ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

PUISSANCES EN ALTERNATIF SINUSOÏDAL MONOPHASÉ

LES DIFFÉRENTES PUISSANCES

LEÇON 17

Électrotechnique I
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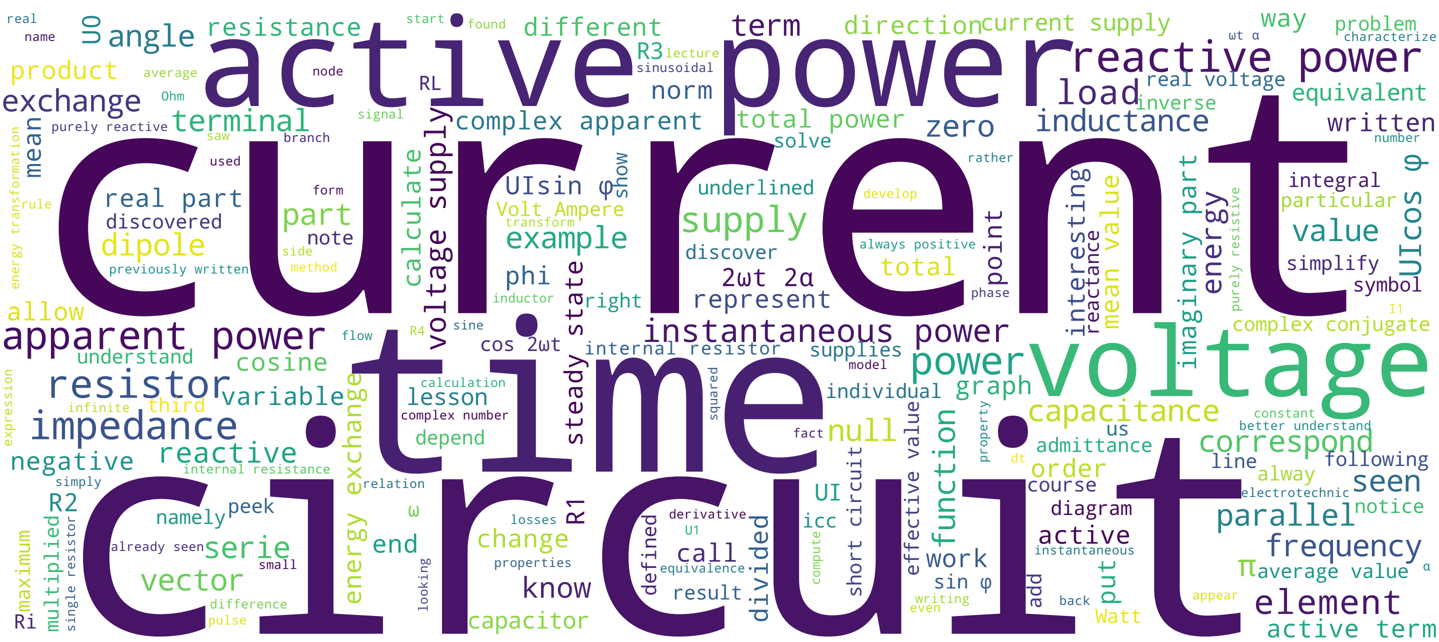
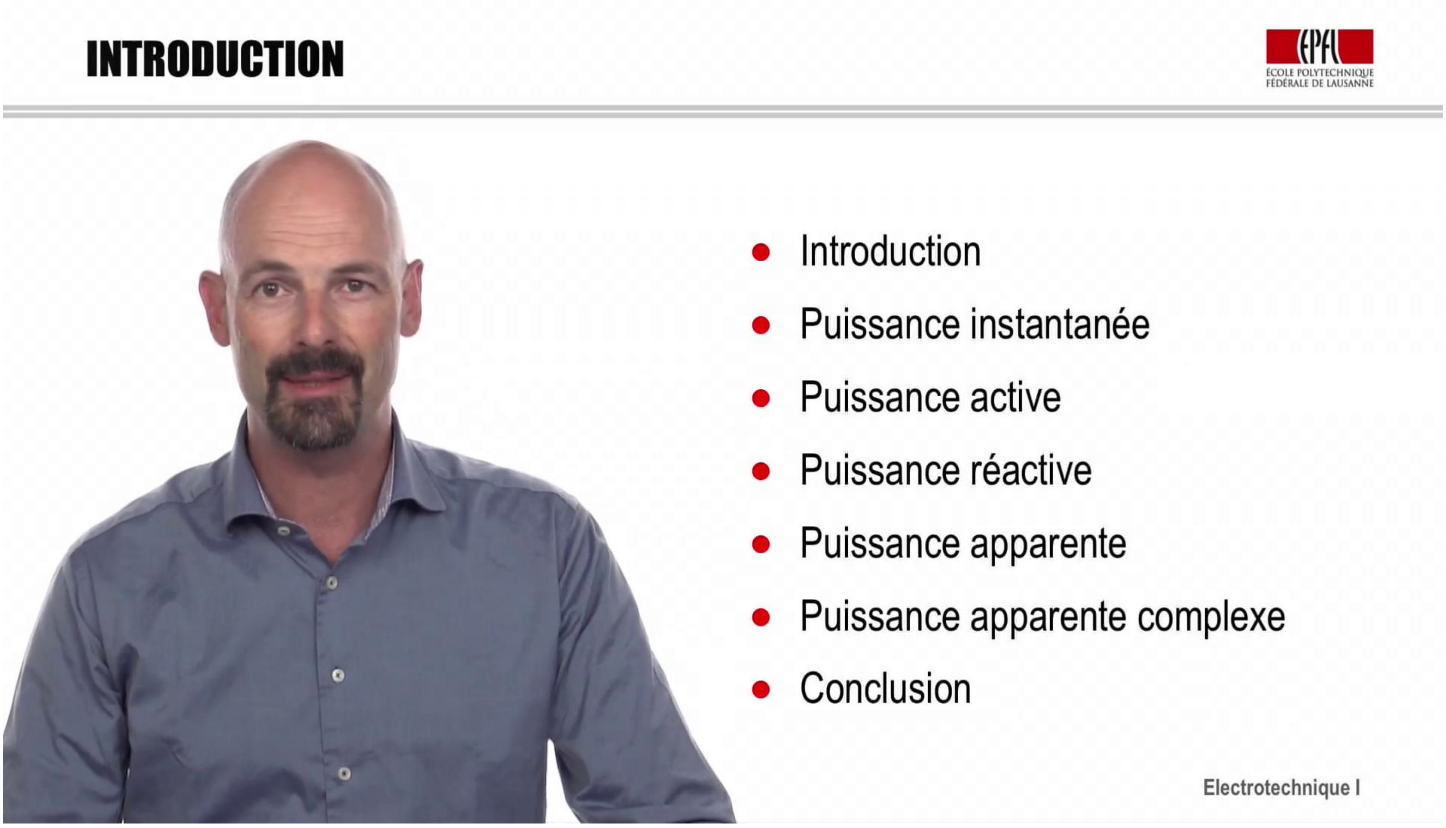


Video

EPFL



Madam, Sir, good morning and welcome to this new lecture about powers. We will discover during this lecture that it exist not only one power with only one name, but several types of powers, starting with the instantaneous power then we will discover what are the active, reactive, apparent, and also the complex apparent powers.

Notes

Summary

0m 03s

$$p(t) = \hat{U}\hat{I}\cos(\omega t + \alpha)\cos(\omega t + \beta)$$

$$= \frac{\hat{U}\hat{I}}{2}\left[\cos(\alpha - \beta) + \cos(2\omega t + \alpha + \beta)\right]$$

$$= UI\cos\varphi + UI\cos(2\omega t + \alpha + \beta) \qquad \text{avec} \quad \varphi = \alpha - \beta$$

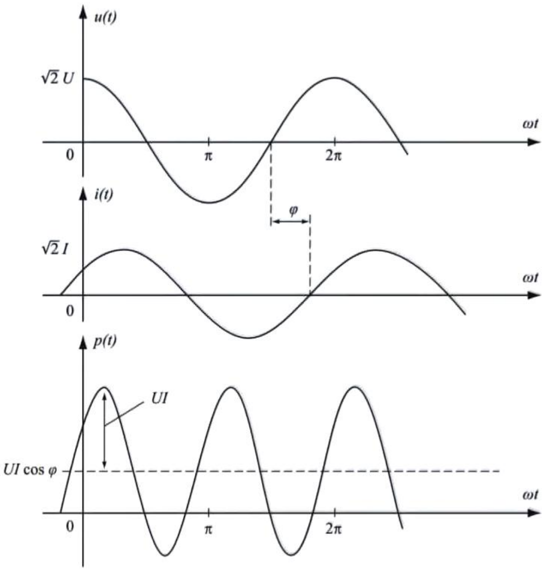
First of all, the instantaneous power. We can right what we already know from a long time ago about the power. The power being defined by the product of the instantaneous values of the voltage and the current. We can then write that this instantaneous power product of the current and the voltage, but in instantaneous values, gives that. After small development, we will end up with an equation or a clearer result in order to understand what means this instantaneous power. First of all, a linked part here not to the time but rather to an angle and then a cosine whose frequency is twice which gives, for the effective values here and not anymore peak values in order to simplify the writing... So, to simplify here, I note that phi, as we've already seen previously, it is the difference of angle between the voltage's angle and the current's angle. This instantaneous power function has a constant component UIcos φ, as you see, it doesn't depend on time, and a sinusoidal component of amplitude UI and of a double frequency. What's interesting to do now, it is to draw the graph of this function in order to better understand what's happening in terms of exchange.

Notes

Summary

0m 25s

PUISSANCE INSTANTANÉE

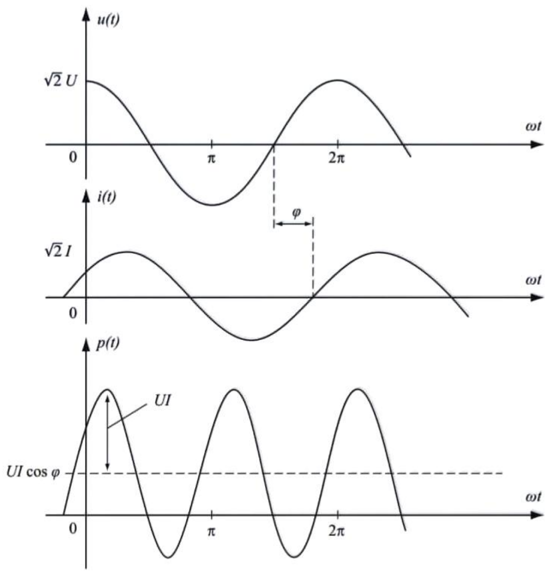


First of all, we've represented here, high above, the graph of the voltage U(t), instantaneous voltage, then, the current graph as a function of time I(t). And finally, that instantaneous power we've just computed. We remark here, the dotted line that corresponds to UIcos φ, it's that constant that we've talked about before, that we've found in the equation of the instantaneous power. And around this constant, an added a cosine at the double frequency. We therefore see that, from time to time, we have a exchange of positive power even, here in this graph, an exchange of negative power, but, in average, we have this exchange here: UIcos φ. It is now interesting, to develop what w have previously written to write it a bit differently. We will bring out two fundamental components of the instantaneous power under sinusoidal steady state. First, we put: the angle beta which is equal to alpha minus phi and we also put the following trigonometric identity: cos (2ωt + 2α - φ) It is equal to : cos φ cos (2ωt + 2α) + sin φ sin (2ωt + 2α).

Notes

Summary

2m 04s



pose $\beta = \alpha - \varphi$, pose :

$$\cos(2\omega t + 2\alpha - \varphi) = \cos\varphi \cos(2\omega t + 2\alpha) + \sin\varphi \sin(2\omega t + 2\alpha)$$

$$p(t) = UI\cos\varphi\left[1 + \cos(2\omega t + 2\alpha)\right] + UI\sin\varphi\,\sin(2\omega t + 2\alpha)$$

Electrotechnique I

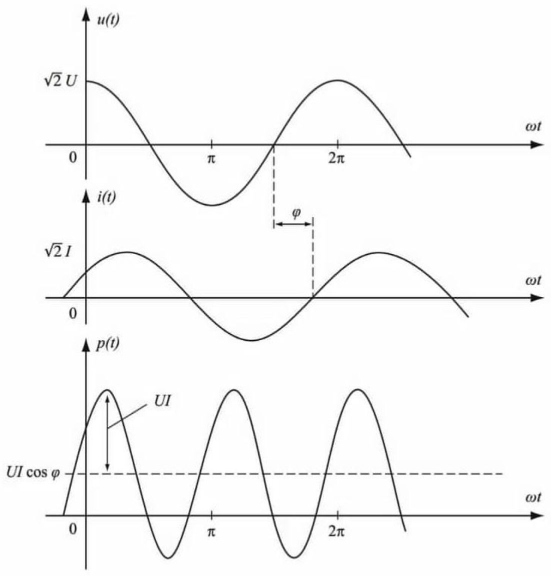
It looks more complicated, it even complicates the equation that we have seen before, but if we replace this identity in the pulsating part of the instantaneous power that we have previously written, we end up with the following result: this instantaneous power that will be equal to to UIcos φ that multiplies [1 + cos (2ωt + 2α)] + UIsin φ sin (2ωt + 2α). So, here we will see something very interesting in these two components, the first one, that corresponds to the first term of the second member, is a component pulsed always positive, it's: cos (2ωt + 2α) always positive, hovering around the average value φ UIcos. It translates an exchange of a unidirectional energy between a supply and a load. Then, the second part corresponds to the second term of the second member, and it is an alternative component that varies sinusoidaly with the amplitude UIsin φ and then, the average value is always equal to zero. It is then alternatively positive and negative, and it shows here, an oscillatory and reversible energy exchange between the supply and the load.

Notes

Summary

3m 42s

PUISSANCE INSTANTANÉE



pose $\beta = \alpha - \varphi$, pose :

$$\cos(2\omega t + 2\alpha - \varphi) = \cos\varphi \cos(2\omega t + 2\alpha) + \sin\varphi \sin(2\omega t + 2\alpha)$$

$$p(t) = \underbrace{UI\cos\varphi\left[1 + \cos(2\omega t + 2\alpha)\right]}_{\text{Puissance active}} + \underbrace{UI\sin\varphi \sin(2\omega t + 2\alpha)}_{\text{Puissance réactive}}$$

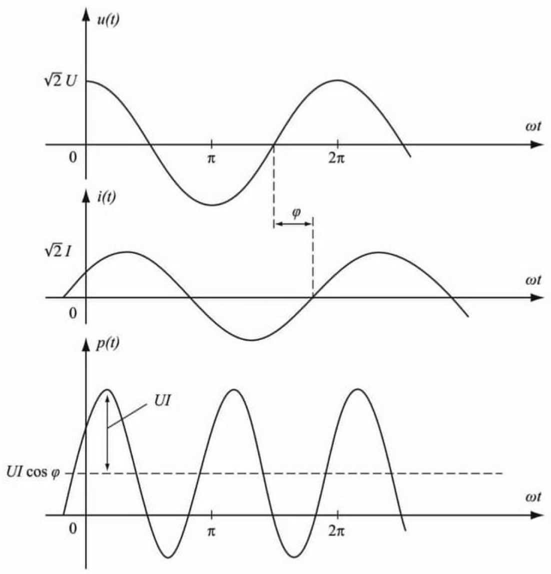
Electrotechnique I

So, we can already observe something, when phi is equal to zero, namely, a load purely resistive, the cosine of phi here will be equal to 1, the sine of phi here will be equal to 0, the average value UIcos φ is therefore maximal and is equal to U time I whereas the second part, as we've already said before, will be null. On the contrary, there is another extreme, when phi is equal to π/2 or -π/2, then the load is purely reactive, an inductance or a capacitance, so in this case, it is this power which will be maximum because sin φ will be equal to 1 or -1 whereas the UIcos φ will be equal to 0 and this part becomes zero. We will then define here this first part as being an active power and this second part. reactive power. This active power indicates an energy transformation. This energy will be transformed, will be used, whereas the reactive power is an energy exchange between the supply and the load but without transformation. It is the fundamental part of our course that we will see during all that lecture and we start by defining now one after the other, that active power and that reactive power.

Notes

Summary

5m 16s



pose $\beta = \alpha - \varphi$, pose :

$$\cos(2\omega t + 2\alpha - \varphi) = \cos\varphi \cos(2\omega t + 2\alpha) + \sin\varphi \sin(2\omega t + 2\alpha)$$

$$p(t) = \underbrace{UI\cos\varphi\left[1 + \cos(2\omega t + 2\alpha)\right]}_{\text{Puissance active}} + \underbrace{UI\sin\varphi \sin(2\omega t + 2\alpha)}_{\text{Puissance réactive}}$$

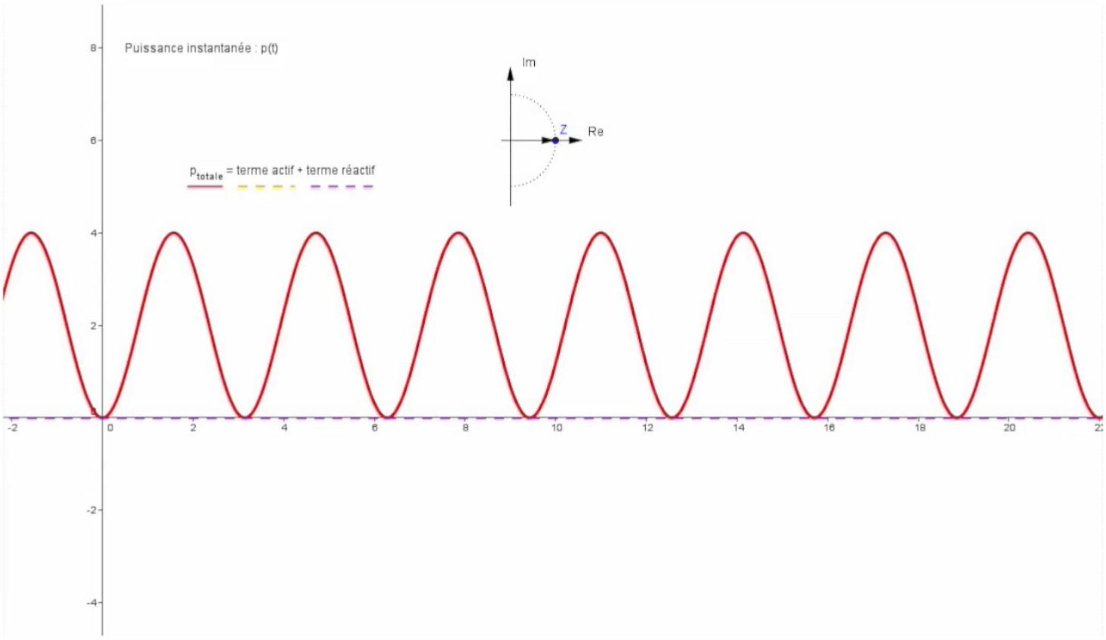
But before that, and to better understand, I would like to show you now a movie that will allow you to visualize that exchange of energy either active or reactive or both at the same time when in the presence a load of both resistive and reactive As you may seen in this graph we have drawing here the total power, with the reactive and active terms and you see actually in the top middle, in the imaginary plane, the vector of the impedance Z.

Notes

Summary

6m 46s

PUISSANCE INSTANTANÉE



Electrotechnique I

We see that impedance Z that changes. Now, that impedance is purely reactive. We then see that the reactive term in dotted purple lines, is maximum whereas the active term is null. We will see when that impedance varies again the change of this power, with once again a passage with an impedance purely reactive, at that moment, the active term is complete and the total power is equal to the active term. And here again a change arriving to a capacitance, because Z has now an angle of -π/2 (capacitor) and you have again the reactive power that completely dominates and the active term which is null. Here again a change, in other to well understand that energy exchange between the supply and the load. And as I mentioned previously, either that exchange is purely alternative in supply and load, or we have then an energy transformation.

Notes

Summary

7m 22s

Définition de la puissance active : la moyenne de $p(t)$

$$\overline{p(t)} = P = \frac{1}{T}\int_0^T p(t)\,dt = UI\cos\varphi$$

a) Résistance : $\varphi = 0 \rightarrow P = UI$

b) Inductance : $\varphi = \frac{\pi}{2} \rightarrow P = 0$

c)

Electrotechnique I

We define now the active power. As we have seen before, this active power is the mean value of the instantaneous power, this is the definition of the active power, the mean of the instantaneous power p(t). We can then write that this average of p(t), which is written as p(t) with a bar, which is equal to capital P, we will define this as a variable now, P will mean the active power, or the mean value of the instantaneous power, and by definition, the mean value is the integral from 0 to T of the function p(t)dt and this mean value if we reconsider the equations previously written, it is only the part that does not vary with time that to say UIcos φ. So, to be very clear, we can maybe already do three small cases. If we take the case of the resistor. A resistor has an angle of the impedance which is null and that will make this active power equals, for only a single resistor, to U times I. For the case of the impedance. The inductance has an angle equals to π/2, which leads us to an active power = 0. The cosine of π/2 will be equal to 0, So here we have an active power which is null. And for the capacitance, the angle is equal to -π/2 and the active power is still equal to 0.

Notes

Summary

8m 29s

PUISSANCE ACTIVE

Définition de la puissance active : la moyenne de $p(t)$

$$\overline{p(t)} = P = \frac{1}{T}\int_0^T p(t)\,dt = UI\cos\varphi$$

a) Résistance : $\varphi = 0 \rightarrow P = UI$

b) Inductance : $\varphi = \frac{\pi}{2} \rightarrow P = 0$

c) Capacité : $\varphi = -\frac{\pi}{2} \rightarrow P = 0$

Electrotechnique I

The active power, measurable by a power meter, corresponds to a real supply of convertible work or heat energy. And, we can see it here, this is possible only with a resistor. The two other elements, inductance and capacitance, can't transform that energy, convert it to work or heat and, as mentioned before, the active power is maximum in case of load purely resistive, this is what we see in this small table.

Notes

Summary

10m 18s

PUISSANCE RÉACTIVE

Définition: $Q = UI \sin\varphi$ [var]

Puissance active $P = UI\cos\varphi$ [W]

" réactive $Q = UI\sin\varphi$ [var]

Electrotechnique I

We can then define now a reactie power. Definition: This reactive power, you have seen it before, there is an exchange of energy. With the active power we notice that, when we try to compute the active power of an inductance or a capacitance, we end up with a result null and yet we know that there is an exchange of energy. We have seen it in the equation, where we clearly see these two different terms. this exchange is described being oscillating around a value UIsin φ. We will define this reactive power. We will choose the symbol or the letter Q as UIsin φ that corresponds to this alternative exchange of energy. So, so that there is no confusion, we will give to it an other variable than the active power, written in Watt, and this reactive power will be written as var for Volt Ampere Reactive. Therefore, the reactive power P, which is equal to UIcos φ, is measured in Watts and the reactive power defined here, is measured in Volt Ampere Reactive in order to avoid any confusion. It's actually here, for the reactive power, a fictive power. It doesn't respond to a real physical definition but it allows us to characterise that nonconvertible exchange of energy that appears in the case of a reactive load: Inductance or capacitance.

Notes

Summary

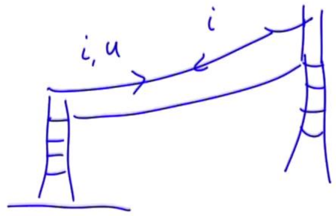
10m 49s

PUISSANCE RÉACTIVE

Définition: $Q = UI \sin \varphi$ [var]

Puissance active $P = UI \cos \varphi$ [W]

" réactive $Q = UI \sin \varphi$ [var]



Electrotechnique I

Although this exchange corresponds to a zero balance after an integer number of periods, it must nevertheless take account the circulation of this energy in particular on a distribution network. If you have here a pylon with wires from one load to another, the energy that flows here, we have a current i, a voltage between the wires, even though this current is going back and forth as function of time, fifty times per second, and even if finally this balance is zero, we will have losses on the line and these losses have an overall impact on the network. So it is necessary to characterize this exchange by this so-called reactive power.

Notes

Summary

12m 46s

PUISSANCE APPARENTE

Définition : $S = UI$ $[VA]$

cette grandeur est liée aux puissances active et réactive :

$$P = UI\cos\varphi$$
$$Q = UI\sin\varphi$$
$$S = \sqrt{P^2 + Q^2}$$

Electrotechnique I

We will again define a third power. If we consider the two first were reactive and active, the third one is called apparent power. It corresponds to the product of the effective values of the voltage and the current. It is expressed conventionally, to distinguish it, in yet another unit as we saw earlier which are the Volt Ampere. This product has the dimension of a power but it does not necessarily provide a work hence the name of apparent power. By definition, this power which will take the S symbol, is the product of U times I. As now said, we will note it in Volt Ampere. Is the amplitude of the fluctuations of the instantaneous power compared to its average value so called apparent power. This variable is finally linked to the two powers that we have seen before: the reactive and the active power, because if I remind you that P is equal to UIcos φ, that Q is equal to UIsin φ then it seems almost obvious here, when we know the trigonometry calculations, that S is equal to $P^2 + Q^2$. Then the apparent powers, corresponding to a module, can not be added algebraically because of this.

Notes

Summary

13m 36s

Définition : $S = UI \quad [VA]$

cette grandeur est liée aux puissances active et réactive :

$$P = UI\cos\varphi \longrightarrow P_{tot} = \sum_{i=0}^{k} P_i$$

$$Q = UI\sin\varphi \longrightarrow Q_{tot} = \sum_{i=0}^{k} Q_i$$

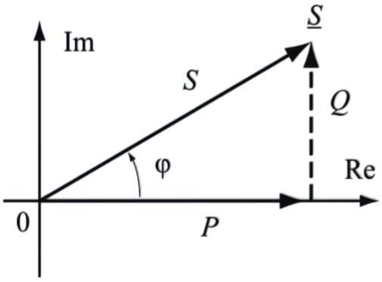
$$S = \sqrt{P^2 + Q^2} \longrightarrow S_{tot} \neq \sum_{i=0}^{k} S_i$$

Contrariwise, one of the properties is that if one is looking at the total power in a circuit, the total power in a circuit is the sum of individual P and the individual is from 0 to k, the number of items that we have in the circuit. We can do the same for the total reactive power, it is the sum of i from 0 to k of individual Q. This is a very good way, that will be seen in another lesson, we can use this property to solve the problems. On the other side, beware the total S is not equal to the algebraic sum from 0 to k of Si. We can only do that on the active and reactive power but not on the apparent power because of this relation that you have here and making things non-linear and therefore impossible to solve.

Notes

Summary

15m 16s



Définition : $\underline{S} = P + jQ = S\,e^{j\varphi}$

$\underline{S}_{tot} = \sum_{i=0}^{K} \underline{S}_i$

To finally allow the possibility of having the sum of these powers corresponds to the sum of the different elements, a new power is invented, another new power called complex apparent power. It is a pure conceptual product to mathematically finally allow to do the sum, this time a vector sum, of the different apparent powers of the elements correspond to the total complex apparent power of the circuit. We will therefore define this complex apparent power as being always S, but now it is vectorial, it is underlined, P + jQ or Se(jφ) Once again, it is purely conceptual. Here you have in the imaginary plane written the vector S with its purely real component P, its purely complex component Q. It brings together the various powers previously defined by making its real part the active power and its imaginary part the reactive power. Here, the property is that this total underlined S, but this is vectorial, this time is equal to the sum, but this time vector sum, of the different apparent powers. It is interesting to see again some properties of the complex apparent power.

Notes

Summary

16m 14s

PUISSANCE APPARENTE COMPLEXE

$\underline{I}^* = I e^{-j\beta}$

$\underline{Z} = R + jX$

On peut écrire : $\underline{S} = \underline{U} \cdot \underline{I}^*$

$$\underline{S} = \underline{Z} \cdot \underline{I} \cdot \underline{I}^* = \underline{Z} \cdot I^2 = \underbrace{R \cdot I^2}_{P_{active}} + j \underbrace{X I^2}_{Q_{réactive}}$$

$P = UI \cos\varphi = R \cdot I^2$

$Q = UI \sin\varphi = X \cdot I^2$

Electrotechnique I

In fact, if we define the complex conjugate of the current as being this current with a negative angle relative to the normal current, we can say that the complex apparent power it is U times I complex conjugate. We will demonstrate here. We take an impedance Z which is equal to R + jX. We will replace in this expression, the expression written here: U times I conjugate complex, replacing U with the equation of the law of Ohm Z times I. This becomes S underlined, we replace U with Z.I that further multiplies the complex conjugate I. One of the well-known relations is that a vector multiplied by its complex conjugate becomes purely real, we have then here Z underlined that multiplies I^2 but which is not a vector anymore, it is just I^2. And if we write it ZI^2 and we develop it it becomes obvious that we have $RI^2 + jXI^2$ What have we just discovered? We've just discovered here the active power, that corresponds to the Joule losses, and here the reactive power, that corresponds to the exchange of energy related to the reactance X which may be here a capacitance or an inductance. We can then write and summarize that P, which is equal to UIcos φ, is also equal to RI^2 and that the reactive Q, which is equal to UIsin φ, is also equal to XI^2, another way to compute the active power or the reactive power if we know for instance the impedance value and, even better, the current value.

Notes

Summary

17m 51s

Type de puissance	P Puissance active [W]	Q Puissance réactive [Var]	S Puissance apparente [VA]
R résistance	UI	0	UI
L inductance	0	UI	UI
C capacitif	0	− UI	UI

Electrotechnique I

I come to the synthesis of our lesson where I summarized here, on this graphic or rather on this table, the active power, the reactive power and the apparent power for a purely inductive, capacitive or resistive load. We summarize and we note that, for a single resistor, the active power, as it is equal to UIcos φ, will be equal to UI. On the other side, as we have already seen, the reactive power UIsin φ, if sin φ is equal to 0, this is null and the apparent power is always equal to UI. Then, the inductance. The inductance, as we have seen for the active power, the angle of this impedance is equal to π/2, we have here an active power null whereas the reactive power is equal to UI, and the apparent power doesn't depend on the angle so it is always UI. And finally for the capacitance where the angle is equal to -π/2, the active power is alway null, the reactive power is negative -UI, and finally the apparent power is equal to UI. We first remark here for those who discover the active and reactive powers, and especially those who are used to the active power which is always strictly positive, they are Watts, we can only see 10, 100, 100 Watts but not negative. The reactive power, cant be negative.

Notes

Summary

20m 14s

Type de puissance	P Puissance active [W]	Q Puissance réactive [Var]	S Puissance apparente [VA]
R résistance	UI	0	UI
L inductance	0	UI	UI
C capacitif	0	− UI	UI

Energie non convertible

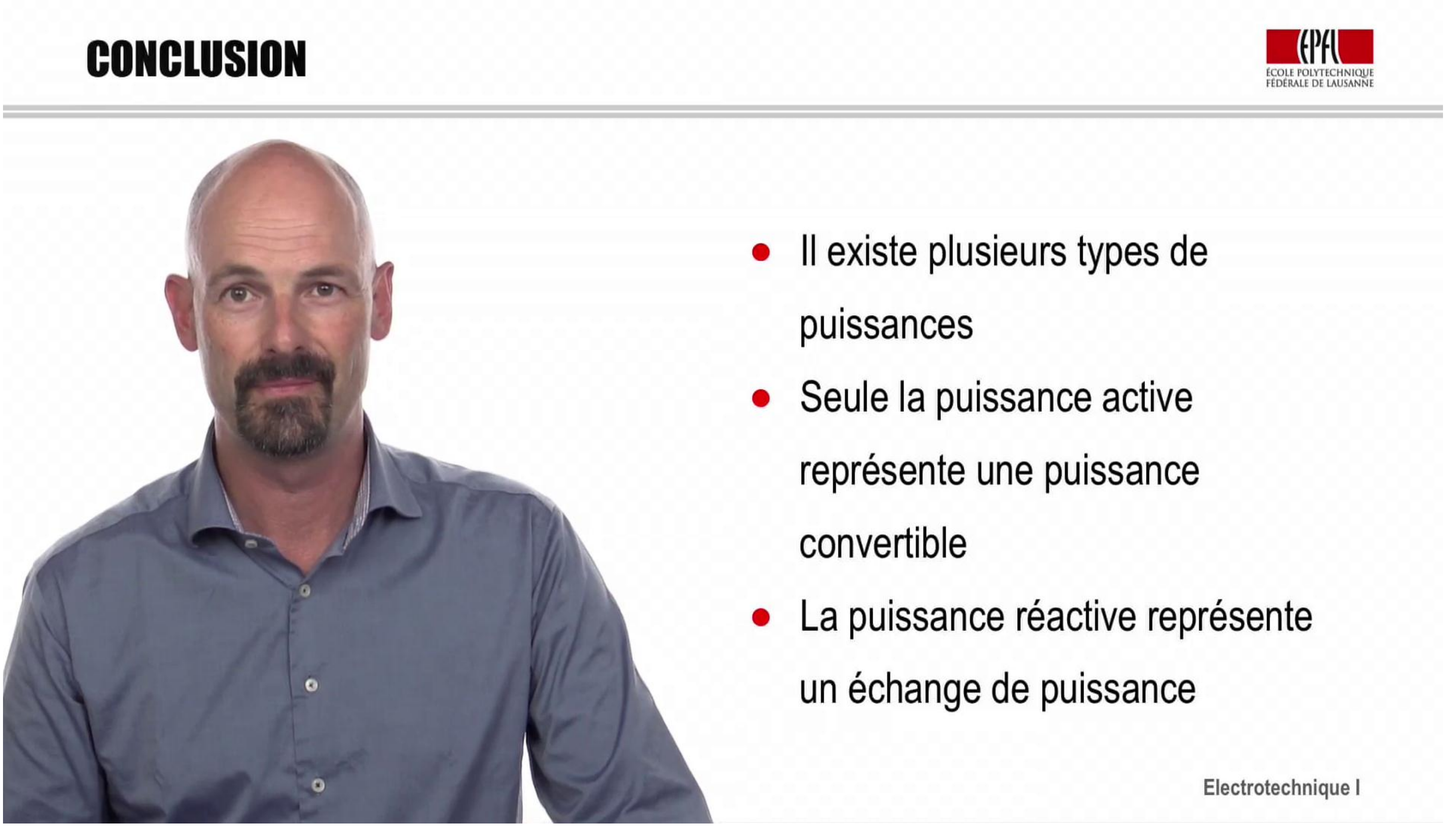
Electrotechnique I

It will characterize the load and its reactive value, either inductance or capacitive depending on the plus or the minus. the apparent power is always positive. We notice then that we have two elements: the inductance and the capacitance, whose power is not convertible, or the energy is not convertible. As we see, the active power being null, this inductance and this capacitance can only create energy exchanges of power exchanges in the circuit. Only the resistance allows this energy transformation into work or heat.

Notes

Summary

21m 46s



That's it, as you have discovered during this lesson, we have seen a multitude of different powers: the instantaneous power which then is characterized by active, reactive, apparent and complex apparent powers. Only the active power represent a power or a convertible energy and the reactive power, corresponds to an energy exchange between the supply and the load.

Notes

Summary

22m 31s