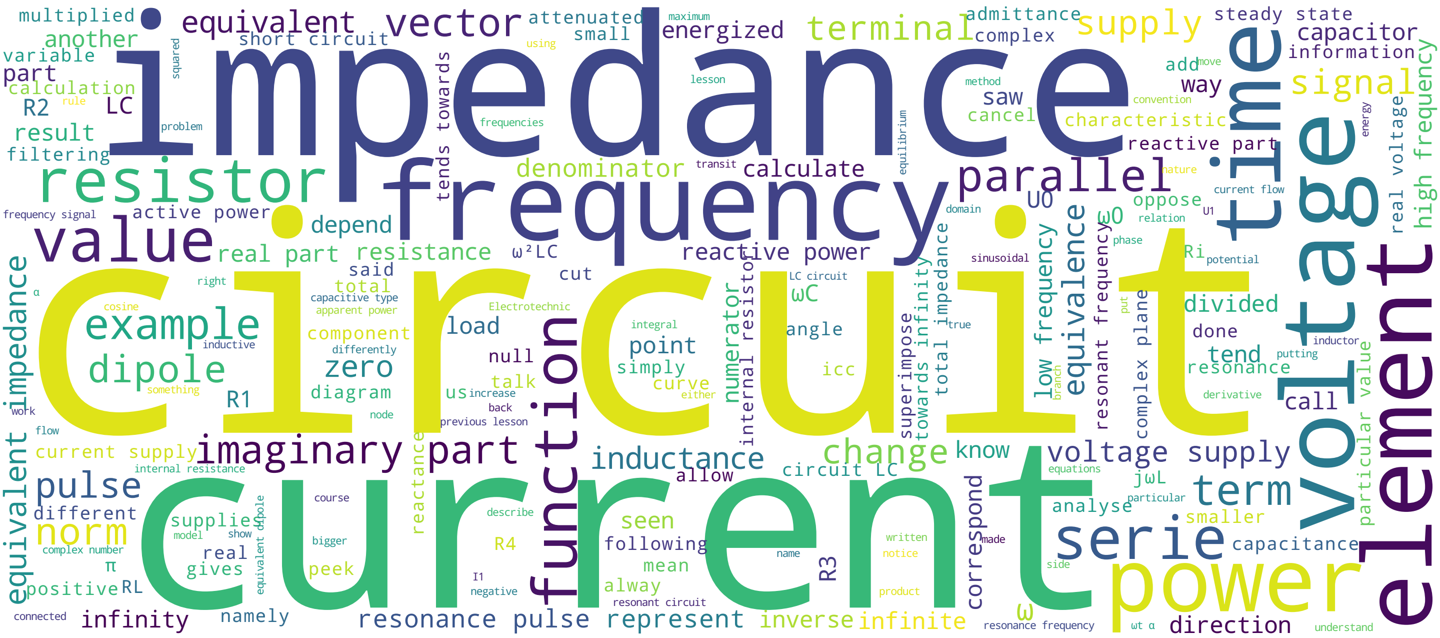
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

COMPORTEMENT FRÉQUENTIEL

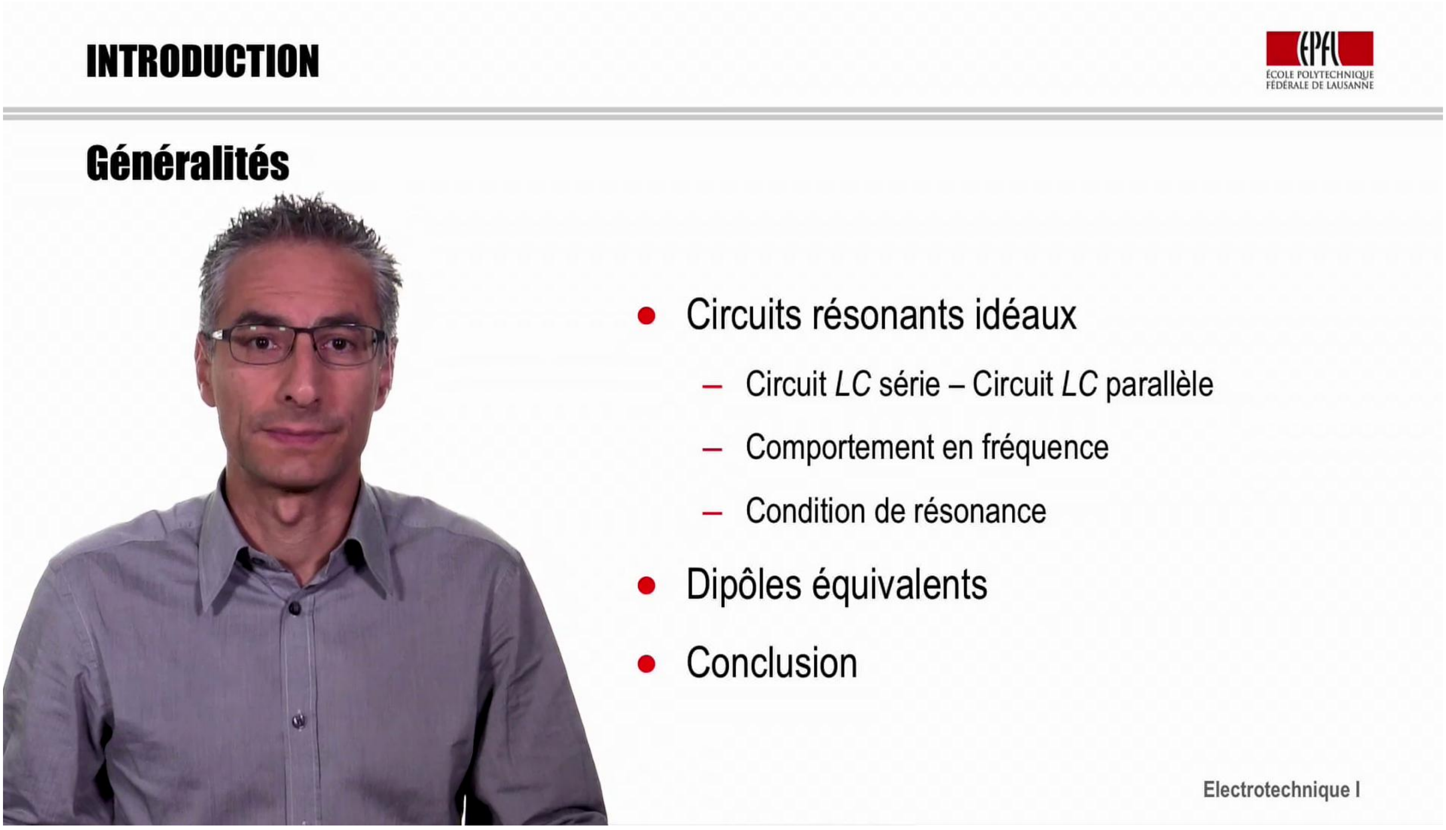
CIRCUITS RÉSONANTS - CONDITIONS DE RÉSONANCE – CIRCUITS ÉQUIVALENTS

LEÇON 21

Électrotechnique I
Yves PERRIARD & Paolo GERMANO
Laboratoire d'Actionneurs Intégrés



Video



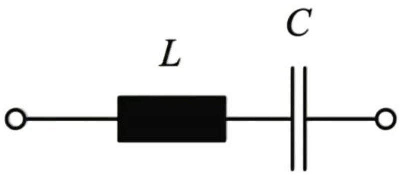
Hello, and welcome to this new lecture of the Electrotechnics course. In the previous lecture, we have studied and constructed the geometric locus of an impedance. We have seen a particular point of this locus, it is the point for which the impedance is purely resistive, without any reactive part. Today, we will study in more details this particularity. We will then talk about resonant circuits, composed of elements L and C. We will see two different type of circuits LC, the case of a circuit LC in series, and the case of a circuit LC in parallel, we will see their behavior in terms of frequency and the resonance condition. Finally, we will focus on the equivalence of this dipole, in series or in parallel, and their frequency dependence.

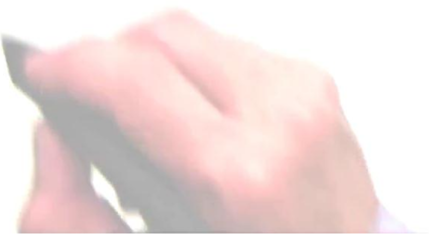
Notes

Summary

0m 04s

Circuits résonants – LC série idéal - Conditions de résonance





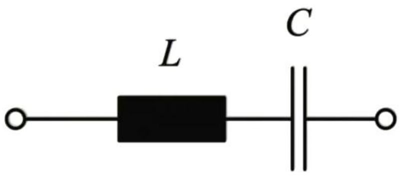
A dipole composed of two elements L and C, connected in series or in parallel, is called resonating circuit. We understand this name if we refer to powers, we have shown that the reactive power is periodically transiting from one element to another, L or C. If we make an analogy with the mechanical field, for example a resonating oscillator, made out of a mass and a spring, in which there is a periodical exchange between potential and kinetic energy between the spring and the mass. If we go back to our electric circuit, as function of the frequency when it is excited, it's properties will be modified, in particular because the impedance changes value. Let's take the example of an inductance and a capacitance in series. They are presented in this figure and we will calculate the total impedance of the circuit, of the norm, and we will represent it in the complex plane. Therefore if we call Zeq the total impedance of the circuit, it consists of a first part, it is the reactive part of the inductance, plus, as it is in series, the reactive part of the capacitor, it is - j 1 over ωC.

Notes

Summary

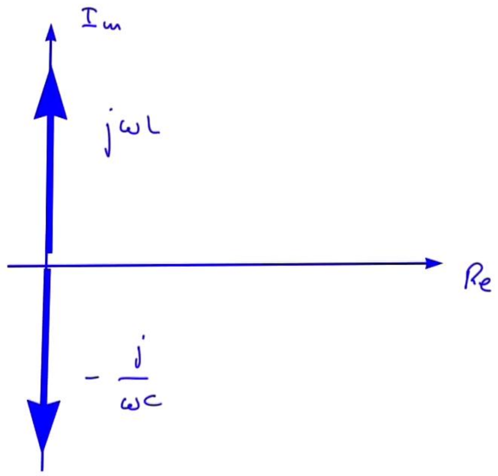
0m 47s

Circuits résonants – LC série idéal - Conditions de résonance



$$\underline{Z}_{eq} = j\omega L - j\frac{1}{\omega C} = j\,\frac{\omega^2 LC - 1}{\omega C}$$

$$Z_{eq} = \left|\frac{\omega^2 LC - 1}{\omega C}\right|$$



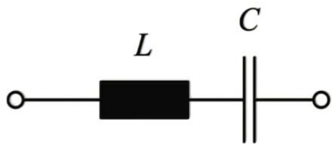
We can write it differently, by putting the two terms under the same denominator, we have j over ωC, and we multiply it by ω²LC-1 We can calculate it's norm, therefore the norm of Zeq is given by the absolute value of ω²LC-1 over ωC The representation in the complex plane, we have already done it many times, is given by the sum of these two vectors, Therefore for the real part and for the imaginary part, we have the vector that is the impedance of C, and the vector that is the impedance of L, that is to say - j over ωC and jωL We see that these two vectors have a length that depends on the pulse or the frequency at which the circuit is energized.

Notes

Summary

2m 09s

Circuits résonants – LC série idéal - Conditions de résonance



$$\underline{Z}_{eq} = \mathrm{j}\omega L - \mathrm{j}\frac{1}{\omega C} = \mathrm{j}\frac{\omega^2 LC - 1}{\omega C}$$

$\omega \to 0 \quad \underline{Z} \to \infty$

$\omega \to \infty \quad \underline{Z} \to \infty$

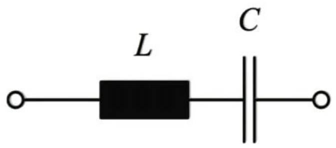
If we analyse this equation that represents the total impedance of the circuit, we see that the pulse which the circuit is energized the circuit tends to 0, The equivalent impedance will tend to infinity, because we have a 0 at the denominator and therefore this value becomes infinite. Contrary to the other extreme, if the pulse tends towards infinity, The equivalent impedance also tends to infinity, because the term ω^2 is at the numerator and therefore the norm of this term will tend to infinity. The resonance condition is achieved for a particular point when the numerator is here equal to 0, that is to say, for a particular value of ω that we will name $\omega 0$, that is equal to 1 over $\sqrt{LC}$, it is this numerator that is equal to 0 The resonance frequency that is proportional to the pulse, is equal to 1 over 2π multiplied by 1 over $\sqrt{LC}$ Therefore $\omega 0$ is the resonance pulse when the numerator is equal to 0 If we analyse this variable as function of the pulse, we see that the circuit is energized at a pulse that is inferior to the resonance pulse, we have that this term, the numerator $\omega^2 LC$ is smaller than 1, we therefore have an impedance $\underline{Z}$ that is a capacitive type.

Notes

Summary

3m 25s

Circuits résonants – LC série idéal - Conditions de résonance



$$\underline{Z}_{eq} = \mathrm{j}\omega L - \mathrm{j}\frac{1}{\omega C} = \mathrm{j}\frac{\omega^2 LC - 1}{\omega C}$$

$= 0$

$\omega \to 0 \qquad \underline{Z} \to \infty$

$\omega \to \infty \qquad \underline{Z} \to \infty$

$\omega_0 = \frac{1}{\sqrt{LC}} \qquad f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

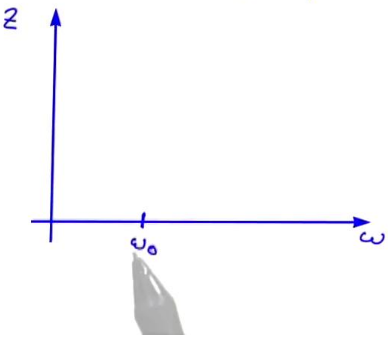
Pulsation de résonance

Si $\omega < \omega_0 \qquad \omega^2 LC < 1$

$\underline{Z} = -jX_1$ "nature capacitive"

Si $\omega > \omega_0 \qquad \omega^2 LC > 1$

$\underline{Z} = jX_2$ "nature inductive"



Electrotechnique I

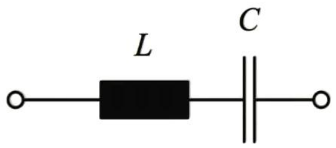
The nature of the equivalent circuit is capacitive type. Therefore the circuit behaves in this case as a capacitor. Contrary if the pulse which energizes the circuit is bigger than ω0 for the resonance pulse, we will have that ω²LC will be bigger than 1, and the equivalent impedance will be jX2 type, and in this case here, the circuit will behave as an inductance, this is positive, we say that the nature of the circuit at this frequency, is inductive. The circuit globally behaves as an inductance. Graphically, if we represent this function in a ωZ plane therefore Z as function of the pulse, we will have that this Z, as function of the pulse ω for a particular value of ω0, is the resonance condition, the numerator is equal to 0, and therefore this impedance Z is equal to 0. However when we get further from this resonance pulse, that is to say towards 0 or infinity, we have the norm of this impedance that tend towards infinity, which gives a curve that approximatively looks like this.

Notes

Summary

5m 41s

Circuits résonants – LC série idéal - Conditions de résonance



$$\underline{Z}_{eq} = \mathrm{j}\omega L - \mathrm{j}\frac{1}{\omega C} = \mathrm{j}\frac{\omega^2 LC - 1}{\omega C}$$

$= 0$

$\omega \to 0 \qquad \underline{Z} \to \infty$

$\omega \to \infty \qquad \underline{Z} \to \infty$

$\omega_0 = \frac{1}{\sqrt{LC}} \qquad f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

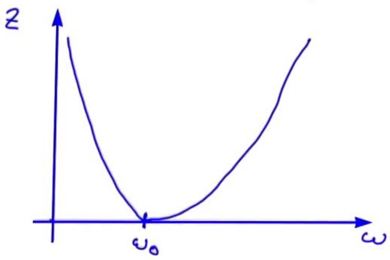
Pulsation de résonance

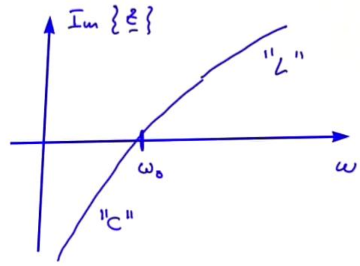
Si $\omega < \omega_0$ $\quad \omega^2 LC < 1$

$\underline{Z} = -j X_1$ "nature capacitive"

Si $\omega > \omega_0$ $\quad \omega^2 LC > 1$

$\underline{Z} = j X_2$ "nature inductive"





Electrotechnique I

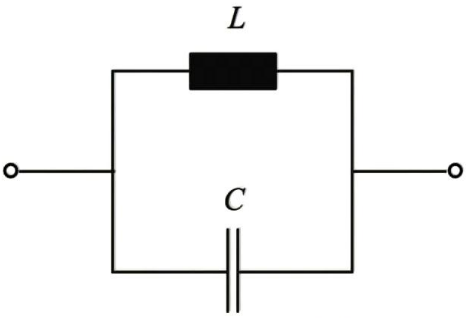
If we look at the imaginary part of Z as function of ω we will have something that describes the following shape, therefore as function of ω, the equivalent imaginary part of Z, we said that the circuit behaves in a capacitive way, under the resonance frequency, and in an inductive way above the resonance frequency, therefore we have a curve that looks like this, thus here a capacitive behaviour, and here an inductive behaviour, the imaginary part is positive.

Notes

Summary

7m 23s

Circuits résonants – LC parallèle idéal - Conditions de résonance



Si $\omega \to 0$ $\quad \underline{Z} \to 0$

Si $\omega \to \infty$ $\quad \underline{Z} \to 0$

$$\underline{Z}_{eq} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{1-\omega^2 LC}$$

Electrotechnique I

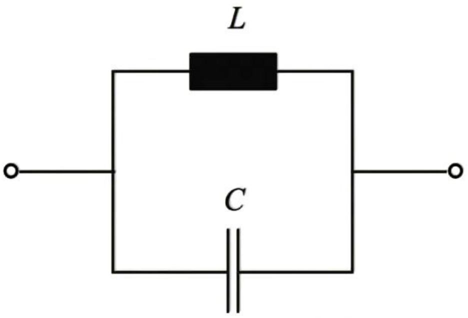
If we now consider the LC resonant circuit but parallel, we have this circuit here, we can again write the equivalent impedance of the circuit, that is equal, as the elements are in parallel, to the inverse of the sum of the impedance's inverse, which gives 1 over jωL, it's the inverse of the inductance impedance, + jωC written differently to show the scalar at the denominator, it gives us this, the denominator is equal to 1 - ω²LC and jωL at the numerator If we analyse again the behaviour of the circuit as function of the frequency, thus when we energize at a frequency ω that is very small, therefore that tends to 0, well we have an impedance that also tends to 0, because this term in the numerator cancels this term here. Conversely if the frequency which we energized the circuit tends to infinity, we also have the impedance that tends towards 0 because the term ω at the denominator becomes very high and cancels this term here. For the particular value that cancels here the denominator, for this particular resonance pulse value, ω is equal to 1 over √LC thus we cancel the denominator, this time the impedance will tend towards infinity.

Notes

Summary

8m 12s

Circuits résonants – LC parallèle idéal - Conditions de résonance



$$\underline{Z}_{eq} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{\underbrace{1-\omega^2 LC}_{\approx 0}}$$

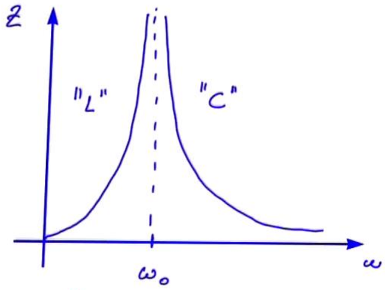
Si $\omega \to 0$ $\quad \underline{Z} \to 0$

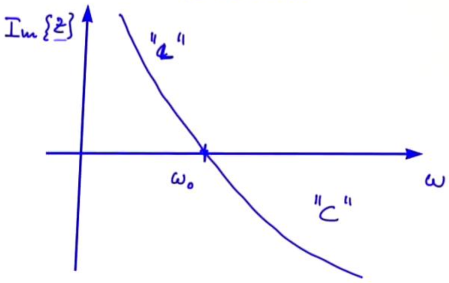
Si $\omega \to \infty$ $\quad \underline{Z} \to 0$

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad \underline{Z} \to \infty$

pulsation de résonance

" le circuit est accordé "





"CIRCUIT BOUCHON"

Electrotechnique I

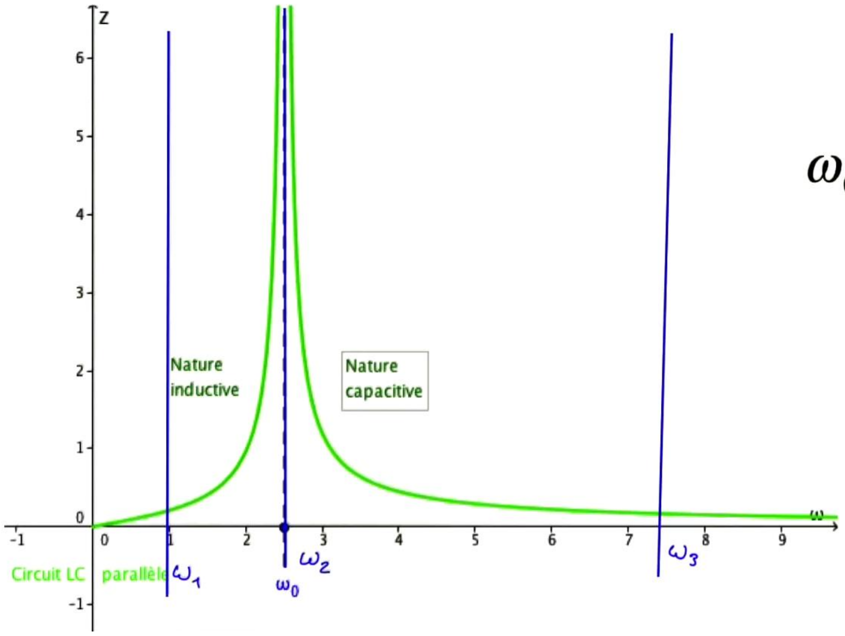
Therefore this is again the resonance pulse, it is said that the circuit is tuned, and it's the exact same condition as in the LC circuit in series Again, if we represent the norm of Z as function of the pulsation to which the circuit is energized, we have this particular value ω0 the resonance pulse, for which the impedance is infinite, and beyond this frequency, the impedance tends towards 0, thus this is the norm of the impedance, we have something schematically, which is like this, with again an inductive behaviour of the circuit, for small values under the pulse ω0, and has a capacitive type when we go above the resonance pulse. So if we look now the imaginary part of the equivalent impedance as function of the pulse, we have again this resonant pulse ω0 for which the imaginary part of Z is zero, and the circuit is of inductive nature, at low frequency, and of capacitive nature, at high frequency. This circuit is also called tank circuit, because at the resonant pulse, its impedance is infinite, and therefore it will oppose to the current flow at this frequency.

Notes

Summary

10m 06s

Circuits résonants - Conditions de résonance – Modification de ω_0



$$\omega_{0_1} = \frac{1}{\sqrt{L_1 C_1}}$$

Electrotechnique I

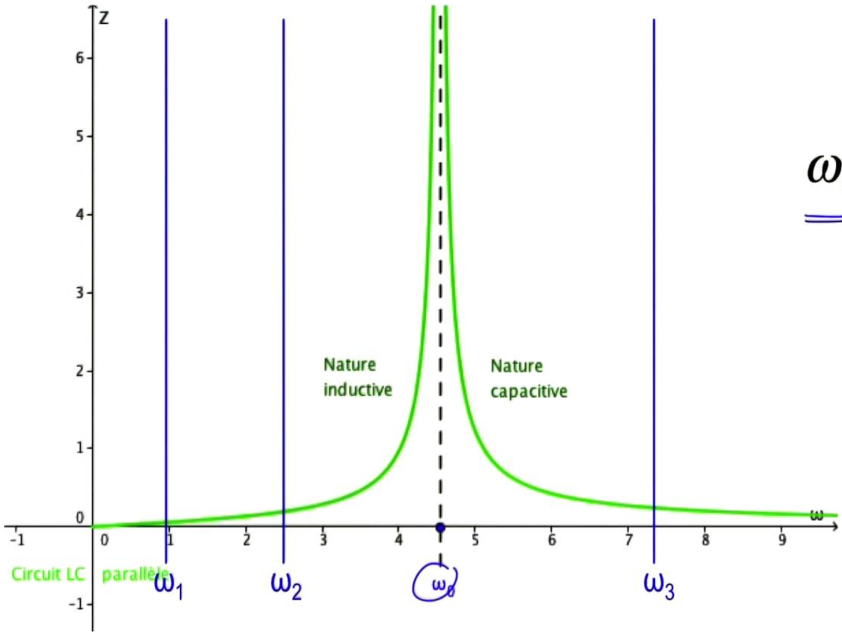
We have calculated by the real case of a LC circuit in parallel, for witch we sketched the characteristic with a simulation software, we can see as we had said earlier, that we have this resonant frequency here ω0 The circuit is inductive for frequencies smaller than ω0 and capacitive for longer frequencies. If we power this circuit at a pulse smaller than ω0 lets call it ω1, the circuit will not oppose to the current flow, because the impedance is low. On the other side, if we power the circuit at a frequency of pulse ω2 which corresponds to the resonant frequency, no current will flow because the impedance is infinite, the circuit opposes to this frequency. If we increase again the excitation pulse of the circuit, till a value, here for example, ω3 again the circuit will not oppose to the transit to this frequency, their will be an output signal, because the impedance is smaller. Now, if we change the circuit parameters, that is to say that we will change the value of LC, and that we lower its values, the feature will move, and will move to a value ω02 that is higher because the LC pair is smaller, and the signal at the pulse ω2 that was cut by the previous circuit, is not for this circuit here because the feature moved, and it's now this frequency that will be attenuated by the circuit.

Notes

Summary

12m 26s

Circuits résonants - Conditions de résonance – Modification de ω_0



$$\omega_{0_2} = \frac{1}{\sqrt{L_2 C_2}}$$

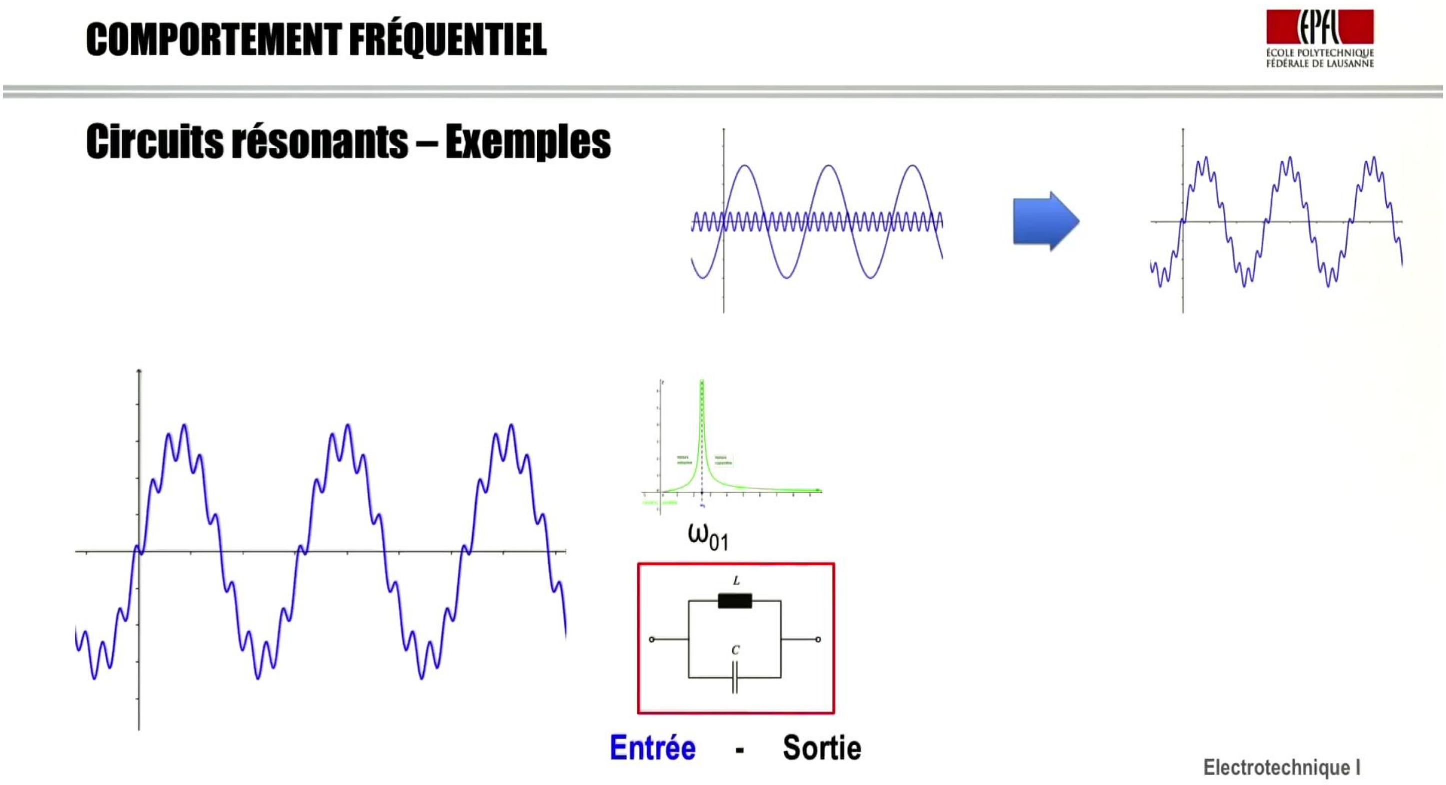
Electrotechnique I

We see that one can use the elements L and C of the circuit, to finally bloc the frequency that we want to. This is the principle of filtering.

Notes

Summary

14m 32s



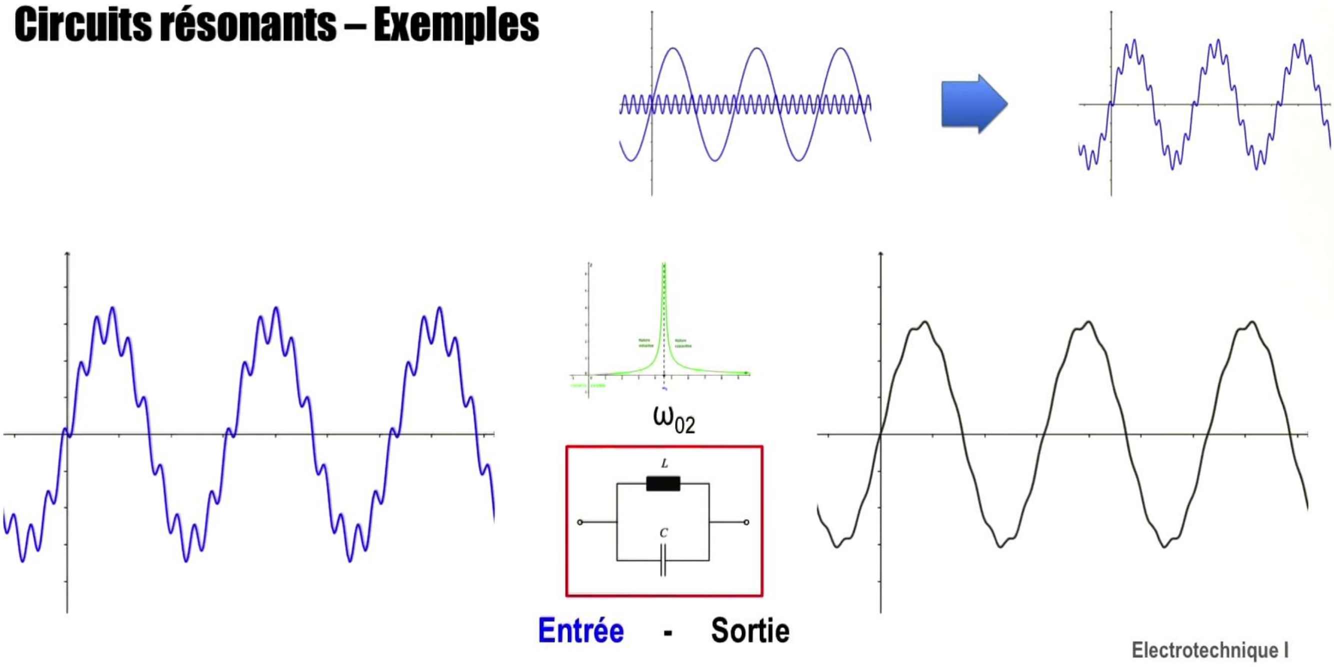
As a concrete case we have a signal which is composed of two voltages, first and second, it can simply be 2 power supplies, on at a frequency ω1, the other at a frequency ω2 that powers our parallel circuit LC, or it can be a more general case, for example an audio signal which would include many frequencies, here we just have two but it could be more, if we add these two signals, we find something like this, a signal that has two or even several components, we see here the first component at a certain frequency, and the second component at another frequency. If now we power the parallel LC circuit where the resonant frequency is ω01 which corresponds in this case, to a low frequency, which matches to this signal here, I power my circuit with the total signal here, that is the sum of both, this frequency ω0 corresponds to the low frequency here, it will be attenuated, cut, and therefore as output result, we will have a signal that will mainly have a high frequency signal, with a small low frequency signal that is attenuated.

Notes

Summary

14m 43s

Circuits résonants – Exemples



If now we change the resonant frequency of the circuit, that is to say we change the parameters L and C, we decrease them so ω02 increases and get the right frequency, the high frequency of the signal, well by powering the circuit with this signal, we will have the high frequency that will be cut, and a t the output, their will only be the low frequency that remains, here that is not attenuated by the circuit.

Notes

Summary

16m 16s

Exemples d'application

Exemple :
ADSL (Asymmetric Digital Subscriber Line)

Exemple :
CPL (Courants Porteurs en Ligne)

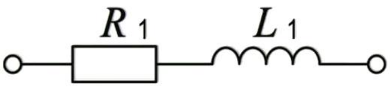
We can name two examples from our everyday lives, the first one is an ADSL link, therefore it's a telephone link, thus the human voice that has a certain number of frequencies between 16 and 15 kilohertz, and on this signal we will superimpose an information signal to transit the information of the network. Well when we talk, we don't want to hear all these frequencies, we would rather only hear the voice frequencies, when we are on the phone, we have this filter here, that allows to cut the high frequencies of the digital signals. A second example, is what we call Power Line Carrier, that is to say that we will superimpose to the 50 hertz domestic supply network, we will superimpose a high frequency signal that is also an information, and then this information, we would like to split it when it comes to the computer for example, we have the information signal and not the 50 Hz signal, thus again we use the filtering, as we have explained previously.

Notes

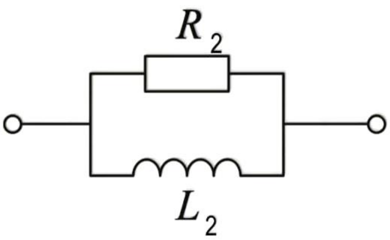
Summary

16m 49s

Dipôles équivalents



$$\underline{Z} = R_1 + j\omega L_1$$



$$\underline{Z} = \frac{R_2\omega^2 L_2^2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

Ces deux circuits sont équivalents si :

$$R_1 = \frac{R_2 \omega^2 L_2^2}{R_2^2 + \omega^2 L_2^2}$$

et :

$$L_1 = \frac{L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

Electrotechnique I

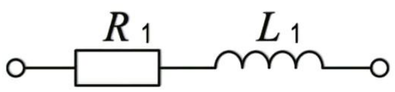
We will address one final point today, those are equivalent dipoles. We have here a dipole that is made of a resistor R1 and an inductance L1 in series, we can determine the equivalent impedance of this dipole, we gave it in the previous lessons, it is represented by the equation here, and the circuit R2 L2 in parallel where the equivalent impedance is given here. You will also find this result in the tables that we have given in the previous lessons. So that these two circuits are equivalent, it is necessary that firstly the resistive portion of R1 must be equal to the resistive part of the other, and on the other hand the imaginary part of one, must be equal to the imaginary part of the other. Therefore R1 must be equal to this, first condition, and second condition, L1 must be equal to this term here. What is interesting to notice, is that this equivalence is true only at one frequency because the term R1 depends on R2 and also the pulse ω same for the equivalent inductance that depends on the pulse, thus this is true only at a given frequency, if we change the frequency, we must change again the parameters R2 and L2 for the equivalence of the two circuits.

Notes

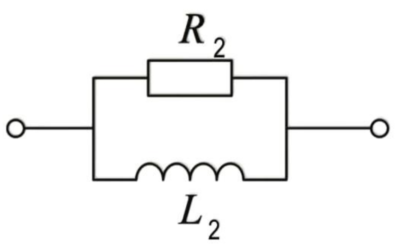
Summary

18m 10s

Dipôles équivalents



$$\boxed{\underline{Z} = R_1 + j\omega L_1}$$



$$\boxed{\underline{Z} = \frac{R_2\omega^2 L_2^2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}}$$

Ces deux circuits sont équivalents si :

$$R_1 = \frac{R_2 \omega^2 L_2^2}{R_2^2 + \omega^2 L_2^2}$$

et :

$$L_1 = \frac{L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

Electrotechnique I

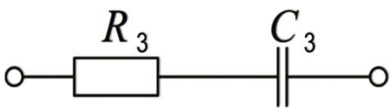
It is clear that the transition is done here in the direction parallel series, but could be done in the other direction, parallel series, by extracting the term R2 of this equation, and for the inductance, extract the term L2 of this equation here.

Notes

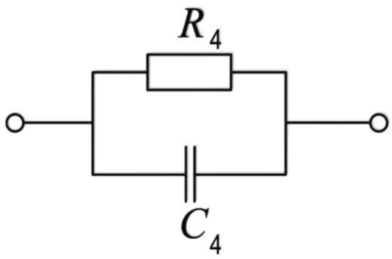
Summary

20m 16s

Dipôles équivalents



$$\underline{Z} = R_3 + \frac{1}{j\omega C_3} = R_3 - j\frac{1}{\omega C_3}$$



$$\underline{Z} = \frac{R_4 - j\omega C_4 R_4^2}{1+\omega^2 R_4^2 C_4^2}$$

Ces deux circuits sont équivalents si :

$$R_3 = \frac{R_4}{1+(\omega R_4 C_4)^2}$$

et :

$$C_3 = \frac{1+(\omega R_4 C_4)^2}{\omega^2 R_4^2 C_4}$$

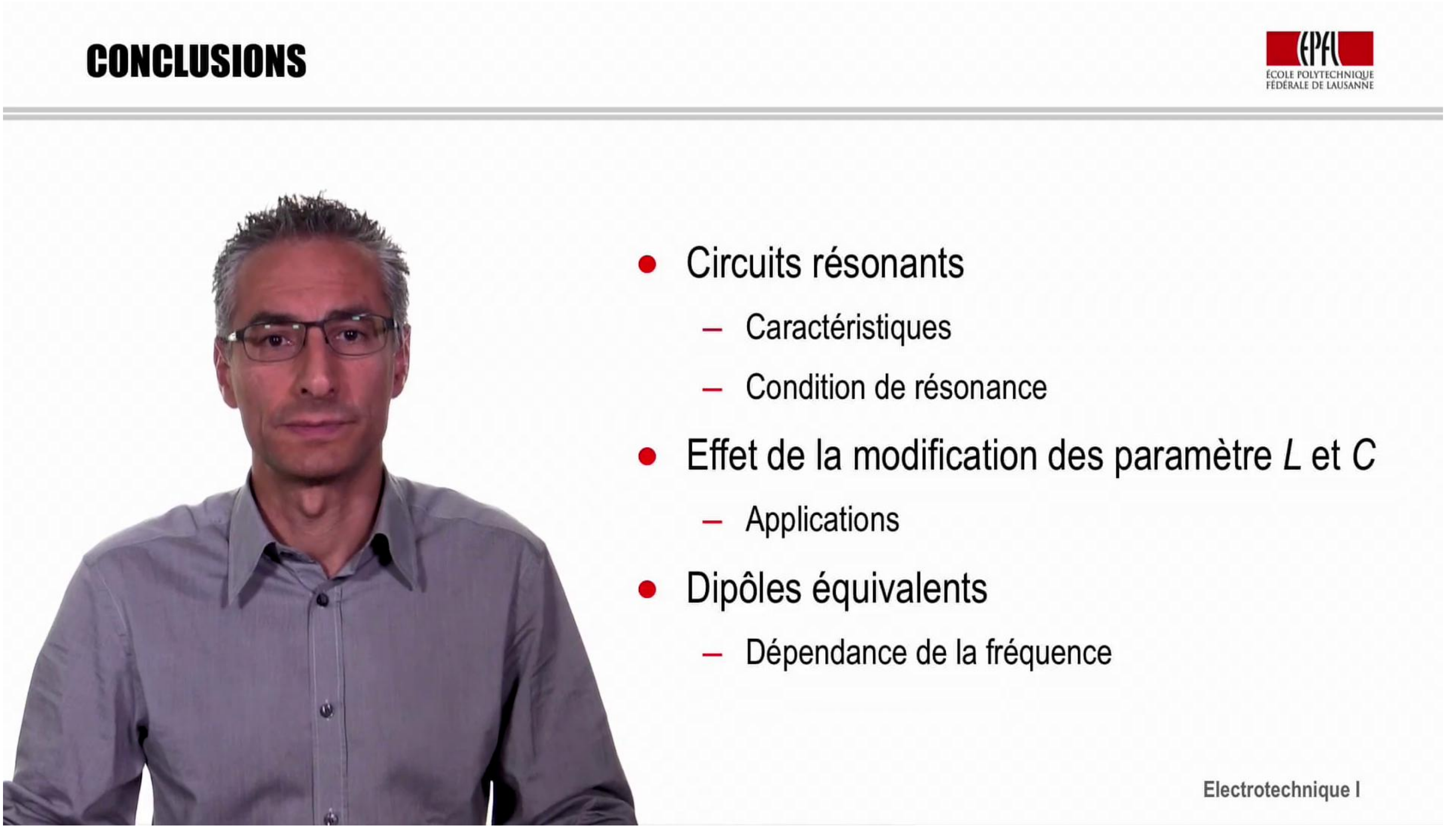
Electrotechnique I

It's the same for the circuit RC serie and RC parallel, if we want those two dipoles to be equivalent, we write the relation that describes R and the equivalent impedance, it's this term here, we can write it differently, for the parallel RC circuit, we have this equation here, and for there to be equivalence, the real part of one must be again equal to the other and the imaginary parts also have to be equal. We can rewrite the term R3 as function of R4 and C4, and we have that R3 is equal to R4 divided by the denominator, and for the equivalence of the imaginary part, C3 must be equal to this term, it's the equivalence condition of these two circuits, but again, at a given pulse or frequency, because this equivalence depends on the pulse.

Notes

Summary

20m 38s



We saw what were resonant circuits, what are their characteristics and under what conditions they are in resonance. We saw how to change the characteristics, and what we can do, especially in the domain of filtering. Finally, we saw the equivalent dipoles, and we saw that they depend on the power supply frequency. Thank you for your attention.

Notes

Summary

22m 12s