ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

RÉGIMES TRIPHASÉS

PUISSANCE EN RÉGIME TRIPHASÉ

LEÇON 5

Électrotechnique II
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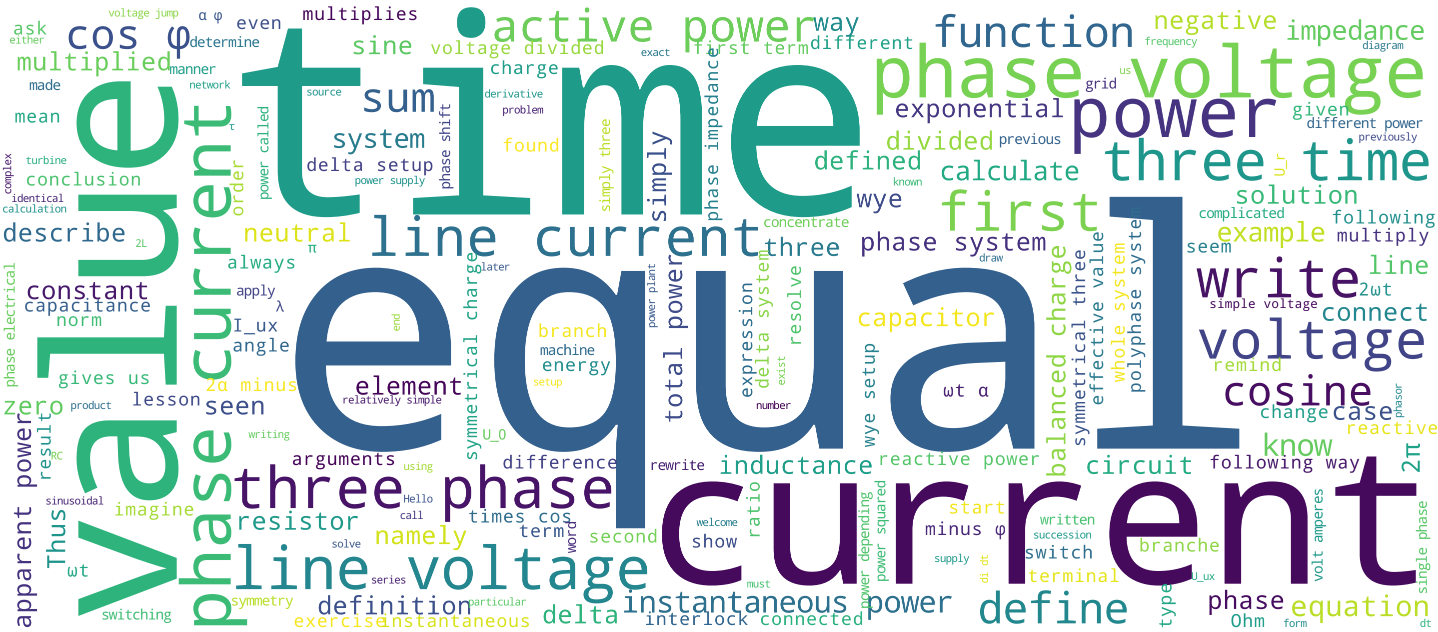
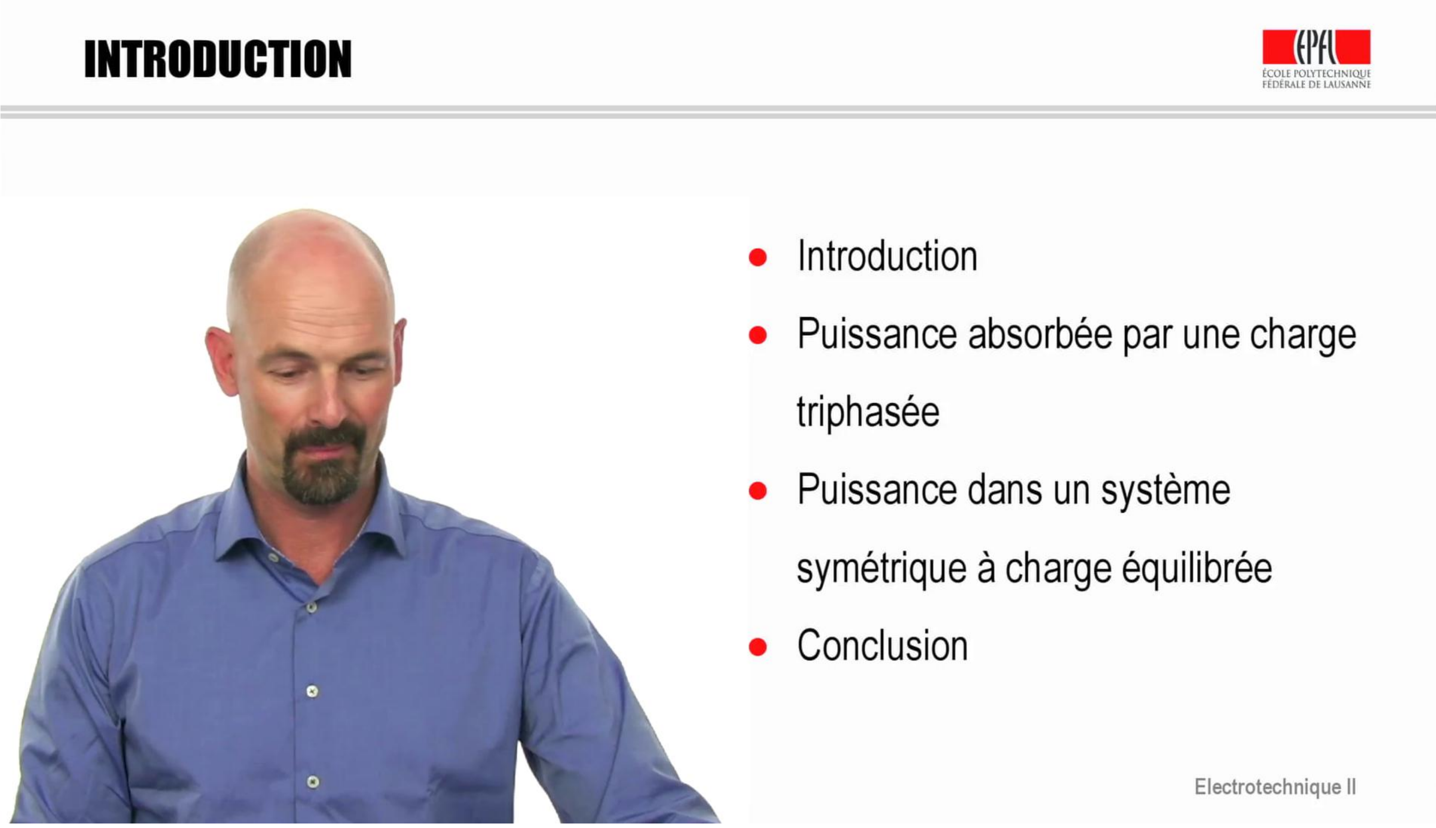
Laboratoire d'Actionneurs Intégrés



EPFL

Video



Hello and welcome to this lesson dedicated to three-phase electric power. We will define what the instantaneous three-phase power is in a symmetric system and will study using two examples the power with a balanced charge, or rather, the powers in a balanced charge.

Notes

Summary

0m 04s

4 types de puissance : $p(t)$ = puissance instantanée

P = puissance active $= \overline{p(t)} = U I \cos\varphi$ [W]

Q = " réactive $= U I \sin\varphi$ [Var]

S = " apparente $= U I$ [VA]

Par symétrie : Puissance totale $\equiv \sum 3 \cdot P_{phase}$

To define the instantaneous power within a symmetric three-phase system, I would like to start with a short reminder of the different types of power that exist. We can count four types of powers, starting with the first type, if one may say, of power. The instantaneous power, which is the power depending on time. We have then defined the active power by the letter P. This active power, we have seen that, by definition, it is the average of the instantaneous power and it is also equal to U*I*cos(φ) and all this is measures in watts. Then, we defined another type of power that defines the round trip of the target non-convertible power called reactive power, and is by definition equal to U*I*sin(φ), written in reactive volt-amperes. Last but not least, the power called apparent power is equal to U*I and is written in volt-amperes. Now, how do we solve the power problems in a three-phase system ? We have seen that by symmetry, all these values are equal in each branch. We also know that the total power in a system is simply the sum of these powers. We can then say that by symmetry we can write the total power as equal to the sum of all three powers in each phase.

Notes

Summary

0m 26s

$$p = u_{ux} \cdot i_{ux} + u_{vy} \cdot i_{vy} + u_{wz} \cdot i_{wz}$$

$$P = U_{ux} I_{ux} \cos\varphi + U_{vy} I_{vy} \cos\varphi + U_{wz} I_{vz} \cos\varphi$$

$$= 3\, U_{ph} \cdot I_{ph} \cdot \cos\varphi$$

Electrotechnique II

We will be able to write for each of these types, or each of these powers Firstly, for the instantaneous power, we can write that it is U_ux which multiplies I_ux, which are here instantaneous values depending on time. We then have the second phase U_vy which multiplies I_vy, and the third phase U_wz which multiplies I_wz. We will see later how this instantaneous power is finally defined and what conclusion we can draw in the end. For the active power, as mentioned earlier, it is three times the active power in each branch. Therefore, we will write U_ux and I_ux which multiply cos(φ), and we have the same for the other two branches U_vy times cos(φ) and U_wz times cos(φ). Three times this which will give us, since these three values by being symmetrical, are equal to three times the phase voltage times the phase current times cos(φ). You will probably remark "yes, but if one of these assemblies is a wye or delta, will that change the result?" Well no, since each time either the line voltage is equal to the phase voltage, or the line current is equal to the phase current. This is always true and can even be replaced by the line voltage and current voltage by writing with the relation √3 U_I I_l, cos(φ).

Notes

Summary

2m 23s

$$p = u_{ux} \cdot i_{ux} + u_{vy} \cdot i_{vy} + u_{wz} \cdot i_{wz}$$

$$P = U_{ux} I_{ux} \cos\varphi + U_{vy} I_{vy} \cos\varphi + U_{wz} I_{wz} \cos\varphi$$

$$= 3\, U_{ph} \cdot I_{ph} \cdot \cos\varphi = \sqrt{3}\, U_\ell I_\ell \cos\varphi$$

$$Q = 3\, U_{ph} \cdot I_{ph} \cdot \sin\varphi = \sqrt{3}\, U_\ell \cdot I_\ell \sin\varphi$$

$$S = 3\, U_{ph} \cdot I_{ph} = \sqrt{3}\, U_\ell \cdot I_\ell$$

Montage en étoile ou en triangle

Electrotechnique II

Therefore, for the active power in a three-phase system, we have a simple relation letting us focus on one phase only and not on the more complicated system since it is simply three times each phase. We can now write, in the exact same way, the reactive power that we have three times the phase voltage: phase current times the sine of the angle and in the same way write this depending on the line voltage and line current. We will then have, for the apparent power, three times the voltage phase times the current phase or √3 times the line voltage times the line current. All this we can say is valid whatever the setup, so wye or delta.

Notes

Summary

4m 09s

$$p = u_{ux} \cdot i_{ux} + u_{vy} \cdot i_{vy} + u_{wz} \cdot i_{wz}$$

$$u_{ux} = \sqrt{2}\, U_{ph} \sin(\omega t + \alpha)$$

$$i_{ux} = \sqrt{2}\, I_{ph} \sin(\omega t + \alpha - \varphi)$$

$$u_{vy} = \sqrt{2}\, U_{ph} \sin\left(\omega t + \alpha - \frac{2\pi}{3}\right)$$

$$i_{vy} = \sqrt{2}\, I_{ph} \sin\left(\omega t + \alpha - \varphi - \frac{2\pi}{3}\right)$$

$$u_{wz} = \sqrt{2}\, U_{ph} \sin\left(\omega t + \alpha - \frac{4\pi}{3}\right)$$

$$i_{wz} = \sqrt{2}\, I_{ph} \sin\left(\omega t + \alpha - \varphi - \frac{4\pi}{3}\right)$$

Electrotechnique II

We wil now find interest in the instantaneous power, as described previously, that I will rewrite here. Each of these values U, I and in each phase corresponds to an equation that describes, depending on time, the voltage and the current. We will concentrate on the first term U_ux and I_ux and describe in a faster way the two other terms that are simply out of phase by 2π/3 and 4π/3 respectively. For the first voltage U_ux, by definition, we know that it is a sine or cosine. I will describe here √2 times the phase voltage, which gives the peak voltage, and for example lets take the sine of ωt with an additional random phase shift which will define the zero of the voltage. We have for the current in the same way, √2 times the phase current times a sine that will by shifted by another angle which will be either β, either (α - φ). We can describe the other two values U_vy and U_vz and the other two values of current I_vy, I_vz in the following way. The instantaneous power is the sum of the product of these two functions of these two functions and these two functions. All this seems very complicated here and we will apply a relatively simple general trigonometry formula that I will show you here.

Notes

Summary

5m 09s

PUISSANCE EN RÉGIME TRIPHASÉ

$$\sin a \cdot \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$p = 3U_{ph} \cdot I_{ph} \cos\varphi - U_{ph} I_{ph} ($$

We know that the sine of a the multiplies the sine of b is really equal to a sum of cosines. It is the cosine of the difference of arguments minus the cosine of the sum of arguments. This can seem more complicated than before, but you will see that it will make everthing easier. By adding up all the terms of previous U,I that we have seen, we get: First of all, we do the difference of the arguments. By doing this, we have the term ωt + α that disappears each time. In each element, we always do (a-b) and the ωt + α is found in each of them, so it disappears. Thus, this instantaneous power that is finally equal to three times the phase voltage times the current phase and, since the ωt plus α disappears each time, there is only the cosine of φ left. We then have a minus sign that appears here and this minus will also be interesting since we have here a phase voltage, a phase current that multiply this time the sum of arguments (a+b), and we have this three times. I will write them down for you. We have the cosine of 2ωt plus 2α minus φ then another cosine of 2ωt plus 2α minus φ, minus 4π/3 and then one more cosine of 2ωt plus 2α, minus φ that is out of phase of -2π/3.

Notes

Summary

7m 20s

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$$\sin a \cdot \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$p = 3U_{ph} \cdot I_{ph} \cos\varphi - U_{ph} I_{ph} \underbrace{\left(\cos(2\omega t + 2\alpha - \varphi) + \cos\left(2\omega t + 2\alpha - \varphi - \frac{4\pi}{3}\right) + \cos\left(2\omega t + 2\alpha - \varphi - \frac{2\pi}{3}\right)\right)}_{0}$$

$$p = 3U_{ph} \cdot I_{ph} \cos\varphi = P$$

$=$ constante égale à la puissance active

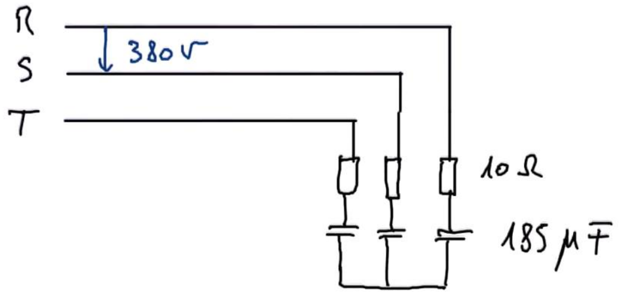
Electrotechnique II

All this, this sum of 3 cosine perfectly out of phase of 2π/3 each, simply gives zero. In conclusion, we see that this instantaneous power, so the power as function of time comes down to because we have here in the first term, three times the phase voltage phase current, cos(φ), and this is none other than the active power that is also, and that is only an observation of this lesson that it is a constant. In other words, the power depending on time is always a constant and equal to the active power. This being so, if we imagine a future use of a three-phase network of a charge for an electromecanical conversion, this means that by converting this electromechanical energy, we can make a system that transfers and transforms a perfectly constant power, so without torque undulation or without a pulsing torque on the machine.

Notes

Summary

9m 33s



$$U_l = 380\ V \qquad U_{ph} = \frac{U_l}{\sqrt{3}} = 220\ V$$

$$f = 50\ Hz$$

$$\underline{I}_{ph} = \frac{\underline{U}_{ph}}{\underline{Z}} = 220\, e^{j0}$$

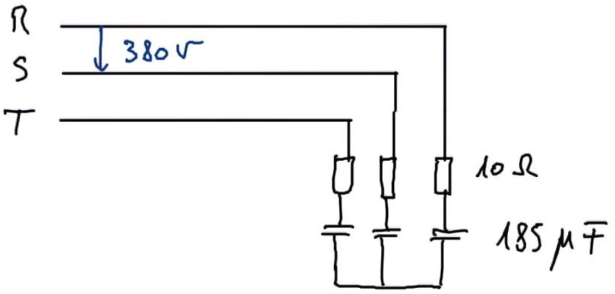
Electrotechnique II

I would now like to give you two examples that will show you how we can calculate in a practical exercise the different powers that we have just described. We will then start with a first example described in the following way: we will connect on the lines R, S and T three completely identical and symmetrical charges. First of all, this charge will be made of three resistors having a value of 10 Ohm followed by a capacitance having a value of 185 µF. We will connect this at a symmetrical three-phase electrical system. We will now give different elements. First of all, we know that the line voltage is equal to 380 V. If the line voltage is equal to 380 V, I remind you that the phase voltage is then equal, in a wye circuit as here, to √3 times less so U_I divided by √3, namely 220 V. We imagine having a frequency of 50 Hz and we ask you to calculate the different powers that we have in this system. First of all, we can define the phase current that circulates in each branche with a different phasor or a different angle, this phase current will be equal to the phase voltage divided by the impedance Z. We can then write 220*e^j(α). We will define α as zero and the reference on Z as the phase impedance.

Notes

Summary

10m 51s



$U_l = 380\ V \qquad U_{ph} = \frac{U_l}{\sqrt{3}} = 220\ V$

$f = 50\ Hz$

$\underline{I}_{ph} = \frac{U_{ph}}{\underline{Z}} = \frac{220\, e^{j0}}{\underline{Z}_{ph}}$

$\underline{Z}_{ph} = 10 - j\frac{1}{\omega \cdot 185 \cdot 10^{-6}} = 10 - 17{,}2\, j = 19{,}9\, e^{j(-60°)}$

$\underline{I}_{ph} = \frac{220}{19{,}9}\, e^{j\frac{\pi}{3}}$

$P_{ph} = R \cdot I^2 = 10 \cdot 121 = 1{,}21\ kW$

$Q_{ph} = X I^2 = -\frac{1}{\omega C} \cdot I^2 = -17.2 \cdot 121 = -2{,}08\ kVar$

Electrotechnique II

We see here that we first need to define this phase impedance. Lets define Z_ph as known, since we know that it is a resistor and a capacitor. namely 10 - j (1/ω) multiplied by 185*10^(-6). This is also equal to 10 - 17.2j or with Euler's formula 19,9*e^j(-60°). It is a negative value and is then a good control since the system being capacitive, we must get a negative phase. The phase current is then equal to 220 V divided by 19.9 and this, e^j times 60° or π/3. What is the power then equal to ? We can define the three types of power : active, reactive and apparent. Thus, the active power in one of the phases is equal to R*I^2 namely 10*121 which gives 1.21 kW. We have the reactive in a phase that is equal to x, the reactance, times I^2. This reactance, defined by the capacitance, so (-1/ωc) that multiplies I^2 and that is equal to, with the values in this example, 17.2*121 is negative, and that is normal since we have a capacitance, the reactive is negative. We now need to determine the total power in the whole system and not only in one of the phases.

Notes

Summary

12m 52s

$$P_{tot} = 3\,P_{ph} = 3{,}63\ \text{kW}$$

$$Q_{tot} = 3\,Q_{ph} = -6{,}24\ \text{kVar}$$

$$S_{tot} = \sqrt{P^2 + Q^2} = 7{,}21\ \text{VA}$$

Electrotechnique II

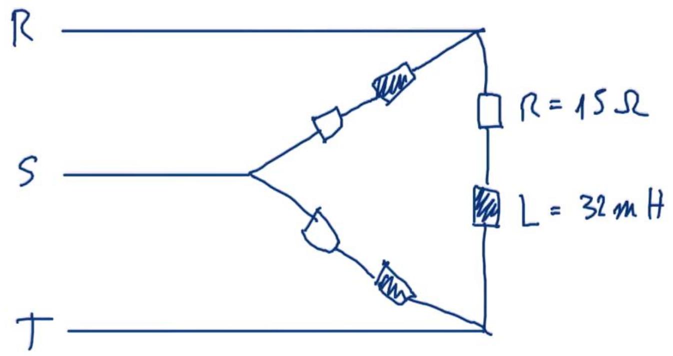
The total power is equal to three times the phase power namely 3.63 kW. The total reactive is simply three time the reactive of a single phase namely -6,24 kVAR. Finally, the total apparent power which, as we know it, is equal to the active power squared plus the reactive power squared, gives us 7,21 VA. We can then see that we just need to calculate the powers on a single phase and to multiply by 3 to get the values of the whole circuit.

Notes

Summary

14m 48s

PUISSANCE EN RÉGIME TRIPHASÉ



$$U_\ell = 380\,V$$

$$\underline{Z} = R + jX = R + j\omega L$$

$$= 15 + 10j$$

$$Z = \sqrt{15^2 + 10^2} = 18\,\Omega$$

$$\cos\varphi = \frac{R}{Z} = \frac{15}{18} = 0{,}83$$

$$P = \sqrt{3}\, U_\ell \cdot I_\ell \cos\varphi$$

Electrotechnique II

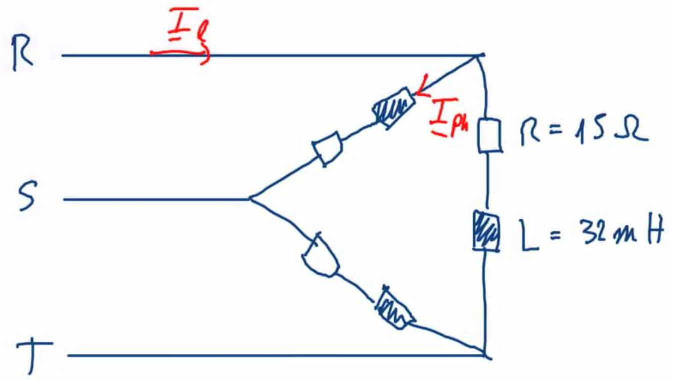
A second example, this time on a delta system, will be suggested. We have here our three-phase electrical circuit on which we will connect a symmetrical charge, composed this time of an inductance and a resistor that are evidently perfectly symmetrical. R will be equal to 15 Ohm and L will be equal to 32 mH. Here too, we ask you to calculate the power given by this system on this charge. Lets write here R, S and T, and lets note that the line voltage is equal to 380 V. We can then define the impedance Z as a starting point, which is equal to R+jx, namely R+jωL and with the numbers of this problem, it gives us the value of 15 plus 10j. The norm of Z is equal to√(15^2 + 10^2), namely about 18 Ohm. The cosine of the impedance, cos(φ) being the real part divided by the total impedance is equal to 15/18 namely 0,83. We know that the active power, which is what is asked in this exercise, is √3 times the line voltage, the line current and cos(φ). We now have the cosine of the angle, the √3 and the line voltage, only the line current is missing. I remind you that the line current is the current circulating for example here in this line, and we are searching for the effective value, that is identical in R, in S or in T.

Notes

Summary

15m 37s

PUISSANCE EN RÉGIME TRIPHASÉ



$$U_\ell = 380\,V$$

$$\underline{Z} = R + jX = R + j\omega L$$

$$= 15 + 10j$$

$$Z = \sqrt{15^2 + 10^2} = 18\,\Omega$$

$$\cos\varphi = \frac{R}{Z} = \frac{15}{18} = 0{,}83$$

$$I_\ell = I_{ph} \cdot \sqrt{3} = \frac{U_{ph}}{Z}\sqrt{3} = \frac{U_\ell}{Z} \cdot \sqrt{3} = 36{,}6A$$

$$P = \sqrt{3}\, U_\ell \cdot I_\ell \cos\varphi$$

$$= 20\,kW$$

Electrotechnique II

We know that it is the ratio between the line voltage and the phase voltage here in the phase, it is √3. We can then write that the line current is equal to, in effective value, the phase current times √3. What is the phase current equal to ? It is the phase voltage divided by Z (the norm) and like always, multiplied by √3. This phase voltage, since we are in a delta system, is equal to the line √3 divided by Z times √3. We have here all the elements and this is equal to 36,6 A. Finally, we can say that our power is equal to exactly 20 kW and this finishes the exercise, where you can see, once again, that by looking at only one impedance of the balanced charge we resolve the whole system.

Notes

Summary

17m 47s

CONCLUSION

- Dans un système symétrique, la puissance instantanée est contante et vaut la puissance active
- Etudier un problème en triphasé symétrique revient à ne considérer qu'une phase

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To conclude, we can describe two important elements. The first one is that in every symmetrical three-phase system connected to a balanced charge, the instantaneous charge is always constant. The second point is that, to resolve a power exercise on a three-phase system, we concentrate on one of the branches and we simply multiply by three the result that we found to get the power of the whole system. See you soon.

Notes

Summary

18m 48s