrotechnique II

Hello, welcome to the 6th lesson, dedicated to conversions between wye and delta systems in a three-phase state, We will see how to adapt a network to a charge that we will be able to change to wye or delta, and adapt to a network having a different voltage or how to convert a wye set up into a delta set up.

Notes

Summary

0m 00s

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

a. Réduction de la puissance absorbée, sans changer ni la tension de ligne, ni les impédances de phase

Etoile (Y)	Triangle (Δ)

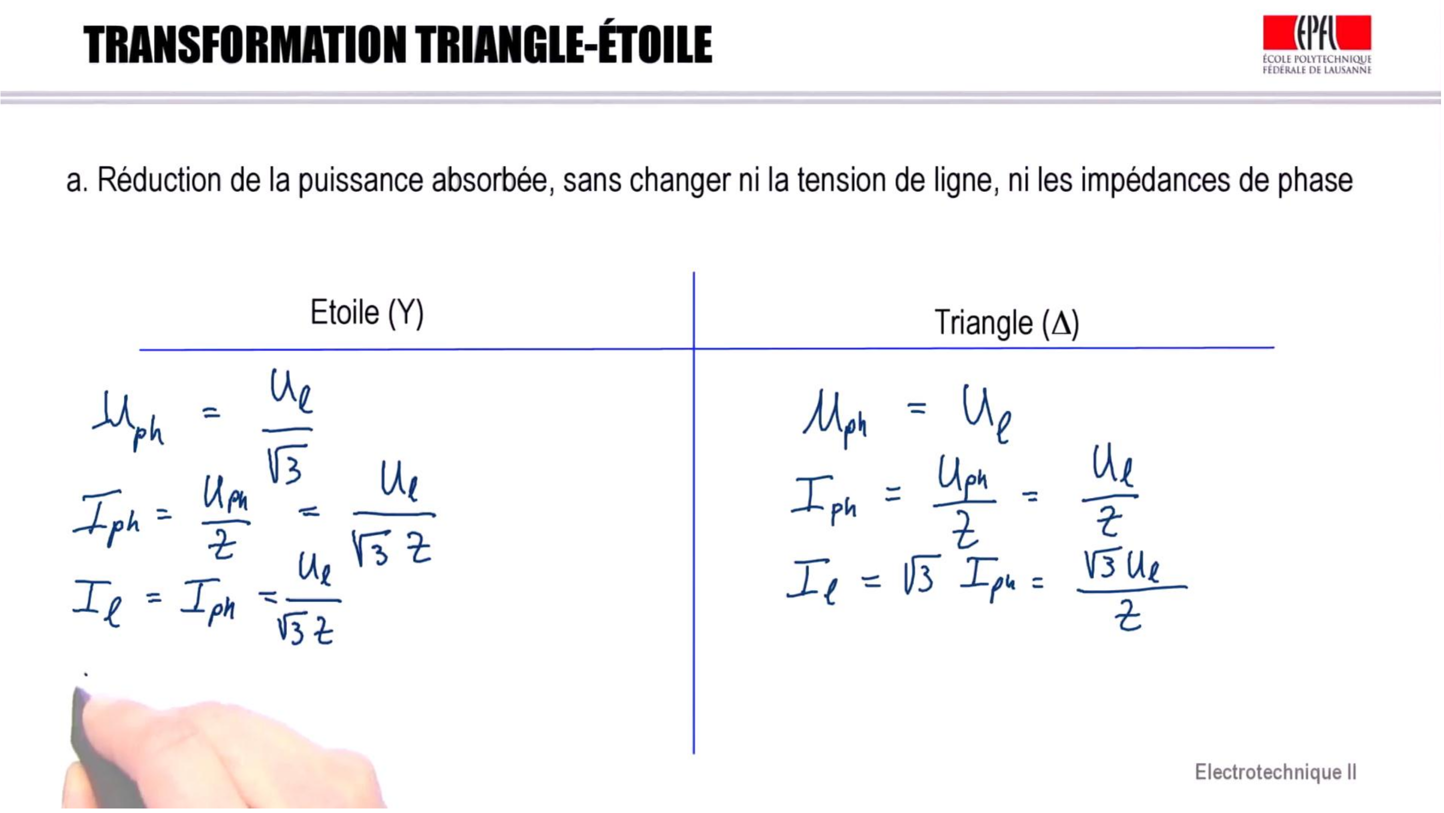
Electrotechnique II

In practice, if the sources are generally connected in a wye set up the user has two different choices for the charge's coupling mechanism. Either wye, or delta, and the switch between the two lets for a brief power reduction or an adjustement to an electrical circuit √3 times higher. Thus, we can say that in practice we have a wye connection and that this wye connection can be transformed into a delta connection. But, we can also have the opposite case and switch from delta to wye. We will now see three different cases that will show us this possible adjustement from one way to the other. Here is the first case that we will study: the power decrease absorbed without changing the line voltage nor the phase impedances. We will see how the power will change during the wye to delta circuit transformation and we will see that the factor change is equal to 3. I have prepared a table here, with on the left all the values that we will analyse in the wye as well as these same values in a delta. We will then start with the phase voltage, which is equal to the line voltage divide by √3 in a wye, as we have already seen in the previous lessons.

Notes

Summary

0m 22s



In the same way, this same voltage phase, but this time in a delta setup will be equal and confused with the line voltage. What is it with the phase currents ? The phase current, by definition, the voltage phase divided by the impedance Z We can now replace the phase voltage by the line voltage that we have just calculated, this gives us U_l/(√3*Z). For the delta setup, lets calculate the phase current and the phase current is equal to the phase voltage divided by Z, simply in our delta the phase voltage is equal to the line voltage, and so we get the following result : the phase current is simply the line voltage divided by the norm of the impedance Z. Lets calculate now the ligne current. The line current, I_I is equal to the phase current. It is a property of wye systems, so by replacing this, since we just found the value, it is the same result as before: line voltage divided by √3*Z for this line current. However, the line current in a delta system, as we know, is √3 times the phase current. By replacing this, we get √3 times the line voltage divided by the norm of the impedance Z. We want to analyse this impedance and compare it between the two setups.

Notes

Summary

2m 09s

TRANSFORMATION TRIANGLE-ÉTOILE

a. Réduction de la puissance absorbée, sans changer ni la tension de ligne, ni les impédances de phase

Etoile (Y)	Triangle (Δ)
$U_{ph} = \frac{U_\ell}{\sqrt{3}}$	$U_{ph} = U_\ell$
$I_{ph} = \frac{U_{ph}}{Z} = \frac{U_\ell}{\sqrt{3}\,Z}$	$I_{ph} = \frac{U_{ph}}{Z} = \frac{U_\ell}{Z}$
$I_\ell = I_{ph} = \frac{U_\ell}{\sqrt{3}\,Z}$	$I_\ell = \sqrt{3}\, I_{ph} = \frac{\sqrt{3}\, U_\ell}{Z}$
$S = \sqrt{3}\, U_\ell \cdot I_\ell = \frac{U_\ell^2}{Z}$	$S = \sqrt{3}\, U_\ell \cdot I_\ell = \frac{3 U_\ell^2}{Z}$

3x

Electrotechnique II

First of all, we will have S that is, by definition, equal to √3 times the line voltage, line current that is equal to, by replacing the line current with the previous calculation, simply U_l^2/Z for a wye setup. We do the exact same thing for the delta by replacing, in the same way, by definition, √3*U_l*I_l. When we replace this, the √3*√3 is at the numerator and we find 3*U_l^2 divided by Z. These two values, are then three times bigger. So, without changing neither the line voltage nor the charge that we connect when passing from one setup to the other the absorbed power of a factor 3 which gives the user all sorts of possiblities to adapt the power or, for example, to start a machine at a lower power than that of its nominal rate.

Notes

Summary

3m 51s

Exemple : Charge $Z = 18\ \Omega$ Triphasée

$U_\ell = 380\ V$ $\cos\varphi = 0{,}8$

$P_\Delta = 20\ kW$

$$\Delta \quad I_\ell = \sqrt{3}\, I_{ph} = \sqrt{3}\, \frac{U_{ph}}{Z} = \frac{\sqrt{3}\, U_\ell}{Z} = 36{,}6\ A$$

$$Y \quad I_\ell = I_{ph} = \frac{U_{ph}}{Z} = \frac{U_\ell}{\sqrt{3}\, Z} = 12{,}2\ A$$

Electrotechnique II

We will now do a small example of a charge that we take symmetric with an impedance norm equal to 18 Ohm. We decide that the electrical circuit's line voltage is equal to 380 V in this example with cos(φ) = 0.8. The active power, in delta, is equal to 20 kW. The question is now to know, if we connect this charge in delta instead of wye, do we have a reduction by a factor 3 on the absorbed power ? First of all, we calculate the line current in delta and this line current is equal to √3 times the phase current. The phase current is then the phase voltage divided by Z, which gives √3*(U_ph/Z) and the phase voltage in delta is the line voltage which is equal to √3*(U_l/Z). The line voltage, just as the impedance norm, are given in the example so we can do the numerical calculation and we get 36.6 A. On the contrary, if we now want to calculate this for a wye conversion, that we often symbolise with Y since it sounds like the letter Y, which is equal to the phase current in this setup. The phase current is equal to U_ph/Z and the phase voltage in a wye is: U_l/(√3*Z). We get 12,2 A. We can already see here that the consumed current is decreased by a factor 3. We can then calculate the power in a wye set up.

Notes

Summary

5m 12s

Exemple : Charge $Z = 18\ \Omega$ Triphasée

$U_\ell = 380\,V$ $\cos\varphi = 0{,}8$

$P_\Delta = 20\ kW$

$$\Delta \quad I_\ell = \sqrt{3}\, I_{ph} = \sqrt{3}\, \frac{U_{ph}}{Z} = \frac{\sqrt{3}\, U_\ell}{Z} = 36{,}6\ A$$

$$Y \quad I_\ell = I_{ph} = \frac{U_{ph}}{Z} = \frac{U_\ell}{\sqrt{3}\, Z} = 12{,}2\ A$$

$$P_Y = \sqrt{3}\, U_\ell \cdot I_\ell \cdot \cos\varphi = 6{,}6\ kW = \frac{P_\Delta}{3}$$

Electrotechnique II

This calculation is relatively simple, we use the definition of the active power, namely √3 line voltage, line current times cos(φ). By replacing these three elements that we have now, either from the data or from the calculations that we have just made, we get 6.6 kW. This is equal to three times the active power that we had in a delta. This can be written in another way. This is equal to the power divided by 3, and shows the adaptation possiblity between a wye and delta setup.

Notes

Summary

7m 41s

b. Adaptation à un réseau plus élevé sans changer ni la puissance, ni les impédances

Etoile (Y)	Triangle (Δ)
$U_{ph} = \frac{U_l}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220V$	$U_{ph} = 220V = U_l$
$I_{ph} = \frac{U_{ph}}{Z}$	

Electrotechnique II

The second case that we will now study is the adaptation to a higher network, without changing neither the powers nor the impedances. This can happen when a machine has been designed or calculated for a given network, and that by changing continent or country, we have a different network voltage that we wish to adapt to the machine of this new network. Then, in the same way, I have created a table that lets us compare a wye or delta setup, and then we will take the netwrok we have in Europe, the classical network, in a wye or delta setup, with a line voltage of 380 V. Lets imagine our phase voltage, that is equal to the line voltage divided by √3 in a wye setup, will be equal to 380 V divided by √3 namely the 220 V that we usually see. Lets admit that we want to connect our machine on a phase voltage, once again at 220 V, but that will then be our line voltage since we connect the same charge in a delta setup. In such a setup, as a reminder, the phase voltage is equal to the line tension. Our phase current, in the wye setup, will be equal to the phase voltage divided by the norm of the impedance. The phase voltage being equal to 220 V, it is simply 220 divided by the norme of Z.

Notes

Summary

8m 42s

TRANSFORMATION TRIANGLE-ÉTOILE

b. Adaptation à un réseau plus élevé sans changer ni la puissance, ni les impédances

Etoile (Y)	Triangle (Δ)
$U_{ph} = \frac{U_\ell}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220\,V$	$U_{ph} = 220\,V = U_\ell$
$I_{ph} = \frac{U_{ph}}{Z} = \frac{220}{Z}$	$I_{ph} = \frac{U_{ph}}{Z} = \frac{220}{Z}$
$S = 3\,U_{ph} \cdot I_{ph} = 3\,\frac{220^2}{Z}$	$S = 3\,U_{ph} \cdot I_{ph} = 3\,\frac{220^2}{Z}$

Electrotechnique II

What about the phase current in a delta setup ? Well, in the same way, it is the phase voltage divided by the norm of the impedance Z, and we realise that it is exactely the same thing. This is what we hope for. We hope, as said beforehand, not to change the absorbed power with the charge. We can also be convinced by calculating the apparent power that will be equal to, by definition, three times the phase voltage, times the phase current and in both cases, we will get three times 220^2 divided by Z, and if we look at a delta setup, we will get exactely the same thing: three times the phase voltage times the phase current, which gives us 3*(220^2)/Z Then, we see that the devices that work on an old network for example 220 V in a delta setup, will be able to be used on a modern network with 380 V line voltage, providing that we pass in a wye setup.

Notes

Summary

10m 13s

Ancien réseau de 220 V en Δ

$\longrightarrow$ peuvent être utilisé sur un nouveau réseau $U_\ell = 380\,V$ mais passage au mode étoile λ

Notes

Summary

11m 08s

TRANSFORMATION TRIANGLE-ÉTOILE

c. Remplacement de la charge triphasée pour obtenir la même puissance lorsque la tension de ligne ne change pas

Etoile (Y)	Triangle (Δ)
$I_\ell = \frac{S}{\sqrt{3}\, U_\ell}$	$I_\ell = \frac{S}{\sqrt{3}\, U_\ell}$
$I_{ph} = I_\ell$	$I_{ph} = I_\ell / \sqrt{3}$

Electrotechnique II

The last case that we will study is the replacement of the three-phase charge to get, this time, the same power when the line voltage does not change. This time, we will change the impedance to keep the power absorbed by the system. In the same way, in the table we will have on one side all the values of a wye setup, and on the other all the values of a delta setup. The calculation of the line voltage is still done in the same way but this time we start with the apparent power, that I remind you needs to be constant in our small example, S and then we have a relation between the line current, the apparent power and the line voltage, in this way. This ratio, or this equation, is strictly identical in a delta setup. We can then write that I_I is also equal, in a delta setup, to S/√3 times the line voltage. For the phase current, now, we are starting to have a certain habit is equal to the line current in the wye setup However, for this same phase current in a delta setup we have a ratio of √3 between the line current and the phase current. It is exactely the opposite for the phase voltage, so here the phase voltage is equal to the line voltage divided by √3.

Notes

Summary

11m 31s

c. Remplacement de la charge triphasée pour obtenir la même puissance lorsque la tension de ligne ne change pas

Etoile (Y)	Triangle (Δ)
$I_\ell = \frac{S}{\sqrt{3}\, U_\ell}$	$I_\ell = \frac{S}{\sqrt{3}\, U_\ell}$
$I_{ph} = I_\ell$	$I_{ph} = I_\ell / \sqrt{3}$
$U_{ph} = \frac{U_\ell}{\sqrt{3}}$	$U_{ph} = U_\ell$
$Z_Y = \frac{U_{ph}}{I_{ph}} = \frac{U_\ell}{\sqrt{3}\, I_\ell}$	$Z_\Delta = \frac{U_{ph}}{I_{ph}} = \frac{U_\ell \cdot \sqrt{3}}{I_\ell}$

$$Z_\Delta = 3\, Z_Y$$

Electrotechnique II

Whereas in the delta setup the phase voltage is equal to the line voltage. All this now lets us, by reassembling these three element, to write that the impedance that we wish to get to maintain and garantee a constant absorbed power. We will then write here that Z in a wye setup that is equal to the phase voltage, by definition on the phase current will be equal to the line voltage divided by √3 times the line current. We calculate exactely the same value, Z but this time for a delta setup. We can write, by definition, that it is the phase voltage divided by the phase current, and by replacing with the elements that we have calculated, (U_l*√3) / I_l. What do we see ? We see that the ratio between Z in a delta setup is equal to three times Z in a wye setup. The ratio can be seen here, you have a ratio of √3 in the numerator of the impedance in a delta setup. It is at the denominator for the impedance in a wye setup. The ratio between the two gives exactely a ratio of 3 between these 2 impedances to get the same power when the voltage does not fluctuate.

Notes

Summary

13m 05s

TRANSFORMATION TRIANGLE-ÉTOILE

$$Z_\Delta = 3 Z_Y \equiv \underline{Z}_\Delta = 3 \underline{Z}_Y$$

Cas d'un batterie de condensateur $\frac{1}{C_\Delta \omega} = \frac{3}{C_Y \cdot \omega}$

$$\Rightarrow C_\Delta = \frac{C_Y}{3}$$

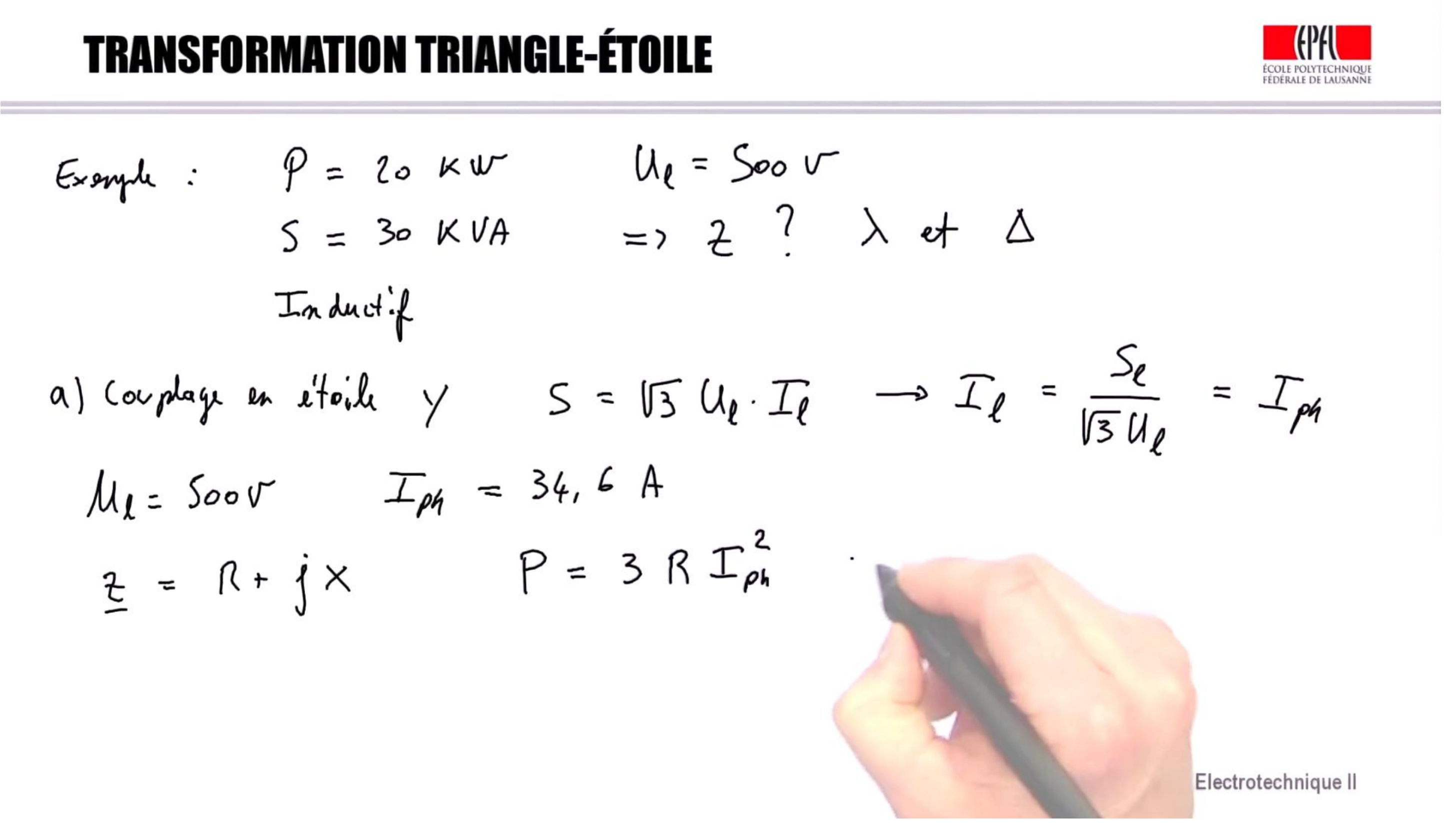
Electrotechnique II

We can even go further and write that the norm that we wrote before, as being the norm in a delta setup equal to three times the norm in a wye setup. We can show that this is also valid for the complex value which means that the complex impedance Z the vector, is equal to three times the complex impedance Z in a wye setup. And then, of for example, we have a battery of capacitors, this battery of capacitors has a reactance equal to 1/(cω). We can write that at this moment, by multiplying by three to get a battery of capacitors this time in a delta setup, times ω. Then, we can say that the capacitance that needs to be chosen in a delta connexion mode will be equal to the capacitance of a wye mode, divided by 3. The ratio 3 exists in this case. We will now do an example, to show you how this can be done in a complete way with a complex impedance and we will observe this ratio 3.

Notes

Summary

14m 52s



For this example, we will then take a symetrical three-phase consumer, whose active power is equal to 20 kW and whose apparent power is equal to 30 kVa. The supply voltage of a line is equal to 500 volts. We ask to calculate the unknown impedance Z in a wye and delta setup. Knowing that we have this inductive system. We then start with a wye coupling. In a wye coupling, we can calculate the apparent power. We start by calculating S. S is equal to √3 times the line voltage, times the line current. We can then extract the line current from this equation, which is equal to S_l / (√3*U_l). And that because, I remind you, we are in a wye coupling. It is also the phase current. So if we have a line current that is equal to, by definition, 500 V, the phase current is equal to, numerically in our case, 34.6 A. We know that, by definition, the impedance Z is R plus j*x, and we also have P, the active power, that is equal to 3*R*I^2. From there we extract the calculation of the resistance that we are looking for since we want to discover R and x to completely determine the impedance Z and we have the active power consumed by the circuit.

Notes

Summary

16m 23s

TRANSFORMATION TRIANGLE-ÉTOILE

Exemple : $P = 20\ kW$ $\quad U_l = 500\ V$

$S = 30\ kVA \quad \Rightarrow \underline{Z}\ ?\ \lambda$ et Δ

Inductif

a) Couplage en étoile Y $\quad S = \sqrt{3}\, U_l \cdot I_l \rightarrow I_l = \dfrac{S_l}{\sqrt{3}\, U_l} = I_{ph}$

$U_l = 500\ V \quad I_{ph} = 34,6\ A$

$\underline{Z} = R + jX \quad P = 3 R I_{ph}^2 \rightarrow R = \dfrac{P}{3 I_{ph}^2} = 5,55\ \Omega$

$Q = \sqrt{S^2 - P^2} = +22,36\ kVAR$

$\underline{Z} = 5,55 + j\,6,21 = 8,33\, e^{j48,2} \quad = 3 X I_{ph}^2 \rightarrow X = \dfrac{Q}{3 I_{ph}^2} = 6,21\ \Omega$

Electrotechnique II

R is therefore equal to, by extracting it from the previous equation, R is equal P divided by three times the phase current squared. We have all the elements needed to calculate this resistance, which is equal to 5,55 Ohm. We can do, in the same way, the calculation of the reactive, and so of the reactance x. Firstly, the reactive is calculated starting from the fact that it is √(S^2-P^2), which gives the reactive Q and we get +22,36 kVAR. I say +22 because we have said previously, and it is given, that the system is inductive, so the reactive must be positive. Then, we also know that Q = 3*x*I_ph^2. We can therefore determine x by writing, in the same way, Q divided by three times the phase current squared, namely 6,21 Ohm. We then have Z, to finish, by substitution, is equal to 5,55 Ohm + j*6,21, namely 8,33*e^j(48,2°). To finish, our impedance has been determined thanks to this wye coupling.

Notes

Summary

18m 25s

b) Couplage Δ $\quad S = \sqrt{3}\, U_\ell \cdot I_\ell$

$$I_{ph} = \frac{I_\ell}{\sqrt{3}} \quad \Rightarrow \quad P = 3\,R \cdot I_{ph}^2 = 3\,R \cdot \frac{I_\ell^2}{3} = R \cdot I_\ell^2$$

$$\Rightarrow \quad R = \frac{P}{I_\ell^2} = \frac{3\,P\,U_\ell^2}{S^2} = 16{,}7\ \Omega$$

$$\rightarrow \quad Q = 3\,X\,I_{ph}^2 \quad \rightarrow \quad X = \frac{Q}{I_\ell^2}$$

Now in a delta coupling, in a delta coupling, a certain number of values will change. We still have by definition that the apparent power is equal to √3 times the line tension, times the line current. However, this time, the phase current has a ratio of √3 with the line current. We can then determine a certain number of values from this starting point. The active power is always calculated from the Joules losses, namely three times R, times the phase current squared. And then 3 times R times the line current squared, divided by √3*√3 that is 3 so finally R*I_l^2. We can then say that R, if we now want to choose and calculate R, is equal to P divided by I_l^2. What is I_I worth? We wrote it just on top here S is equal to √3*U_l*I_l. S squared is 3 times U_l^2, times I_l^2. And we can then replace this here which gives us 3*P*U_l^2 / S^2. We have all the elements to calculate and we get 16,7 Ohm. We directly see that it is equal to 3 times the resistance that we had previously with the wye coupling. We can of course do exactely the same thing, by calculating with the reactive x that is 3*x*I_ph^2 et we will get x = Q/(I_l^2). Finally, we will discover that this is also equal to 18,6 Ohm.

Notes

Summary

20m 10s

b) Couplage Δ $\quad S = \sqrt{3}\, U_\ell \cdot I_\ell$

$$I_{ph} = \frac{I_\ell}{\sqrt{3}} \quad \Rightarrow \quad P = 3\, R \cdot I_{ph}^2 = 3\, R \cdot \frac{I_\ell^2}{3} = R \cdot I_\ell^2$$

$$\Rightarrow \quad R = \frac{P}{I_\ell^2} = \frac{3\, P\, U_\ell^2}{S^2} = 16{,}7\ \Omega$$

$$\rightarrow \quad Q = 3\, X\, I_{ph}^2 \quad \rightarrow \quad X = \frac{Q}{I_\ell^2} = 18{,}6\ \Omega$$

$$\underline{Z}_\Delta = 16{,}7 + 18{,}6j = 25 e^{j\, 48{,}2} = 3\, \underline{Z}_Y$$

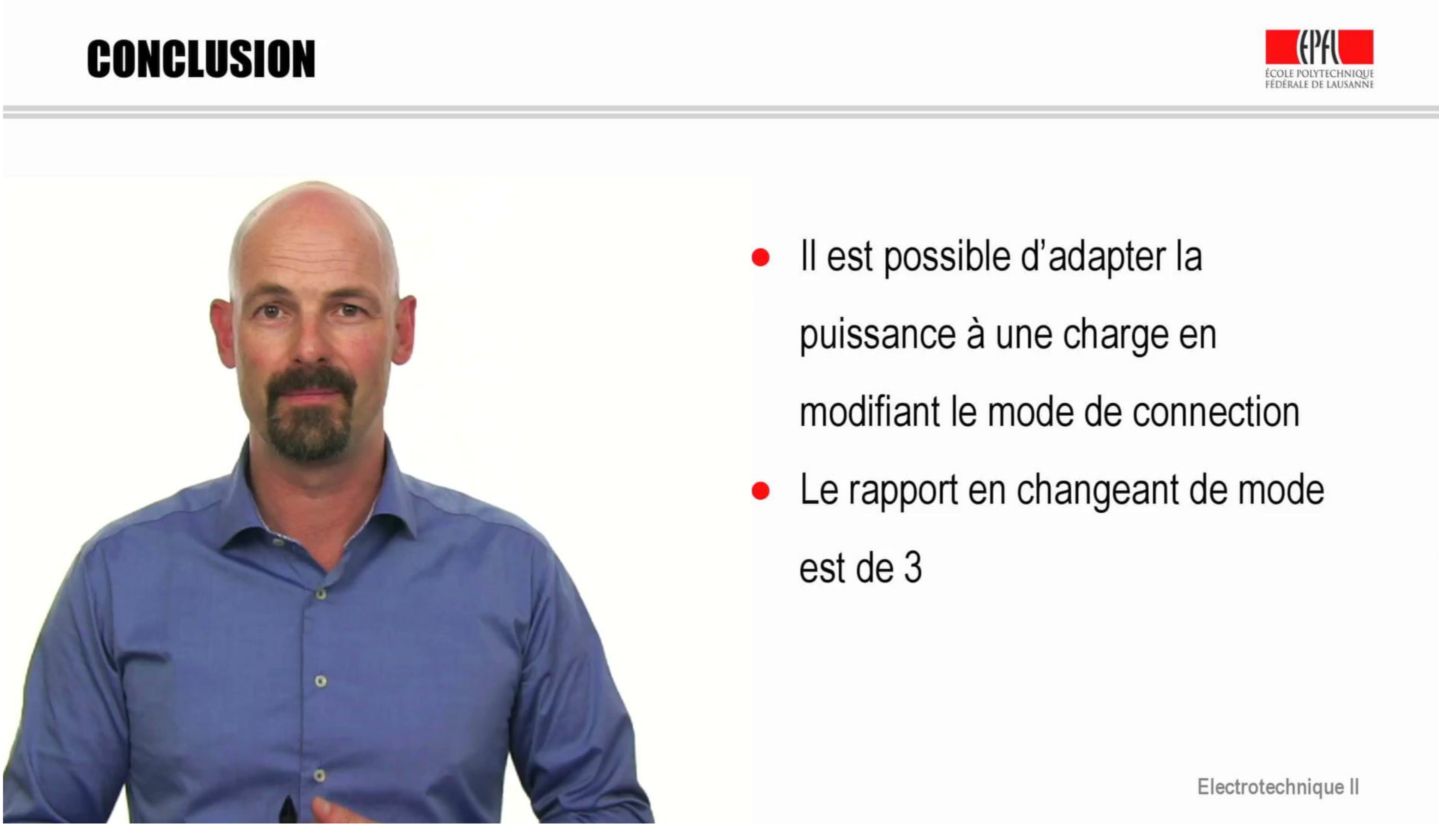
Electrotechnique II

We obtain our new impedance Z, this time in a delta setup and coupling, which is equal to 16,7 + 18,6j, namely 25*e^j(48,2°). We see that this impedance in delta is equal to 3 times the impedance Z that we have just calculated preivously in a wye setup. We also remark that these two vectors are collinear, the angle is exactely identical, the norm is just multiplied by three, as we have seen here with the change between R and x that generally have a ratio of 3 between the wye and delta setups.

Notes

Summary

22m 22s



To conclude, we have seen three different modes that show you the possibility when changing from a wye or delta coupling, to either adapt the charge to a different network or to modify the power, or even to adapt the charge in a change of mode from wye to delta all the while keeping exactely the same absorbed power with an identical network.

Notes

Summary

23m 10s