COMPORTEMENT FRÉQUENTIEL

LIEUX GÉOMÉTRIQUES

LEÇON 20

Électrotechnique I
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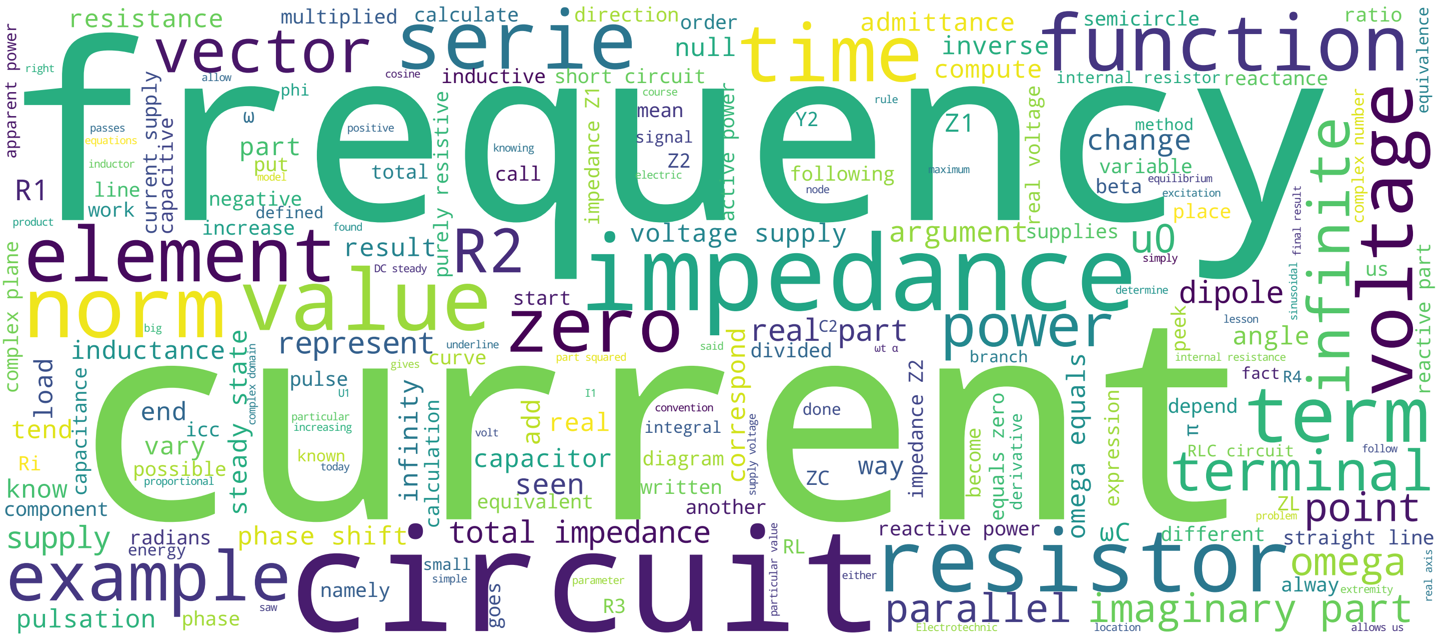
Laboratoire d'Actionneurs Intégrés



Video

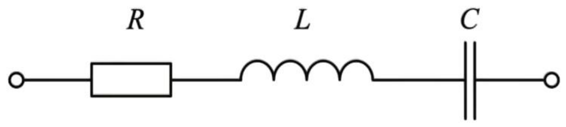
Hello. Welcome to this new lecture of the Electrotechnics course. So far, we have studied many aspects of an electrical circuit but always with a power supply either in DC steady state, or at a certain frequency, eventually at multiple frequencies but simultaneous, it was the case of the principle of superposition. We will see how a circuit behaves over a big range of frequencies. Today we will start by defining what a geometric locus is, then we will see the behaviour of an impedance while varying one of its parameters, in this instance, the frequency, we will see an example, and finally we will look at the current in the circuit, because if we change the impedance the current will also change, we will then compute the current as a function of the frequency.

Notes

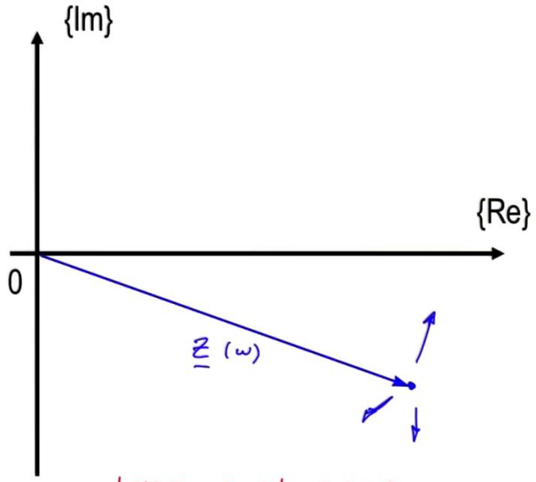
Summary

0m 03s

Lieux géométriques



$$\underline{Z} = R + \mathrm{j}\left(\omega L - \frac{1}{\omega C}\right)$$



Déf. : Lieu complexe relatif à une grandeur complexe quelconque : Lieu ou (courbe) décrit par l'extrémité du vecteur représentant cette grandeur lorsqu'on fait varier un paramètre du réseau (ω)

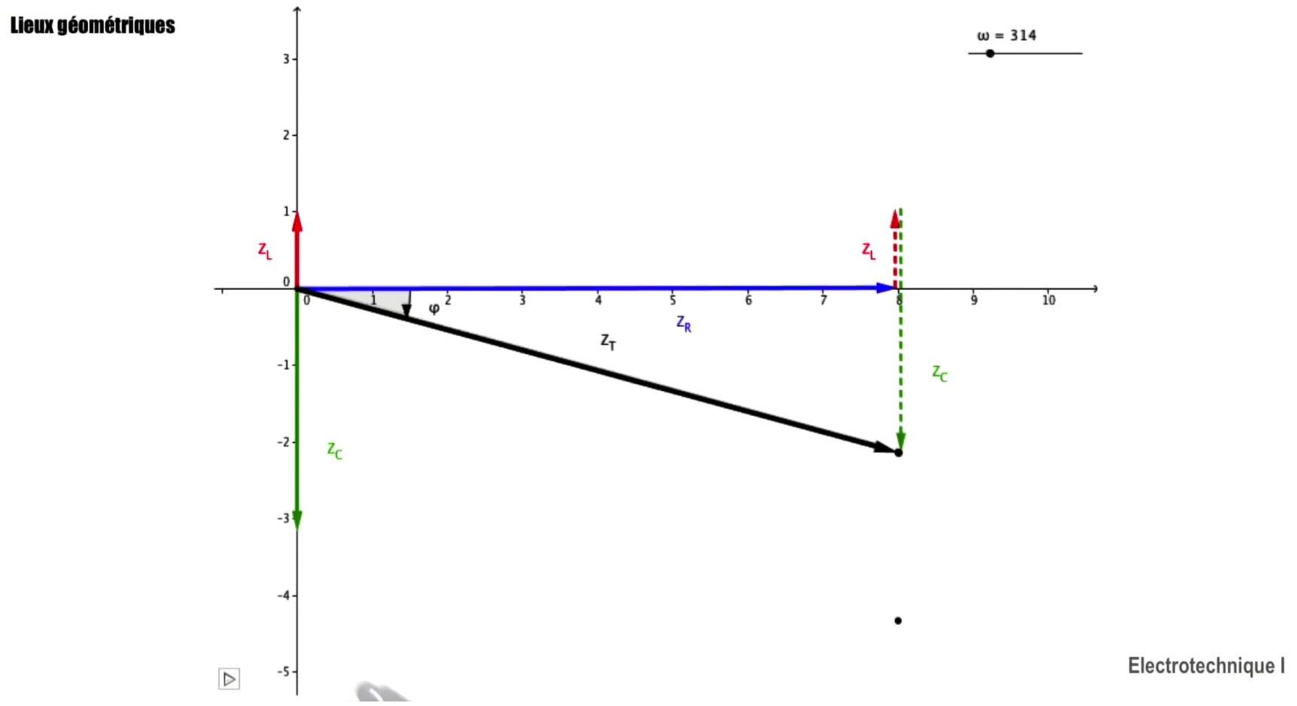
Electrotechnique I

We start then by a definition, the definition of the complex domain. We call complex domain relative to any complex variable, here it would be the impedance Z, thus any complex variable, the location, or the curve, the path, described by the extremity of the vector representing this variable when we vary a parameter of the network, usually it is the pulsation, the frequency, but it could be the resistor, the inductance or the capacitor And today, we will only look at the pulsation. For a given circuit, for example this RLC circuit in series, we can write the expression of its total impedance that has a real part and an imaginary part. We can represent it on the complex plane. We have here Z. Z of omega. And we would like to know how varies this extremity of the vector Z when we move, modify the frequency. For a given circuit, powered by an AC supply, the goal is to compute the curve covered here by the total impedance when we vary the frequency.

Notes

Summary

0m 52s



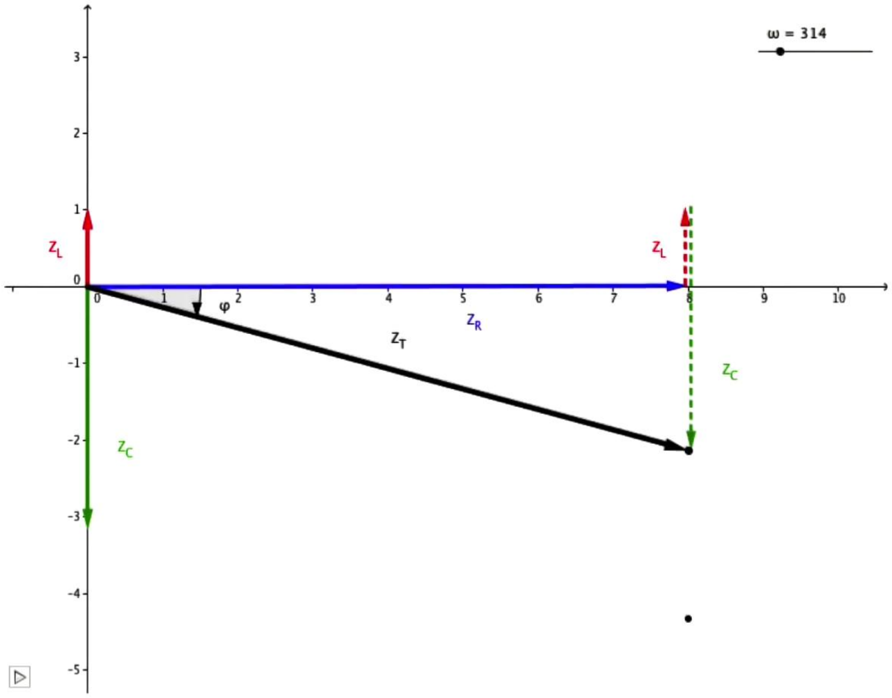
So in the given example, the RLC circuit in series, we are facing three components, because there are three elements. We have the impedance ZR which is equal to R, which doesn't depend on the frequency. We have the component ZL here. ZL which is equal to jωL. And the last component ZC. ZC is equal to -j(1/ωC). The two impedances ZL and ZC are not dependent on the frequency ZC, the inverse of the frequency, and ZL is proportional to the frequency. To get the total impedance of the circuit we will conduct a sum, since they are in series, these three impedances. Therefore, the sum of these three impedances, using the property of the vector addition, we have this first impedance ZR to which we will add ZL, which is represented here, We add and we add again ZC which is negative in order to get the total impedance. This total impedance has a norm and an angle, a phase shift, with respect to the real axis and the end of this vector is given here for a frequency of, for example here, omega equal 200 radians per second. If now we change this frequency value and raise it, that's it, we are here at omega equals 314 radians per second. We see that ZL increased slightly, and linearly with Omega because ZL is ωL.

Notes

Summary

2m 37s

Lieux géométriques



Electrotechnique I

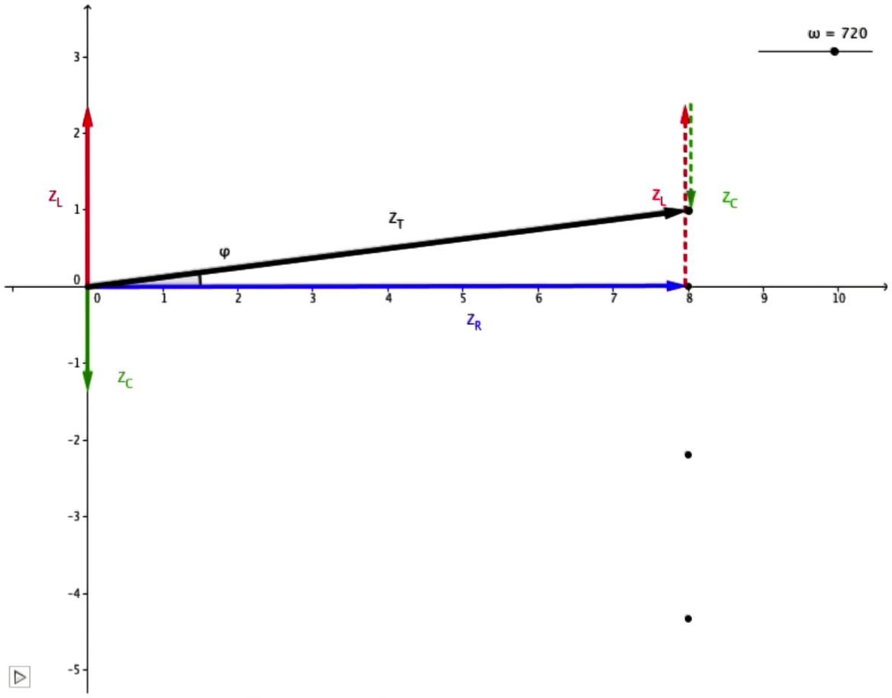
ZC, has decreased because its expression is 1 / ωC, and then the final result, the sum of these three vectors, give a new point here for omega equals 314 radians per second, a new point that corresponds to the new norm and a new phase shift of the impedance with respect to the real axis.

Notes

Summary

4m 29s

Lieux géométriques



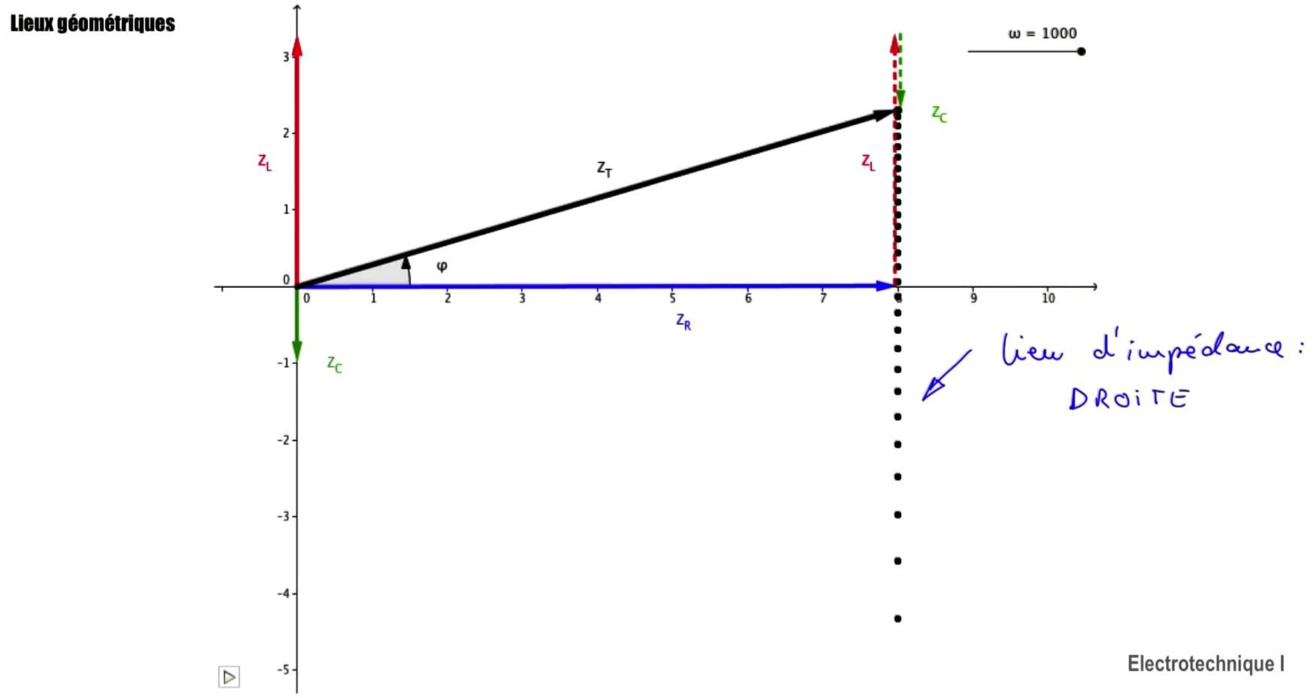
Electrotechnique I

If we increase again the frequency, we have increased once again the frequency or the pulsation, this time we are at 550 radians per second. We see that ZL has again increased and ZC decreased in order to end up with a value so that both offset each other and we end up here with a total impedance, this point here, at 550 radians per second, a total impedance purely resistive. We will threat this particular case in the next lecture. Increasing again the frequency, this time we have omega which is at 720 radians per second for example, so ZL becomes bigger than ZC. Thus the nature of the impedance has become inductive, with a positive reactive part, whereas until now it was capacitive.

Notes

Summary

4m 56s



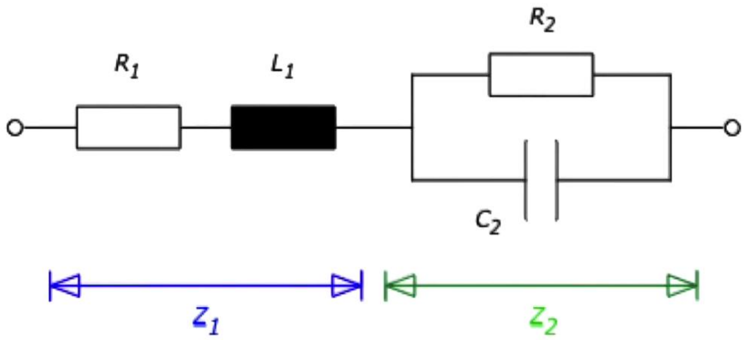
And increasing again the frequency we finally get a total impedance, ZT which this time even more inductive than before and see our spot, namely the curve described by the end of the vector Z as a function of the frequency, that is taking shape. I represent it but for more values of omega and we see that the spot of our impedance which is a RLC circuit in series is finally a straight line. This straight line goes from minus infinity to plus infinity on the imaginary axis, when the pulsation is null we have in fact the term 1/ωC which goes to infinity and when the frequency is infinite we have the term ZL that is worth ωL and tends to infinity.

Notes

Summary

5m 55s

Lieux géométriques
Exemple



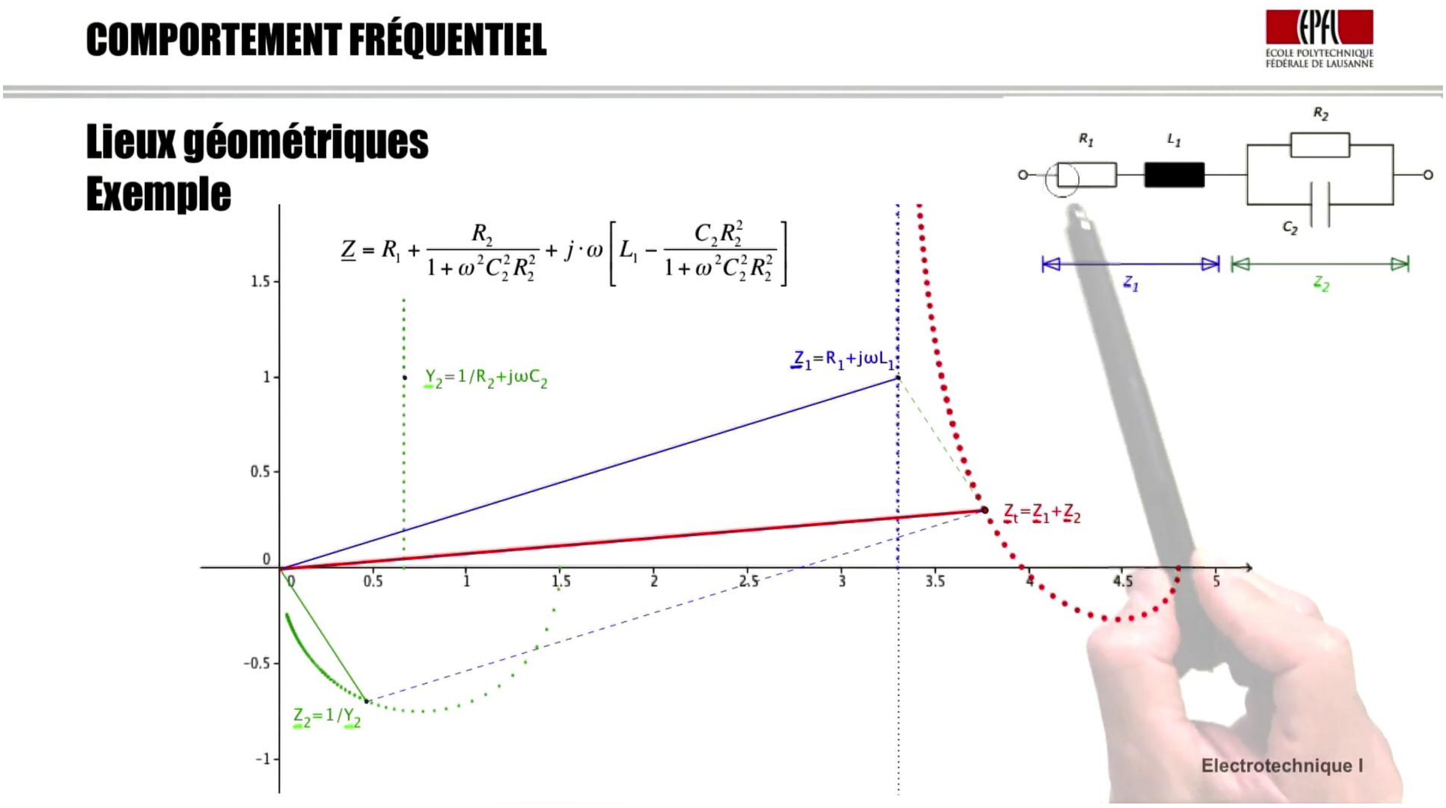
$$\underline{Z} = R_1 + \frac{R_2}{1+\omega^2 C_2^2 R_2^2} + j\cdot\omega\left[L_1 - \frac{C_2 R_2^2}{1+\omega^2 C_2^2 R_2^2}\right]$$

That's it, we will work on a concrete case. We will take this circuit example where we see we have a resistor in series with an inductance, and all in series with a resistor in parallel with a capacitor. So we will define this first part as Z1 and this second part as Z2. These two impedances, Z1 and Z2, are in series. That's it, we can write the equivalent impedance of all this circuit, of all these impedances in series. We see that we have a term that corresponds to Z1 with its part, here, resistive, R1, and the reactive part, jωL1. Then for the Z2 part, you can refer to the table that we've given you in the previous lectures, but we can rewrite the real part, here, of this impedance as being this plus the reactive part which is equal to jω times this term here. Now, we would like to compute or determine the place of the impedance Z when we vary the pulse, the frequency, omega. The result, we give it to you in the following slide.

Notes

Summary

7m 04s



We have here this result that i will comment. So we represent here the impedance Z1 and the impedance Z2. The first impedance here, Z1, is given by, here, this Z1 which has a real part and an imaginary part, it is this term R1 plus jωL1. A small remark because of the software i use that is not able to underline, we have actually to write correctly this equation, we have to underline here the Y2, the Z2 times all the impedances, even these ones to be correct. Thus I said that the first part here of the impedance Z1, which corresponds to this term R1 and this term jωL1, indeed, we find it here if we vary the frequency from zero to infinite. This term does not vary, we still have R1 which is this point here, and the impedance will increase gradually with the increase of the frequency, it's the term jωL1, of a resistor and an inductance in series, it is a straight line that tends from zero to j infinity.

Notes

Summary

8m 30s

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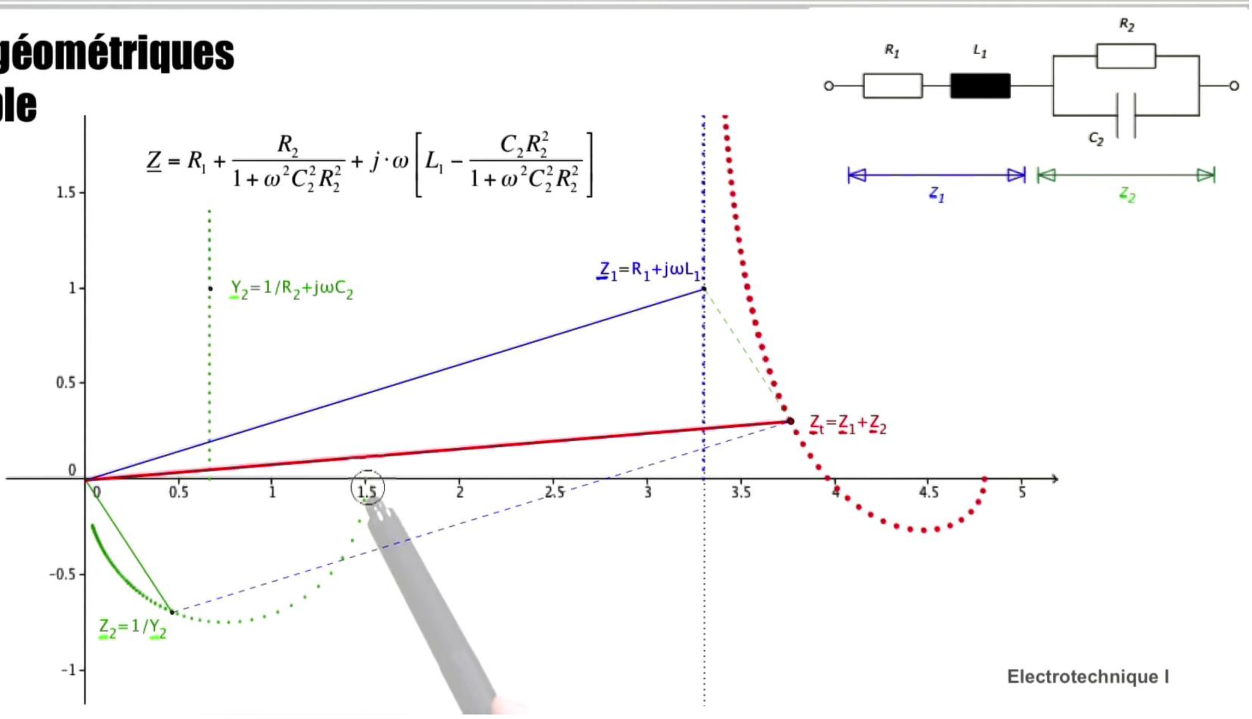
Lieux géométriques
Exemple



Now for the second part of the circuit, the impedance Z2 here, we can advantageously use the admittances, because we known that the parallel admittances can be added, and it allows us to describe the location of Y2, that is to say that Y2 is the sum of these two admittances, the admittance of R2 is here, it is 1/R2, it is the inverse, and the admittance of C2 it is jωC2 and we see that this term here, these terms, are very similar to these two terms here, something constant for the real case, we have this term here, 1/R2, and we add an imaginary term which is proportional to the frequency. It also gives us a place, here, which is half-right that goes from j null to j infinite. So we can do the mathematical demonstration but we will not do it here. Making the mathematical demonstration that the inverse of a line in the complex plane, so that goes from j0 to j infinity, from the zero imaginary part to an infinite imaginary part, is a semicircle. So the image or the inverse of a straight line is a semicircle that starts from this value here for omega equals to zero and tends to this value here.

Notes

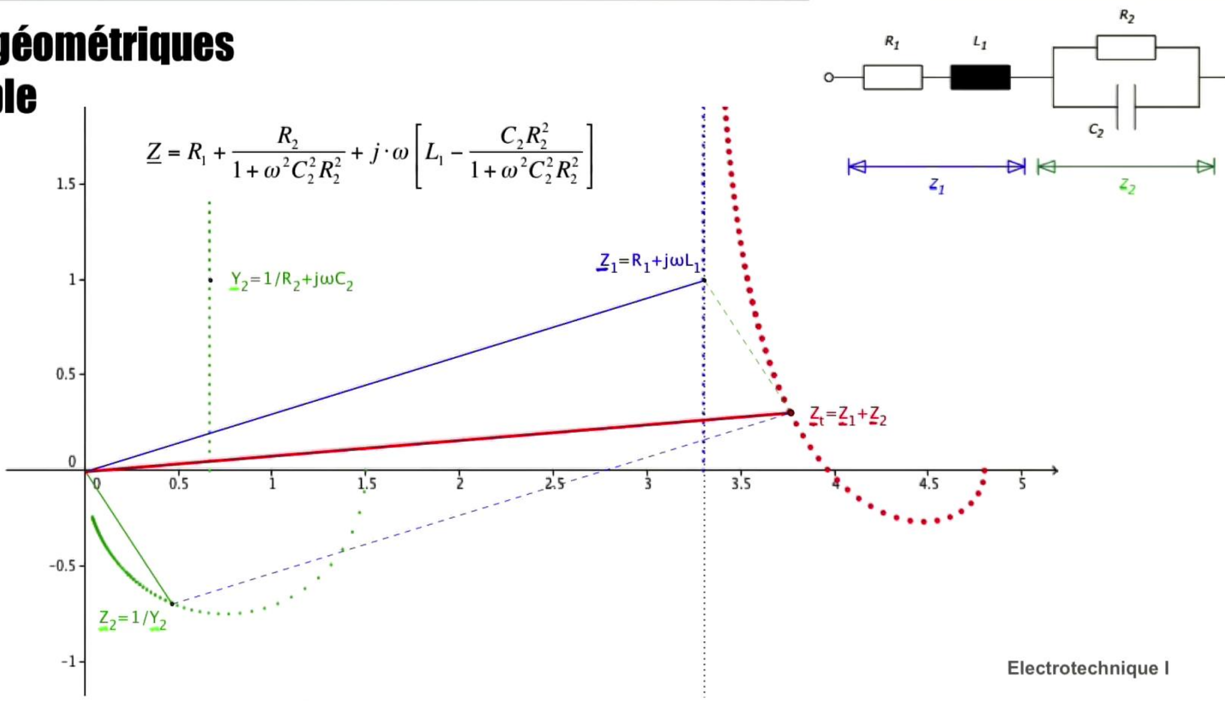
Summary

10m 01s

Lieux géométriques

Exemple

$$\underline{Z} = R_1 + \frac{R_2}{1+\omega^2 C_2^2 R_2^2} + j\cdot\omega\left[L_1 - \frac{C_2 R_2^2}{1+\omega^2 C_2^2 R_2^2}\right]$$



Electrotechnique I

To get the final result, one must add the impedance Z1, is this place here, and this impedance Z2 we found in doing the opposite of this straight line here, it's this semicircle here, the sum of the two, is this impedance plus this one, which is here, which gives this result: Z1 plus Z2. The sun of these two impedances for omega equals zero, we go from this point here where we have the sun of R1 plus R2. And increasing the frequency, first we will be negative, which means capacitive, because at low frequency the impedance Z2 is very high, it is capacitive. So we will first be capacitive, we arrive at a particular value where we are purely resistive and when the frequency increases further, we will become very inductive to follow this curve here.

Notes

Summary

11m 31s

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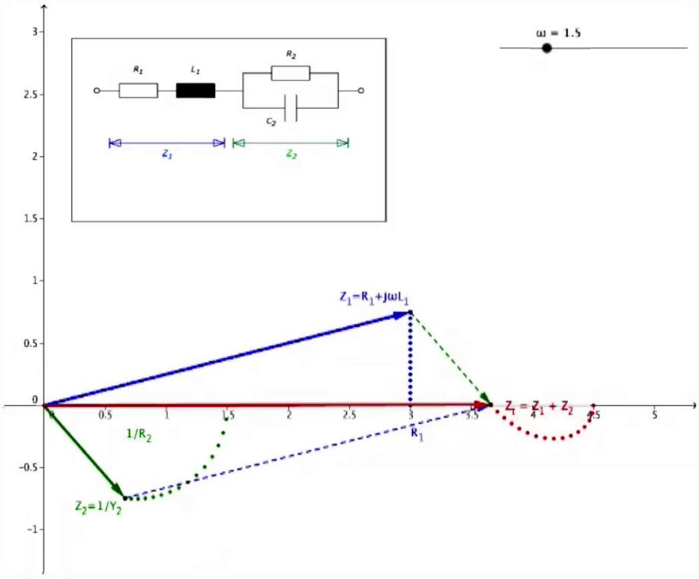
Lieux géométriques
Exemple animé
$0 < \omega < \omega_{max}$



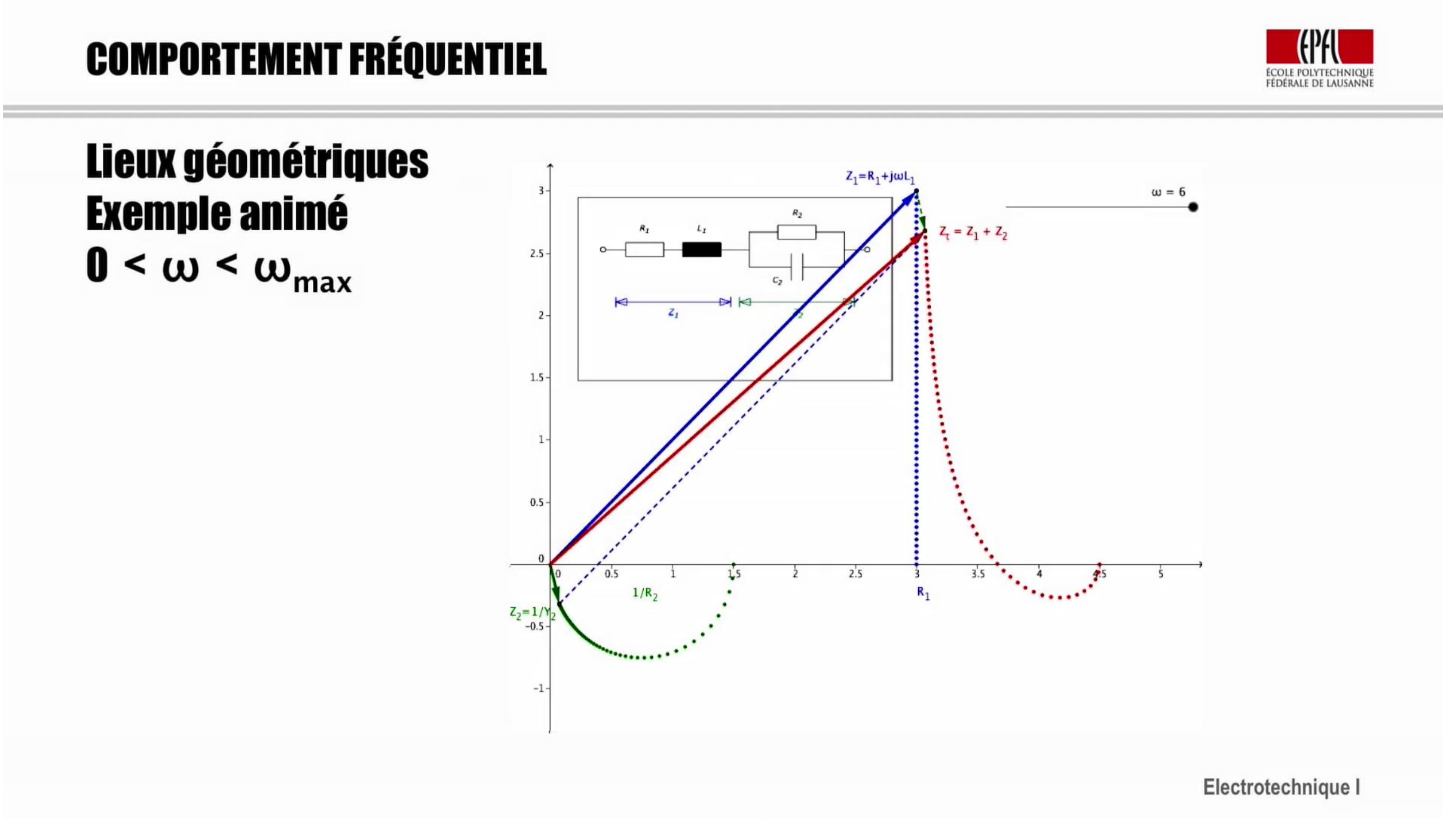
Electrotechnique I

So, we start with a total impedance value for a frequency, a pulsation, null from this point here. We have actually the sum of the two resistors R1 and R2. Why? Because jωL1 is null and 1/ωC is infinite, So we have an open circuit here. Instead of C2 we have only R1 plus R2, it is this starting point.

Notes

Summary

12m 37s



That's it, i launch the video and we see that the impedance is first capacitive, resistive, and she becomes more and more inductive progressively as the frequency increases.

Notes

Summary

13m 02s

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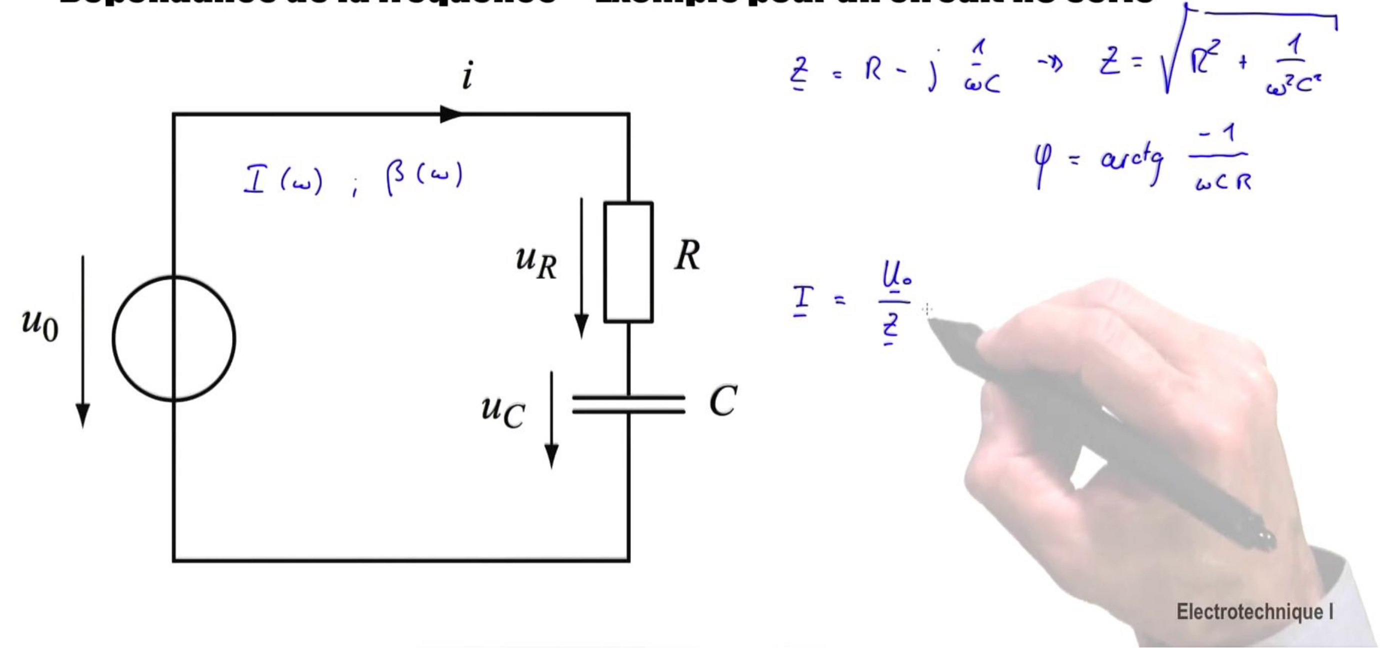
Dépendance de la fréquence – Exemple pour un circuit RC série



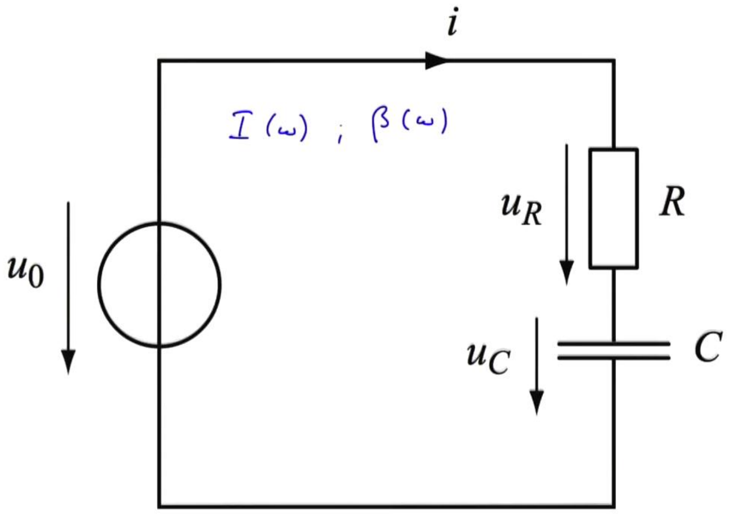
We have seen what happens at the impedance, at its norm, at its phase in the complex plan when we increase the frequency of the circuit. We will now, for this example given here, get interested in the current, namely, its norm and its phase, once again depending on the frequency. So we try to compute I of omega and beta, the current argument. So to know the current which passes through the circuit, we need, knowing here the supply voltage U0, we need to know the impedance here of the RC circuit in series. So we can write that Z = R - j(1/ωC) and then that its norm is equal to, by Pythagoras, to the real part squared plus the imaginary part squared. So we have that the norm of Z it is √R² + (1/ω²C²). This is the norm. Now the phase shift. The phase shift of the impedance, it is given by the arc tangent of the imaginary part over the real part, namely the imaginary part 1/ωC divided by the real part R. That's it, we have completely defined here the impedance Z, and now to compute the current. The current is given by the ratio of u0 to Z. So, if I express u0 in function of the argument, we have u0 times e^jα, usually we put α equal to zero, and the impedance Z which is given by the norm we have just computed multiplied by e^jφ.

Notes

Summary

13m 16s

Dépendance de la fréquence – Exemple pour un circuit RC série



$$\underline{Z} = R - j\frac{1}{\omega C} \rightarrow Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\varphi = \mathrm{arctg}\,\frac{-1}{\omega C R}$$

$$\underline{I} = \frac{\underline{U_0}}{\underline{Z}} = \frac{U_0 \cdot e^{j\alpha}}{Z \cdot e^{j\varphi}} = \underbrace{\frac{U_0}{Z}}_{I} \cdot e^{\overbrace{j(\alpha-\varphi)}^{}}$$

$$\underbrace{\qquad}_{\beta}$$

Electrotechnique I

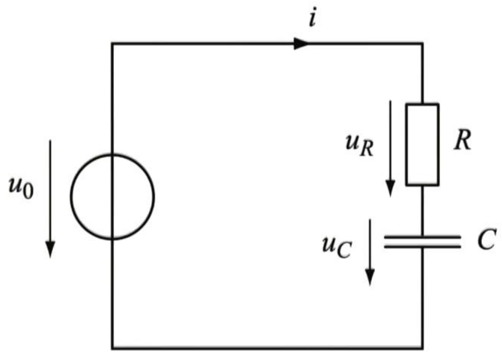
From this equation we can write the norm of the current, which is u0 over Z, multiplied by e^j(α - φ). With identification, we have that this term here is the norm of I and this term here, is the argument of I, it is beta.

Notes

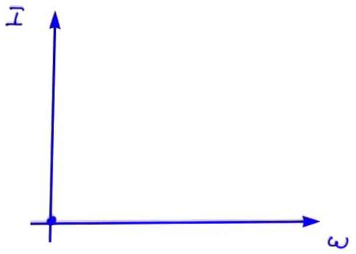
Summary

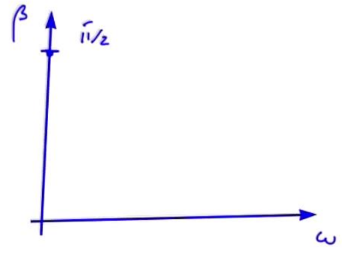
15m 22s

Dépendance de la fréquence – Exemple pour un circuit RC série



$\omega = 0 \Rightarrow I = 0 \qquad \beta = \pi/2$





$$I = \frac{U_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \; ; \; \beta = -\varphi = \mathrm{atg}\left(\frac{1}{\omega RC}\right)$$

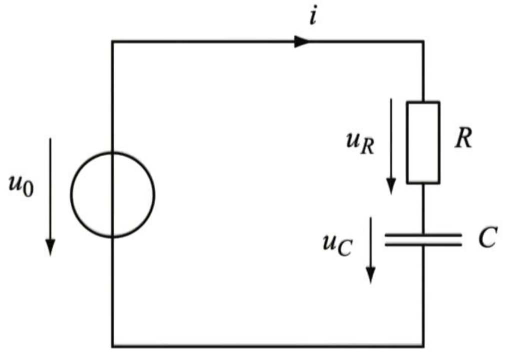
We can express the norm of I, we develop. We have that I it's u0, the supply voltage, divided by the norm of Z, namely $R^2 + (1/\omega^2C^2)$, and the angle beta, the argument of the current, it is given by $-\varphi$, namely tangent arc of $1/\omega RC$. We will analyse now these two functions, the norm of I and the argument of I in function of omega and we will represent them in a plan omega is the variable we would like to represent. We will first represent the norm of I in function of omega. And the argument of I in function of omega. For a pulsation omega equals zero, we have that I is equal to zero. why? Because this term becomes very big and then the ratio of u over something very big is equal to zero. So for a null pulsation, we have a norm of the current that goes from this point here. Concerning the phase, we have that beta is equal to pi over 2. Why? Because the vector ZC is infinite as long as omega is equal to zero, therefore the norm of C is infinite and we have an angle of minus pi over 2. and since beta is equal to minus phi, we go from this value that worth pi over two for a null frequency. In contrast, when the pulsation is infinite, the current tends to a value that is u0/R.

Notes

Summary

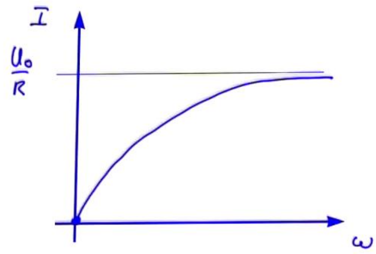
15m 47s

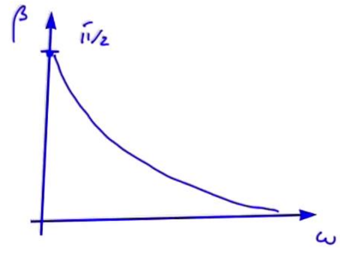
Dépendance de la fréquence – Exemple pour un circuit RC série



$\omega = 0 \Rightarrow I = 0 \qquad \beta = \pi/2$

$\omega = \infty \Rightarrow I = \frac{U_0}{R} \qquad \beta = 0$





$$I = \frac{U_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad ; \quad \beta = -\varphi = \mathrm{atg}\left(\frac{1}{\omega R C}\right)$$

We see that in this equation. The pulsation being infinite, this term is equal to zero and we have then u0/√R², is u0/R. The current, when omega equals infinite, tends to u0/R. In between, we will increase that way. For the argument of the current, beta at infinite equals zero. Why? Because for an infinite value, the capacitor behaves as a Short Circuit and this term here tends to zero. The curve of the argument goes from phi over two and tends to zero for omega equals infinite. We have completely defined the behavior of the current which passes through the circuit as a function of the frequency in terms of the norm and the argument. Thus a steady-state capacitor acts as an impedance infinite, then no current. We are at this point here.

Notes

Summary

18m 06s

And a capacitor at an infinite frequency behaves as a short-circuit and then we only have the resistive part We have defined and built the geometric locus of an impedance, here in function of the frequency, but we could also have done it in function of R, L or C for example if we wanted to take into account the aging of the components where R, L or C may change, or in function of the load that would be put in this circuit. We have seen that a circuit could change in nature depending on the frequency to which it is supplied, so either capacitive, purely resistive or inductive, and there is a particular value for which it is purely resistive, that is what we will see in the next lecture. We have seen that it is possible to delete the reactive part of an impedance but this is only possible at one given frequency. Consequently, one can also remove the reactive power in a circuit combining the element L and the element C. Finally, we have seen that if the impedance Z changes with the frequency, the current also changes with the frequency, we have shown how it evolves in terms of its norm and its phase shift as a function of the frequency. Thank you for your attention.

Notes

Summary

19m 19s