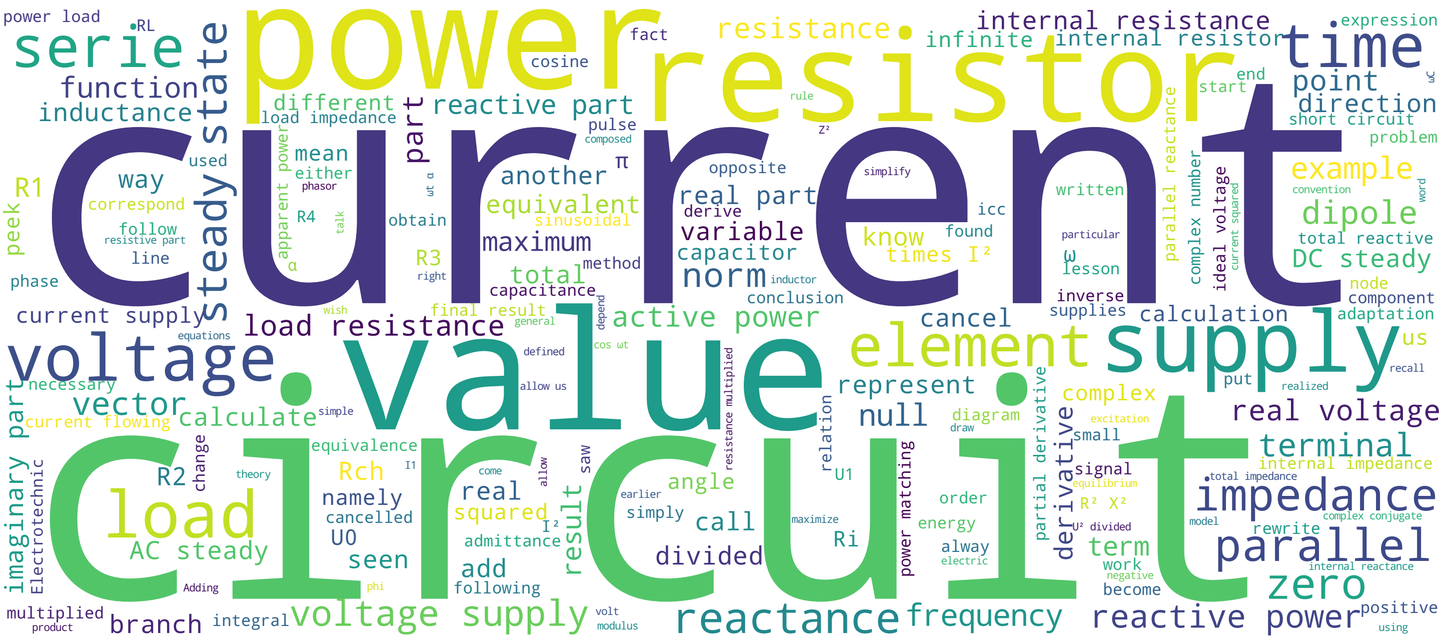
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

MÉTHODES EN RÉGIME ALTERNATIF

PUISSANCE MAXIMALE ET ADAPTATION

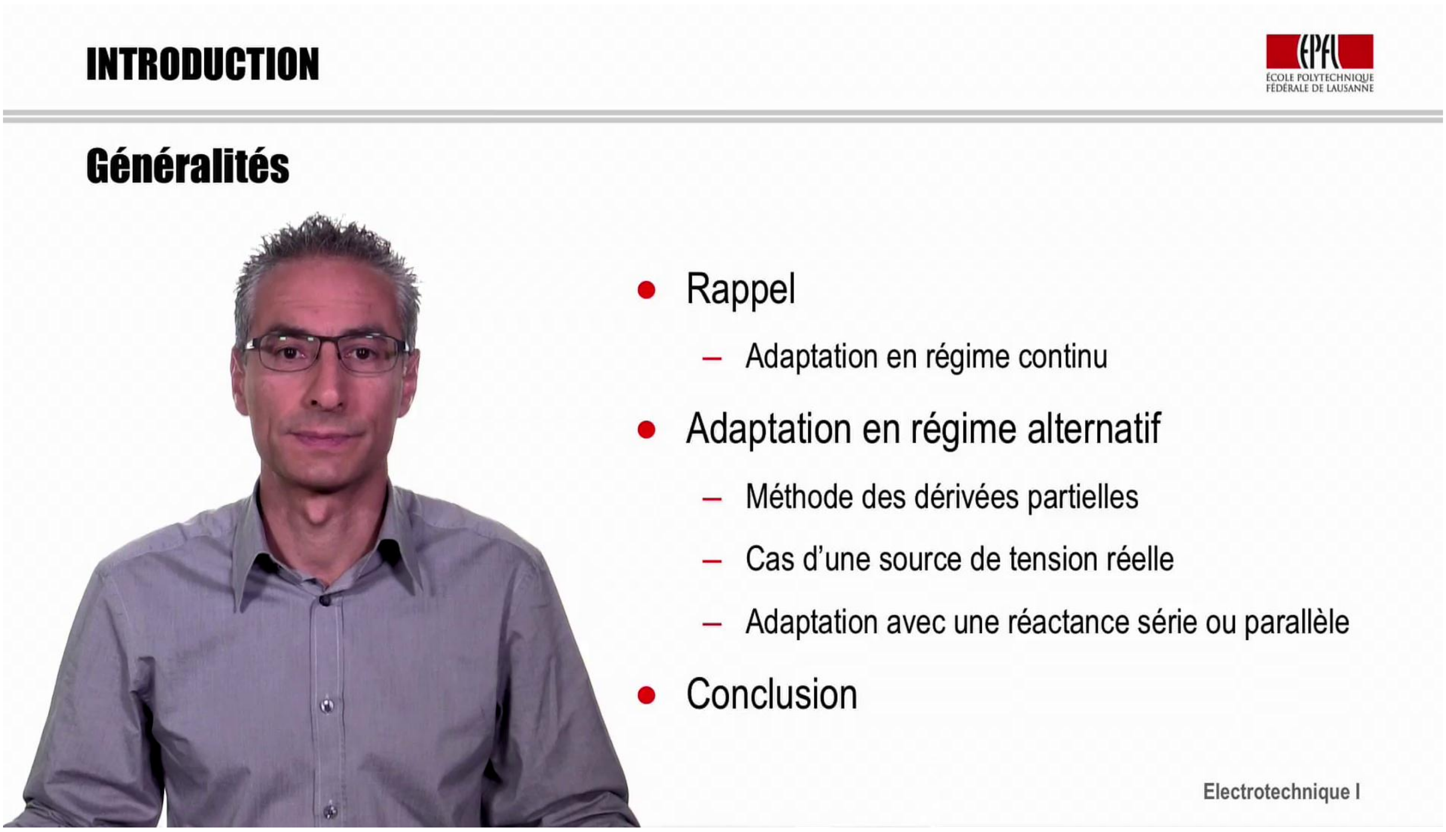
LEÇON 19

Électrotechnique I
Yves PERRIARD & Paolo GERMANO
Laboratoire d'Actionneurs Intégrés



EPFL

Video



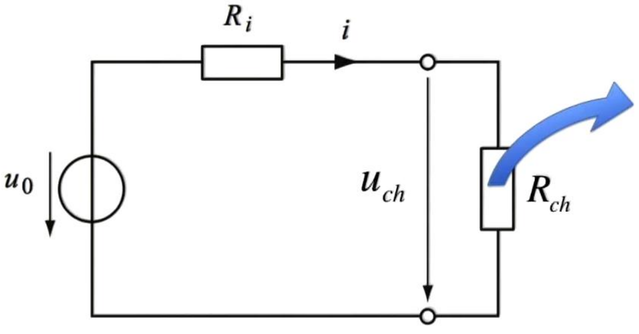
Good morning, welcome to this new lecture of the Electrotechnics course. Today, we will talk about a last point of theory that we have developed in the context of the supplied circuits under a DC steady state, and we will deal with it for the AC steady state. It comes to the power adjustment. We will first start by a recall of the results that we have found for the DC steady state. We will then develop the same theory, but adjusted to the AC steady state. We will see the case of a real voltage supply that feeds a load impedance, namely, with a reactive part. Finally, we will see the case of an adaptation with a reactance in series and a case with a reactance in parallel.

Notes

Summary

0m 03s

Puissance maximale et adaptation - Rappel



$$P_{ch} = \frac{u_0^2 \cdot R_{ch}}{\left(R_{ch} + R_i\right)^2} \qquad Max \rightarrow \frac{\mathrm{d}P_{ch}}{\mathrm{d}R_{ch}} = 0$$

Electrotechnique I

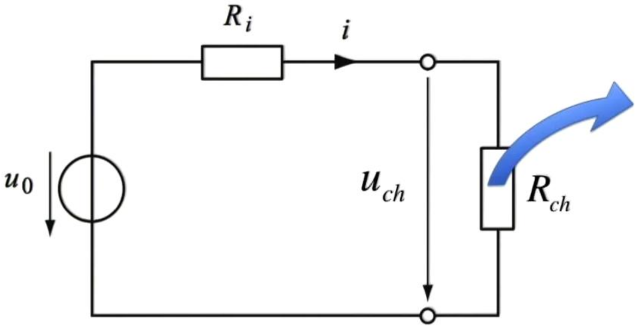
So, here we recall the circuit we have used for the DC steady state. It is a real voltage supply which is composed of an ideal voltage supply and of an internal resistor. This real voltage supply supplies a resistive load. Therefore, a current flows in the circuit. This current is here. What interested us was to calculate the power which was dissipated in the load resistance. To do so, we calculate this value of power load which is, in fact, the resistance multiplied by the current squared. We represent it here in this equation, the resistor is here, and the current squared, it is the voltage divided by the sum of the resistors, they are in series here. Then I², is equal to u over total R squared. We can see in this equation we have two extreme cases. A power which is null when the load resistance is null. So if Rch is null, the power is also null. An extreme case: f the load resistance is infinite, then the current is null. There is no current flowing in the circuit, the power is also null. Between these two points, there is a maximum, and to find that maximum, what do we do? We take this function, a load power, and we will derive it in relation to the variable, that is to say Rch.

Notes

Summary

0m 45s

Puissance maximale et adaptation - Rappel



$$P_{ch} = \frac{u_0^2 \cdot R_{ch}}{\left(R_{ch} + R_i\right)^2} \qquad Max \rightarrow \frac{\mathrm{d}P_{ch}}{\mathrm{d}R_{ch}} = 0$$

Condition : $R_{ch} = R_i$

La condition d'adaptation de puissance est donc réalisée lorsque la valeur de la résistance de charge et celle de la résistance interne de la source sont égales

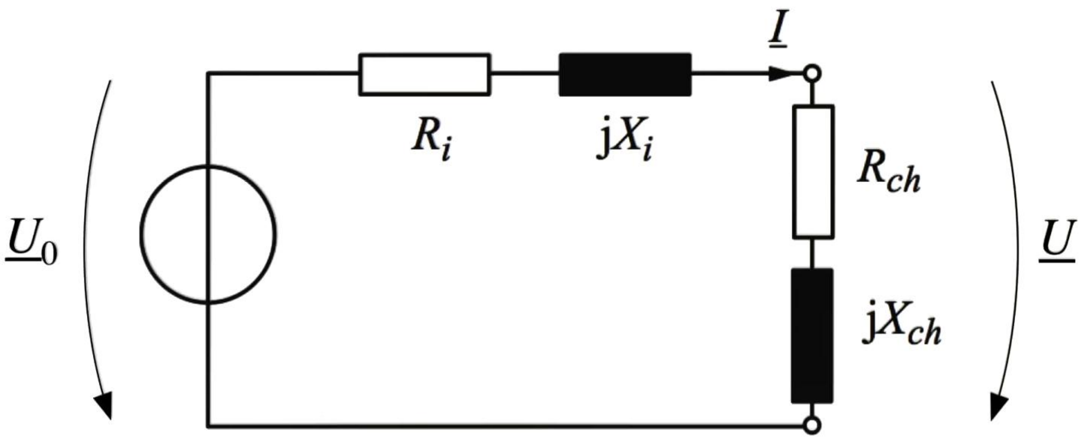
And we look for what values of Rch this derivative is zero, and it will allow us to find the value Rch for which the power is maximum. We have found that in order that the power is maximal, we also speak of power matching, the condition was that the value of the internal resistance of the supply and the value of the load resistances must be equal. And the power matching condition is therefore realized when the value of the load resistance and of the internal resistance are equal.

Notes

Summary

2m 21s

Puissance maximale et adaptation - AC



Electrotechnique I

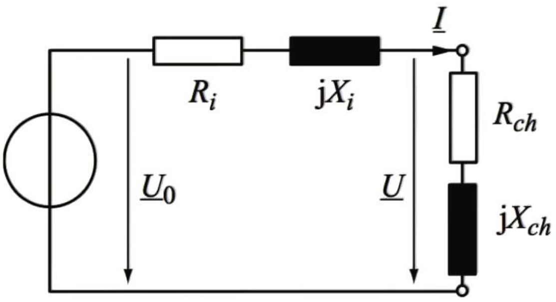
As part of a circuit under an AC steady state, we perform the same approach, considering, to stay in a general case, a real voltage supply with a load with a resistive part, the internal resistance, here, of the supply, and the resistance of the load, first thing. And a reactive part, it is an internal reactance of the supply and the reactance of the load.

Notes

Summary

2m 59s

Puissance maximale et adaptation - AC



$$P_{ch} = R_{ch} \cdot I^2 = \frac{U_0^2 R_{ch}}{(R_i + R_{ch})^2 + (X_i + X_{ch})^2}$$

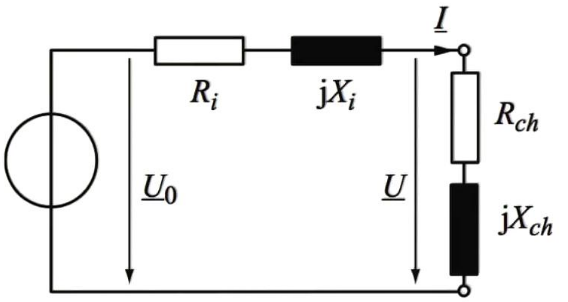
We write now the expression of the active power in the load, here, Pch, it is the variable which one wishes to find the maximum value. This value is given by Rch times I², namely: Rch multiplied by I², and therefore the modulus of I², and the modulus of I is given by: the voltage U0 divided by the norm of the total impedance of the circuit, namely, U0², divided by (Ri + Rch), the resistive part, squared, then the reactive part of the supply, plus the reactive part of the load, squared. So this is Z², the norm of the square of the impedance vector. We rewrite here the equation a little bit cleaner, and we derive this expression of power load that we would like to maximize, with respect to two variables of the load, that is to say Rch and XCH, we will have to make a partial derivative. First, relative to the load resistance Rch the result is the same as in DC steady state, we won't do it again here. And second, a partial derivative with respect to another variable, which is the load reactance. So, we will write the partial derivative of the power load, with respect to the variable Xch, which must be zero.

Notes

Summary

3m 27s

Puissance maximale et adaptation - AC



$$P_{ch} = R_{ch} \cdot I^2 = \frac{U_0^2 R_{ch}}{(R_i + R_{ch})^2 + (X_i + X_{ch})^2}$$

$$\frac{\partial P_{ch}}{\partial X_{ch}} = 0 \quad \rightarrow \quad \dots \quad \rightarrow \quad X_i = -X_{ch}$$

$$\boxed{\text{Condition :} \quad \underline{Z}_{ch} = R_{ch} + jX_{ch} = R_i - jX_i = \underline{Z}_i^*}$$

En régime alternatif, la condition d'adaptation de puissance est réalisée lorsque la valeur de l'impédance de charge et celle de l'impédance interne de la source sont conjugués complexes

Electrotechnique I

So, we won't do again the same calculation, because it is exactly the same as the one for the derivative with respect to Rch, but we obtain a result as follows, which says that the derivative is zero for the value of X is equal to minus Xch. That is to say that the derivative is zero, so the power is maximum, when the internal reactance is equal to the opposite of the load reactance. That's it, based on this final result, here, one can draw the conclusion, is that under the AC steady state, the power matching condition is realized when the load impedance value, here, the load impedance, this elements here, and the value of the internal impedance of the supply, this part here, this part here, are complex conjugates. Namely, the real part is the same and the imaginary part is opposed. The reactive power, since both reactances are cancelled, is therefore also cancelled.

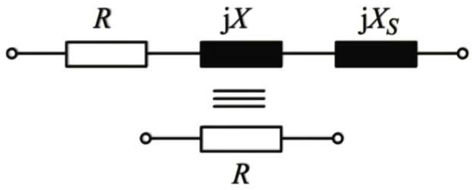
Notes

Summary

5m 12s

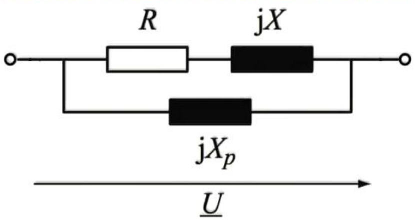
Adaptation par adjonction d'une . . .

- **réactance série :**



$$X_s = -X$$

- **réactance parallèle :**



$$P = \frac{R\,U^2}{R^2 + X^2}$$

Electrotechnique I

The adaptation of the impedance tends to make it minimal, so its norm, by adding a reactance in series or in parallel. The purpose is therefore to cancel the reactive part to make the resulting impedance purely resistive. By canceling the total reactance, the current is then maximum because the norm of the impedance is minimal, and therefore, the active power will be maximal. Adding here a reactance in series the solution is easily found. We only need to write Xs, the reactance in series must be equal to -X. We therefore have these two terms which cancel, it only remains the resistance R. Adding a reactance in parallel, here, we can advantageously work with the powers. For the dipole considered, which is the same as the one earlier, we will add the reactance in parallel, and we wish to cancel the total reactive power of the dipole. In the base branch, this one, we can write the two powers, the active power which is equal to R times I^2, of the current flowing through this branch, therefore we have R times I^2, and the value of I, is U over the norm of Z, namely, U^2 divided by $R^2 + X^2$. For the reactive power, once again, in this branch, we can write that Q is equal to X times I^2 then the resistance multiplied by I^2 which has the same value, it's U^2 divided by the norm of Z^2.

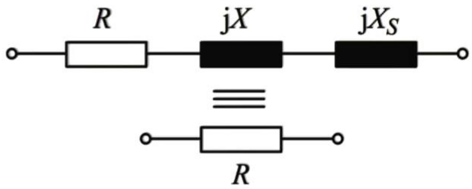
Notes

Summary

6m 29s

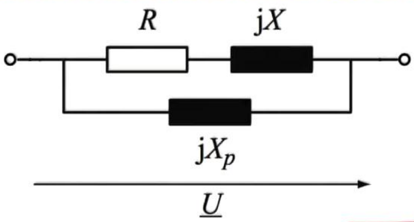
Adaptation par adjonction d'une . . .

- **réactance série :**



$X_s = -X$

- **réactance parallèle :**



$$\begin{cases} P = \dfrac{R\,U^2}{R^2+X^2} \\ Q = \dfrac{X\,U^2}{R^2+X^2} \end{cases}$$

$Q_p = \dfrac{U^2}{X_p}$ $\quad Q_{tot} = Q + Q_p = 0$

$\cdot \dfrac{1}{U^2} \quad \rightarrow \dfrac{X}{R^2+X^2} = -\dfrac{1}{X_p} \quad \longrightarrow \quad \boxed{X_p = -\dfrac{R^2+X^2}{X}}$

Electrotechnique I

So what interests us is mainly this point. This is valid for this branch. At the reactance here in parallel, we can write that the power Qp which passes in the parallel reactance and it is simply U²/Xp. And what we want is that these two reactive powers cancel out so that the sum is equal to zero. We can then write that the total reactive power which is equal to Q + Qp, is equal to zero. If we rewrite this, it is simplified by U², we obtain that X/(R² + X²), must be equal to -1/Xp. And it follows that the value of the parallel reactance we should add to cancel the reactive part, the reactive power, must be equal to -R² + X² divided by X. So for this parallel reactance that we add in order to adjust the supply, we have the final result in this window Xp.

Notes

Summary

8m 30s

CONCLUSIONS

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

- En régime alternatif, pour adapter une charge à une source, il faut réunir deux conditions :
  - $R_i = R_{ch}$
  - $X_i = -X_{ch}$

→ L'impédance $\underline{Z}_i$ de la source et l'impédance $\underline{Z}_{ch}$ de la charge doivent être <u>conjugués complexes</u>

- Adapter la charge à la source : supprimer la puissance réactive Q

Electrotechnique I

That's it, in conclusion, we can say that in the AC steady state, to adapt a load to a supply and therefore maximize the output active power, we need two conditions: as in DC steady state, it is necessary that the values of internal resistance and of load resistance are equal, it's this point here. Also, we must delete the total reagent, The reactance of the supply and of the load should then be opposite, it is this equation here. In the end, it is necessary that the internal impedances, of the supply and the load, are complex conjugates. Therefore, adjust the load to a supply or vice versa, also means removing the reactive power, here, we have seen it in the example treated. Thank you for your attention.

Notes

Summary

10m 13s