MÉTHODES EN RÉGIME ALTERNATIF

THÉORÈMES DE THÉVENIN ET DE NORTON – PRINCIPE DE SUPERPOSITION

LEÇON 16

Électrotechnique I

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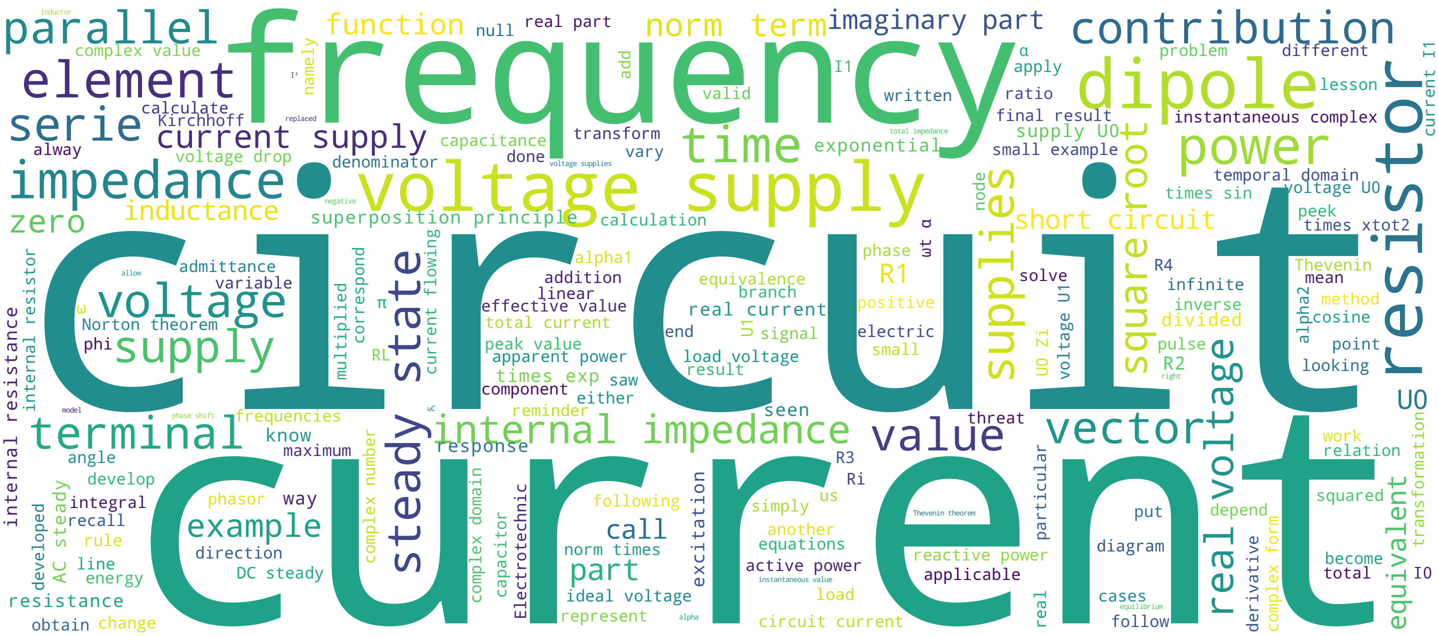
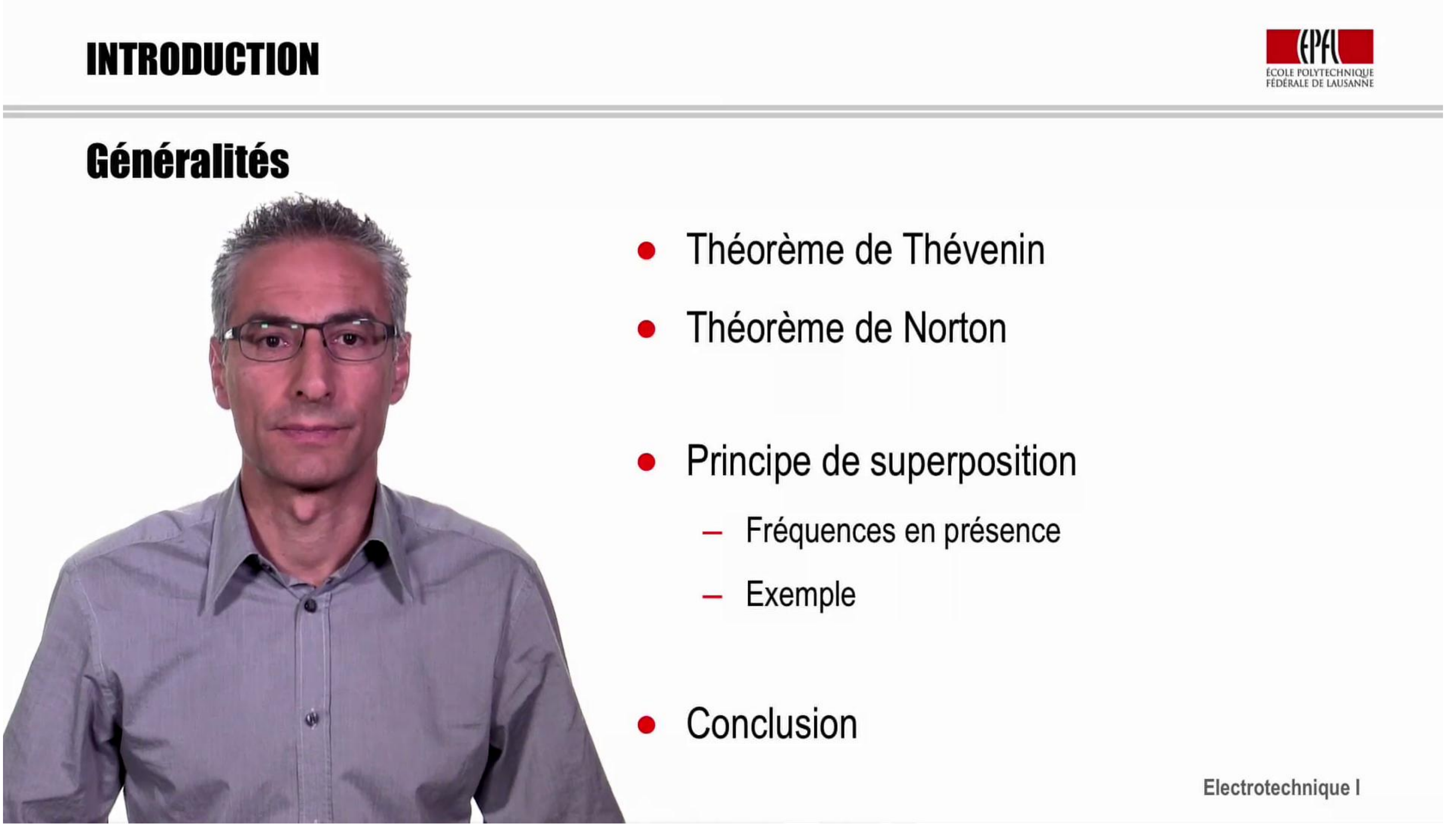


Video

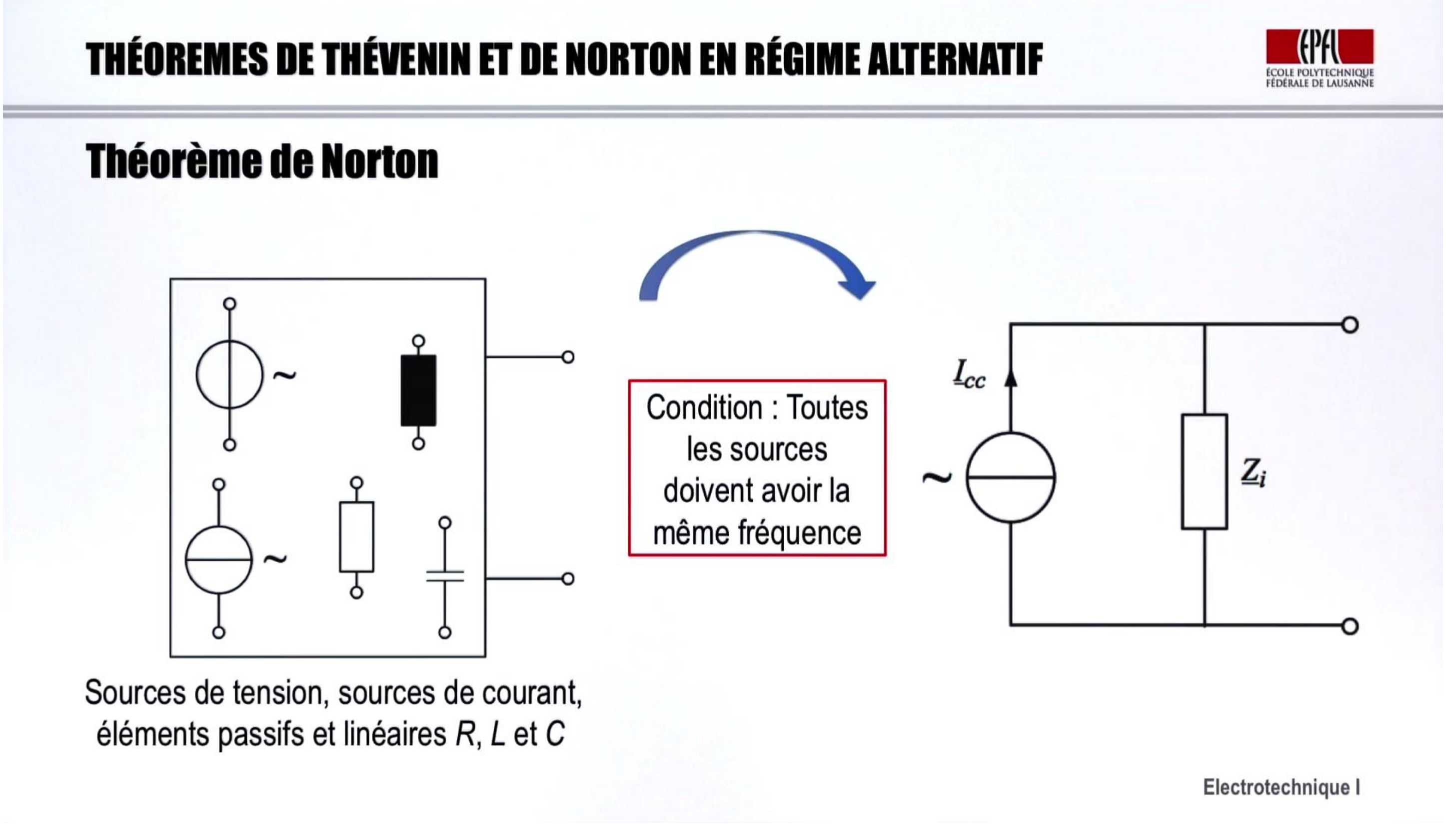


Hello, welcome to this new lesson of the Electrotechnics Course. Today, we will discuss about two methods that we have developed in the DC steady state and we will adopt for the AC steady state. First of all, we will initially see the Thevenin and Norton theorems applied to a circuit under the AC steady state. Secondly, we will see under which conditions we can apply the superposition principle in the AC steady state and we will then see a small example.

Notes

Summary

0m 03s

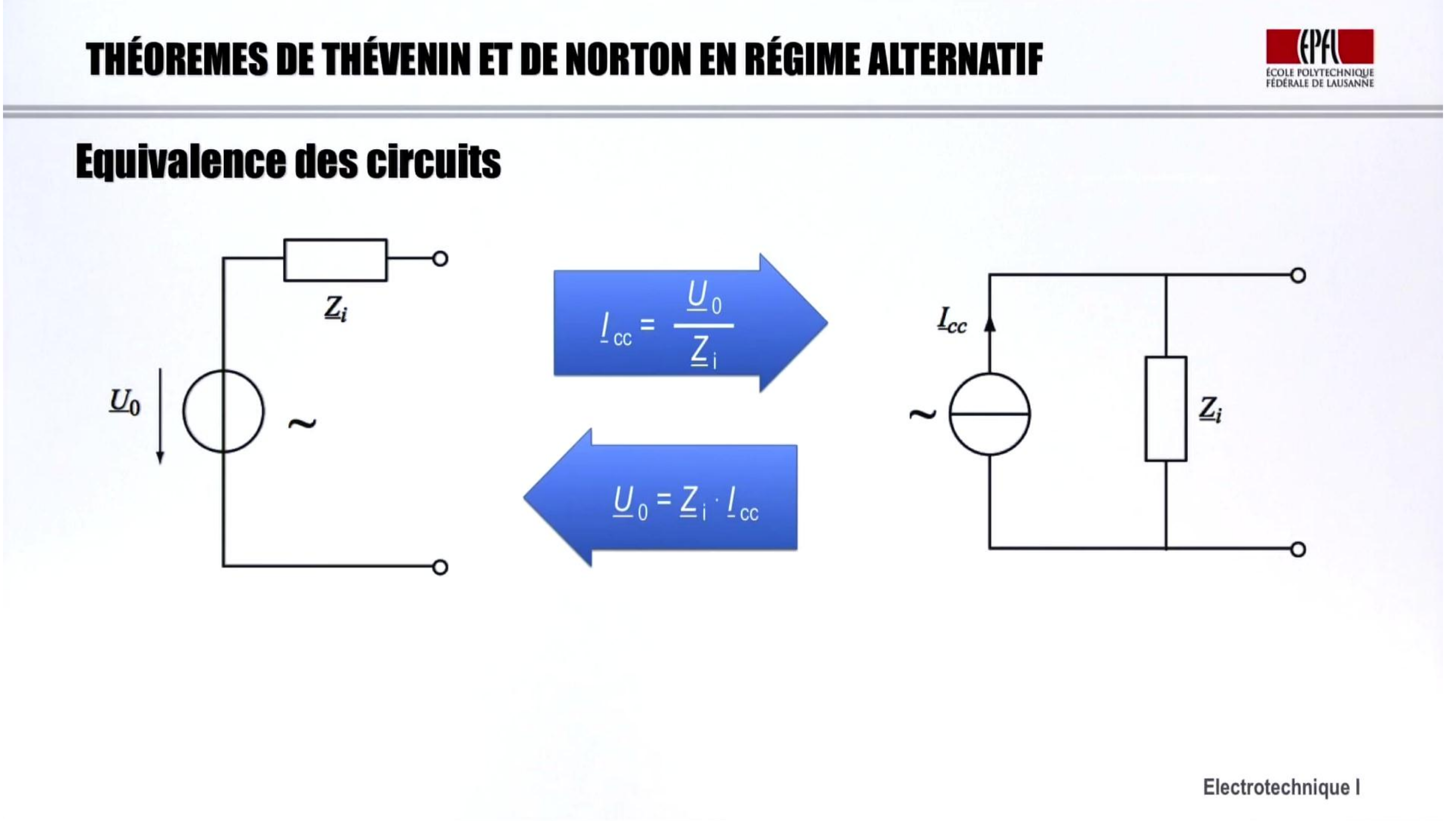


To recall, the Thevenin theorem says that any dipole, here we have an example of dipole with elements inside and we have here the two dipoles. Therefore, any dipole could be replaced by a real voltage supply, then an ideal voltage supply, in series with an internal impedance. A real voltage supply when the voltage U0, here, is the no-load voltage of the dipole and that the internal impedance Zi correspond to the ratio of the open-circuit voltage and short-circuit current. Therefore Z, here, is equal to U0 the non-load voltage divided by the short-circuit current when we short the dipole. Same, the Norton theorem says that any dipole can be replaced by a real current supply, then, here, by an ideal current supply, in parallel with an internal impedance. Therefore, a real current supply with the current Icc, here, is the short-circuit current of the dipole and with an internal impedance equal to the one of the real voltage supply U0 which is equal to Zi*Icc The condition so that the Thevenin theorem and Norton theorem are applicable is that all the supplies in the dipole should have the same frequency.

Notes

Summary

0m 35s

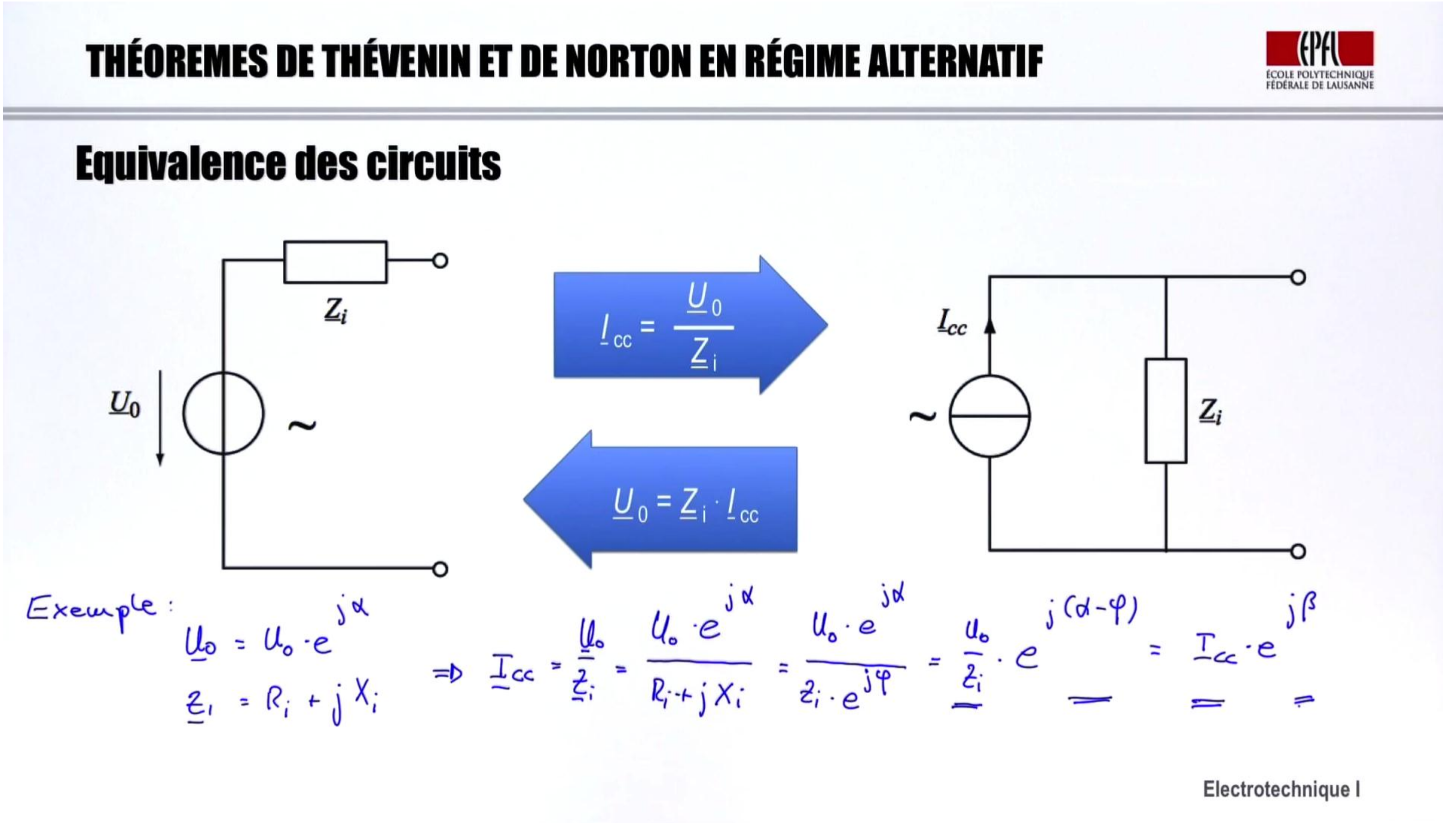


If the dipole is equivalent to a real voltage supply or an ideal voltage supply, then, there is also an equivalence between a real voltage supply and a real current supply. If we replace the dipole, or the circuit, by this real voltage supply, or only part of the circuit by this real voltage supply, we obtain the equivalence applying this relation, namely, the ideal voltage supply is equal to the ratio of the no-load voltage U0 divided by the internal impedance and the internal impedance, which is in series with the voltage supply, is here in parallel with the current supply. The inverse operation, that is to say if we isolate in the circuit or if replace the circuit, this dipole, we find out the equivalence of the real current supply with the real voltage supply replacing the tension U0 by the product Zi*Icc. And we replace the internal impedance in parallel by an internal impedance in series. As we've already seen in the DC steady state, we can also determine the internal impedance removing all voltage supplies and computing the impedance seen from outside, the dipole impedance, and we therefore obtain the internal impedance. We will now see a small example.

Notes

Summary

2m 12s



If the voltage supply is equal to U0 that we write under the exponential complex form, then a norm times an exponent exp(j alpha) and that the internal impedance Zi is equal to an internal resistance plus an internal reactance Xi, the current for the transformation, the short-circuit current will be given under Icc=U0/Zi which is equal to the norm times exp(j alpha) divided by the internal resistance plus the internal j X. We can see at that stage of development that it is not easy to solve this kind of fraction. We will then use the exponential properties, we will transform this part, which is in cartesian form, to an expression in polar form. Then we can rewrite the identical numerator exp(j alpha) and then the denominator that we will write as a norm times exp(j phi), an argument, knowing that Zi, using Pythagoras, is equal to square root of $Ri^2 + Xi^2$ and that the argument phi is given by tangent arc of the imaginary part, the reactance Xi, divided by the internal resistance. Written like that, we can simply rewrite the equation as U0/Zi*exp(j(alpha-phi)) and this is the current expression, namely, Icc*exp(j beta) and, by analogy, we see that the norm of Icc is U0/Zi and the argument beta is equal to alpha minus phi. Then, the mathematical operations must be carried out using the rules of Complex calculation.

Notes

Summary

4m 00s

PRINCIPE DE SUPERPOSITION EN RÉGIME ALTERNATIF

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

A - Cas pour lequel toutes les sources ont la même fréquence

- On considère successivement chaque source prise individuellement pour en connaître la réponse pour la grandeur demandée ;
- La grandeur définitive est la <u>somme vectorielle</u> des contributions individuelles de chaque source ;
- Soit :

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In this second part of the lecture, we will see how could we apply The superposition principle in the AC steady state. As a reminder, the superposition principle says that the response of a circuit, namely a current somewhere in the circuit or a voltage across an element of the circuit, the circuit response to a sum of excitations is equal to the sum of the responses of each excitation. taken individually. As in the DC steady state, the superposition principle is applicable but as long as, the steady state is linear. As a reminder, the linear term means that the value R of a resistor doesn't vary with the current flowing through it. Therefore, the relation U = R*I is still valid. Same for an inductance, its value L will not vary with the current flowing through it, so it does not saturate. And, finally, the value C of a capacitor doesn't vary with the voltage which is present at its terminals. We will then threat these two cases: The first case for which all the supplies of voltage and current have the same frequency. We successively consider each individual supply to know its response for the requested variable. The final variable is the vector sum of individual contributions of each supply.

Notes

Summary

6m 40s

A - Cas pour lequel toutes les sources ont la même fréquence

- On considère successivement chaque source prise individuellement pour en connaître la réponse pour la grandeur demandée ;
- La grandeur définitive est la somme vectorielle des contributions individuelles de chaque source ;
- Soit : $\boxed{\underline{X}_{tot} = \sum_{j=1}^{n} \underline{X}_j}$ Rem. : $\underline{U}$ ✓ $\underline{I}$ ✓ P ✗ (fonction quadratique)

$P_1 = R I_1^2$

$P_2 = R I_2^2$

mais $P_{tot} = P_1 + P_2 \neq R(I_1 + I_2)^2 = R(I_1^2 + \underline{2 I_1 I_2} + I_2^2)$

So for a variable that we call here X, The variable Xtot is the contribution, the vector sum of each contribution of each supply. The variable X in this equation can be either a voltage or a current but not a power. Why? Because the power is a quadratic function and then not linear, so the superposition principle is not applicable in the case of powers. We can demonstrate, namely, if we write the power P1 that will be dissipated in the resistor R due to the supply 1, this power would be equal to $R*I1^2$. The power P2 that would be the dissipated power in the resistor due to supply 2, would be equal to $R*I^2$. But the total power is not equal to this, why? Because Ptot, which is the sum of two powers, is not equal to R times the total current I1 + I2 squared. Why? Because the last expression can be developed as follows... where you can see this double product, here, showing that for the powers the superposition principle is not applicable.

Notes

Summary

8m 13s

PRINCIPE DE SUPERPOSITION EN RÉGIME ALTERNATIF

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

B - Cas pour lequel toutes les sources n'ont pas la même fréquence

- On regroupe les sources par fréquence ;
- Pour chaque groupe de fréquence, on applique la méthode vue au cas A
- Une somme vectorielle de contributions pour chaque <u>fréquence</u> :

  $f_1 \rightarrow \underline{X}_{\text{tot } 1}$, $f_2 \rightarrow \underline{X}_{\text{tot } 2}$, $f_n \rightarrow \underline{X}_{\text{tot } n}$

- L'addition des sommes vectorielles des contributions doit se faire dans le domaine temporel :

Electrotechnique I

We threat now a second case. It is the case for which all the supplies of voltage and current don't have the same frequency and the method is: We will first regroup the supplies based on their frequencies, that is to say we take all the supplies that have the same frequency and we will put them together and for each group of frequency, we will apply the method seen in case A. Therefore, we will have a vector sum of contributions for each frequency, it is expressed here. For the first frequency we have a first contribution of the supplies at that frequency and for this frequency, we have again a sum of contributions, and so forth for all the frequencies. Finally, the addition of the vector sums, of the sums here, of the contributions has to be done, finally, in the temporal domain.

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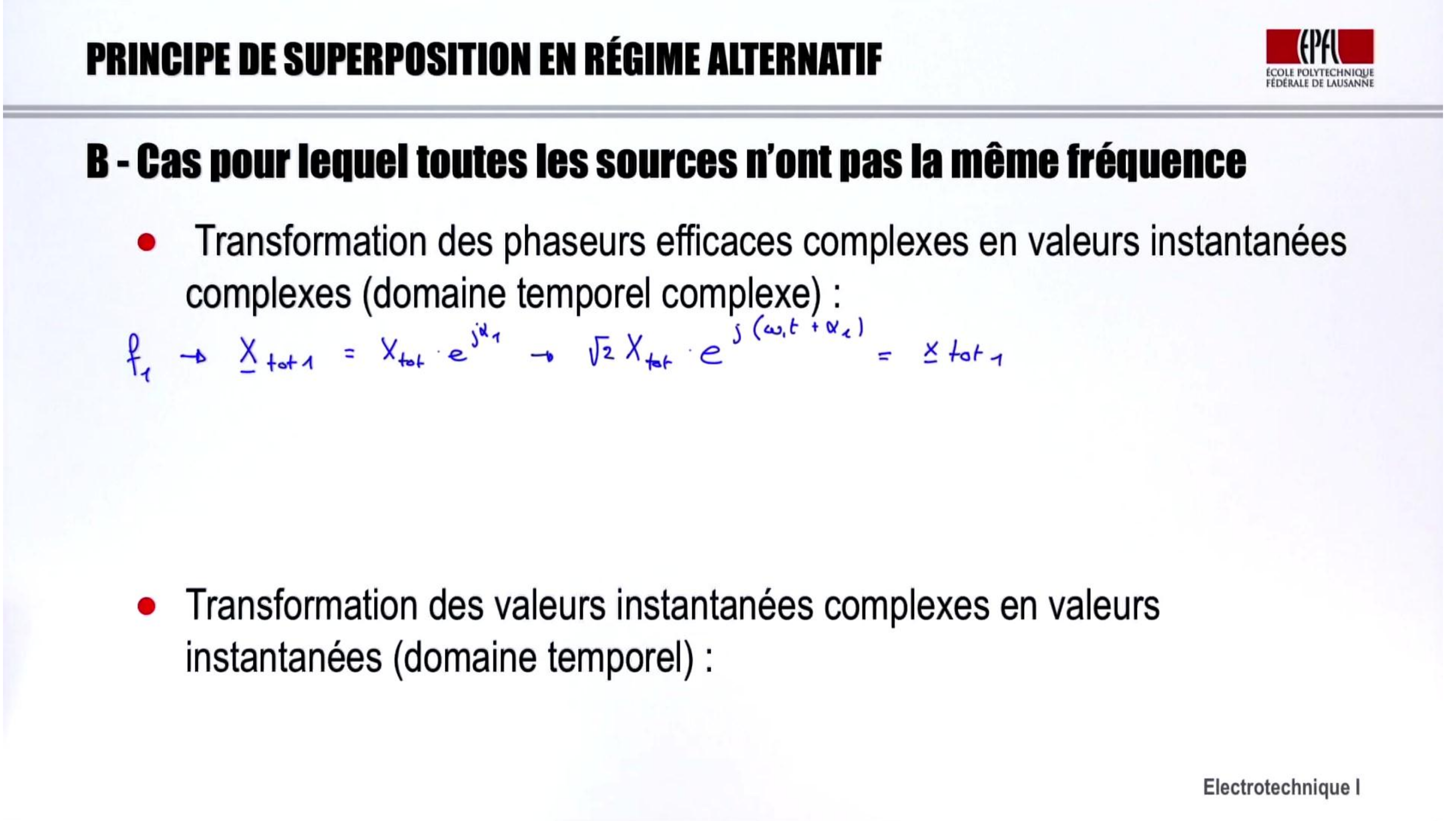
Summary

10m 14s



This transformation in the temporal domain we will do it now, we will threat only a simpler case, a general case where we will have only two frequencies. For the first frequency f1, we will have a sum of all the contributions that we call Xtot1, that i can write under the exponential form with a norm: Xtot*exp(j alpha1) and I will transform this effective complex phasor in an instantaneous complex value, in the temporal complex domain. The norm which is here under effective form, I will take its peak value, namely a square root of 2 times Xtot, times exp(j), we reintroduce the omega1 pulsation, omega1 times t plus alpha1, and this is equal to small x, lower case x, because it depends on the time, xtot1. Same for the second frequency we've identified where we have the sum of the contributions Xtot2 under exponential complex form we can write as Xtot2 -- sorry I've forgotten here the 1 -- times exp(j alpha2), that we transform in a instantaneous complex value namely a square root of 2 times Xtot2, -- sorry I've forgotten again the 1 here -- then the peak value times exp(j oméga2*t + alpha2) and this is equal to xtot2.

Notes

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11m 10s

PRINCIPE DE SUPERPOSITION EN RÉGIME ALTERNATIF

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

B - Cas pour lequel toutes les sources n'ont pas la même fréquence

- Transformation des phaseurs efficaces complexes en valeurs instantanées complexes (domaine temporel complexe) :

$$f_1 \rightarrow \underline{X}_{tot\,1} = X_{tot_1} \cdot e^{j\alpha_1} \rightarrow \sqrt{2}\, X_{tot_1} \cdot e^{j(\omega_1 t + \alpha_1)} = \underline{x}_{tot\,1}$$

$$f_2 \rightarrow \underline{X}_{tot\,2} = X_{tot_2} \cdot e^{j\alpha_2} \rightarrow \sqrt{2}\, X_{tot_2} \cdot e^{j(\omega_2 t + \alpha_2)} = \underline{x}_{tot\,2}$$

$$\underline{x}_{final} = \underline{x}_{tot_1} + \underline{x}_{tot_2}$$

$$\underline{x}_{final} = \sqrt{2}\, X_{tot_1} \cdot e^{j(\omega_1 t + \alpha_1)} + \sqrt{2}\, X_{tot_2} \cdot e^{j(\omega_2 t + \alpha_2)}$$

- Transformation des valeurs instantanées complexes en valeurs instantanées (domaine temporel) :

$$x_{final} = \sqrt{2}\, X_{tot_1} \cdot \sin(\omega_1 t + \alpha_1) + \sqrt{2}\, X_{tot_2} \cdot \sin(\omega_2 t + \alpha_2)$$

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Now, to have the final result, we will add these two partial results, we have that xfinal, the instantaneous complex value in the temporal domain, is equal to xtot1 + xtot2. If we develop, we have xfinal, as function of time, equals to square root of 2 times xtot1 times the exponential. And the second term for the second frequency, square root of 2 times xtot2 times the exponential. The exponential sum is not well suited for the additions so we will transform these instantaneous complex values in instantaneous values in the temporal domain and then the final value.... will be equal to square root of 2 times xtot1 times the sin (oméga1 t + alpha1), plus the second contribution, square root of 2 times xtot2 times sin (oméga2 t + alpha2).

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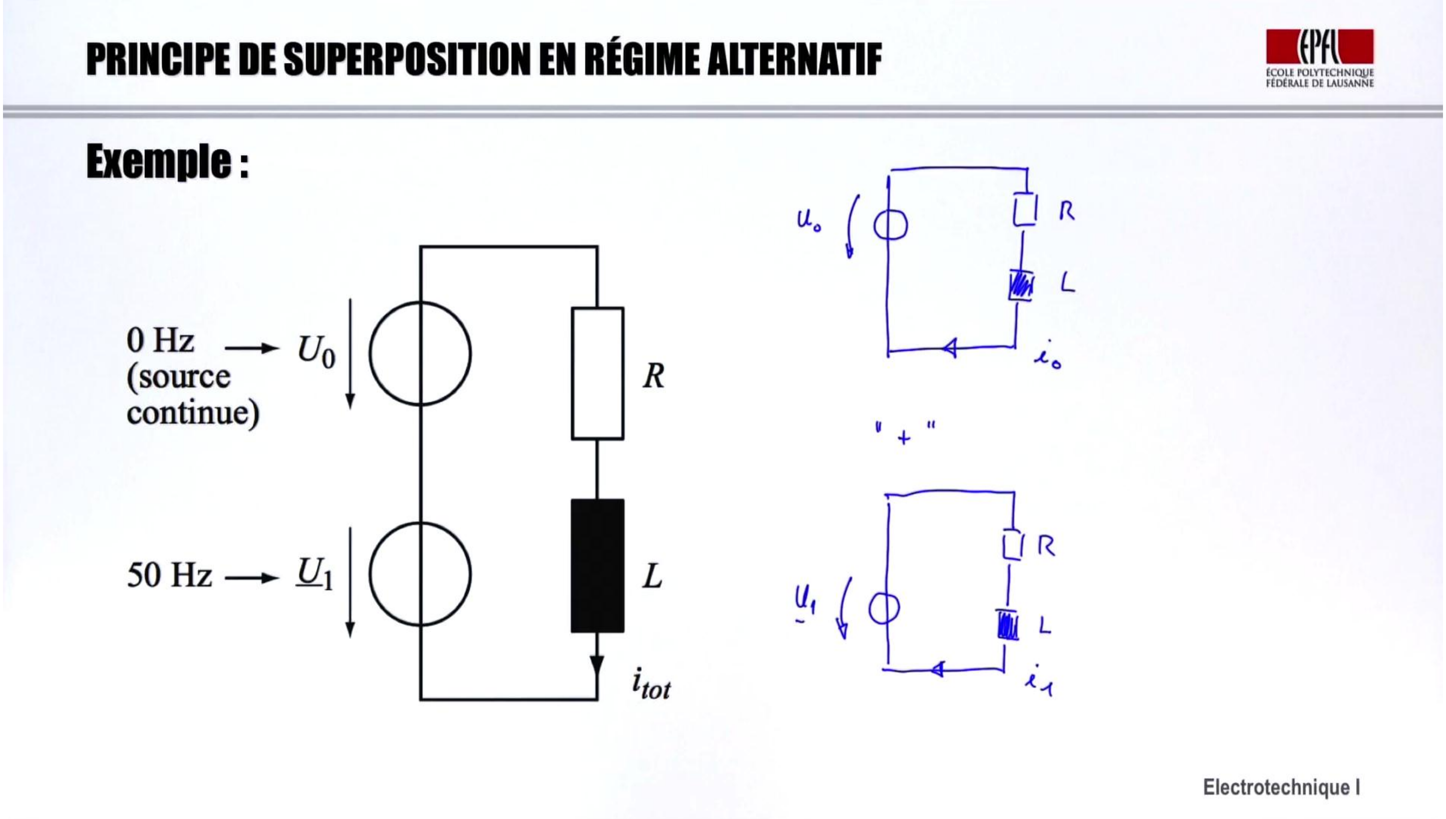
Summary

13m 18s



Let's consider now this small example where an RL circuit is supplied, the circuit will be excited using two supplies: a first supply U0 which has a certain frequency, here a frequency null, it is a continuous supply, and a second excitation, it is the voltage U1 which has a second frequency equal to 50Hz. Therefore, the excitation is caused by these two supplies in this circuit and we are looking for the response, here in this case the total current. We can then decompose this circuit into two sub-circuits, the first one where we consider only the voltage supply U0 and we will have the following result: the voltage supply U0, the resistor R, the inductance L and the contribution of Itot, the contribution of U0 that we call here I0. We will add the contribution of the second excitation, of the second supply which lead us to the following circuit: a voltage U1 which feeds the circuit RL and whose answer is the current I1. The total current will be the sum of these two contributions I0 and I1.

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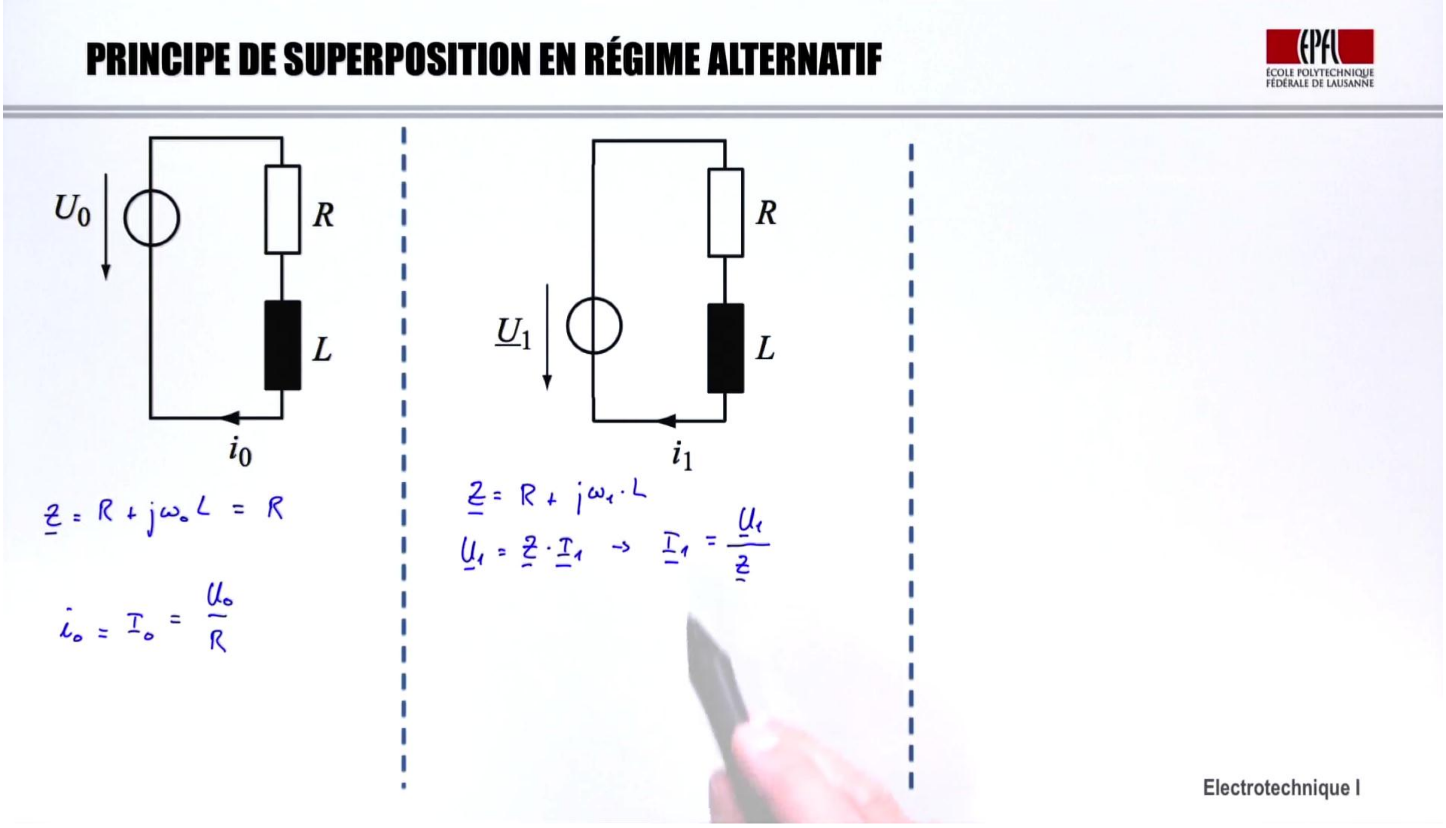
Summary

15m 02s

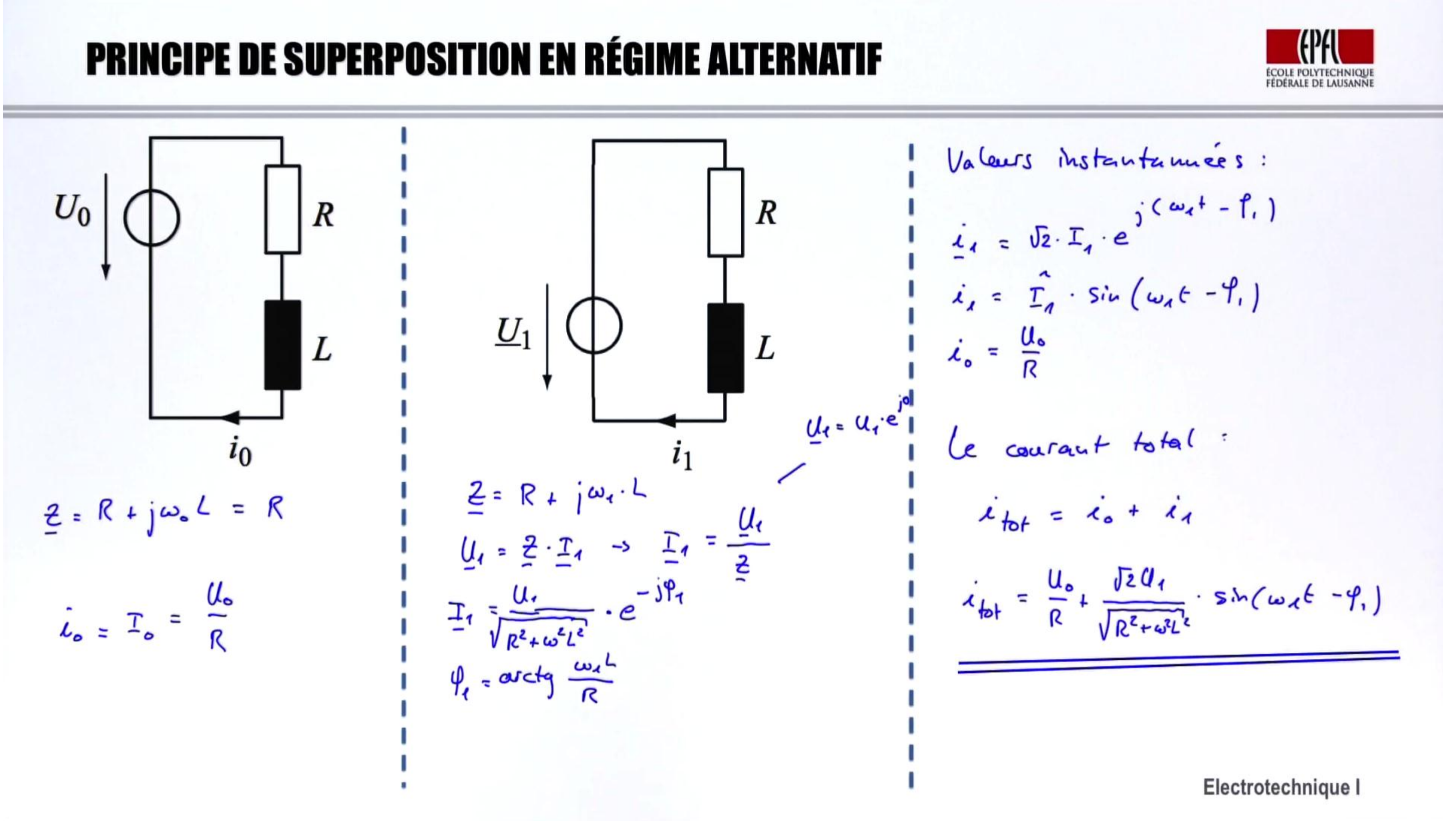


So let's develop now the equations. the impedance Z of this circuit is given by the real term R plus the imaginary term j omega0 times L, and omega0 is the frequency of the supply U0 which is equal to zero so this term here is null, it remains only R. The current I0 resulting therefrom is given by: I0 equals to U divided by the total impedance, then R. It is our first result. Concerning the second excitation, once again we have the impedance Z which is equal to R, the real part, plus the imaginary part here that is equal to j omega1 times L, this time omega1 is not equal to zero, we will have to take this into account in the calculations. and we can write by Kirchhoff that this voltage is equal to the voltage drop across the two elements, we have then U1 equals to the impedance Z, which are in series, times I1. And then, I1 is the ratio of U1 over Z. We develop this term and we obtain that I1 is equal to U1, we divide the normes, divided by the norm of Z, here using Pythagoras, we find that it is equal to square root of $R^2 + X^2$, i directly replace X, with omega square L X2 times the exponentials.

Notes

Summary

16m 50s

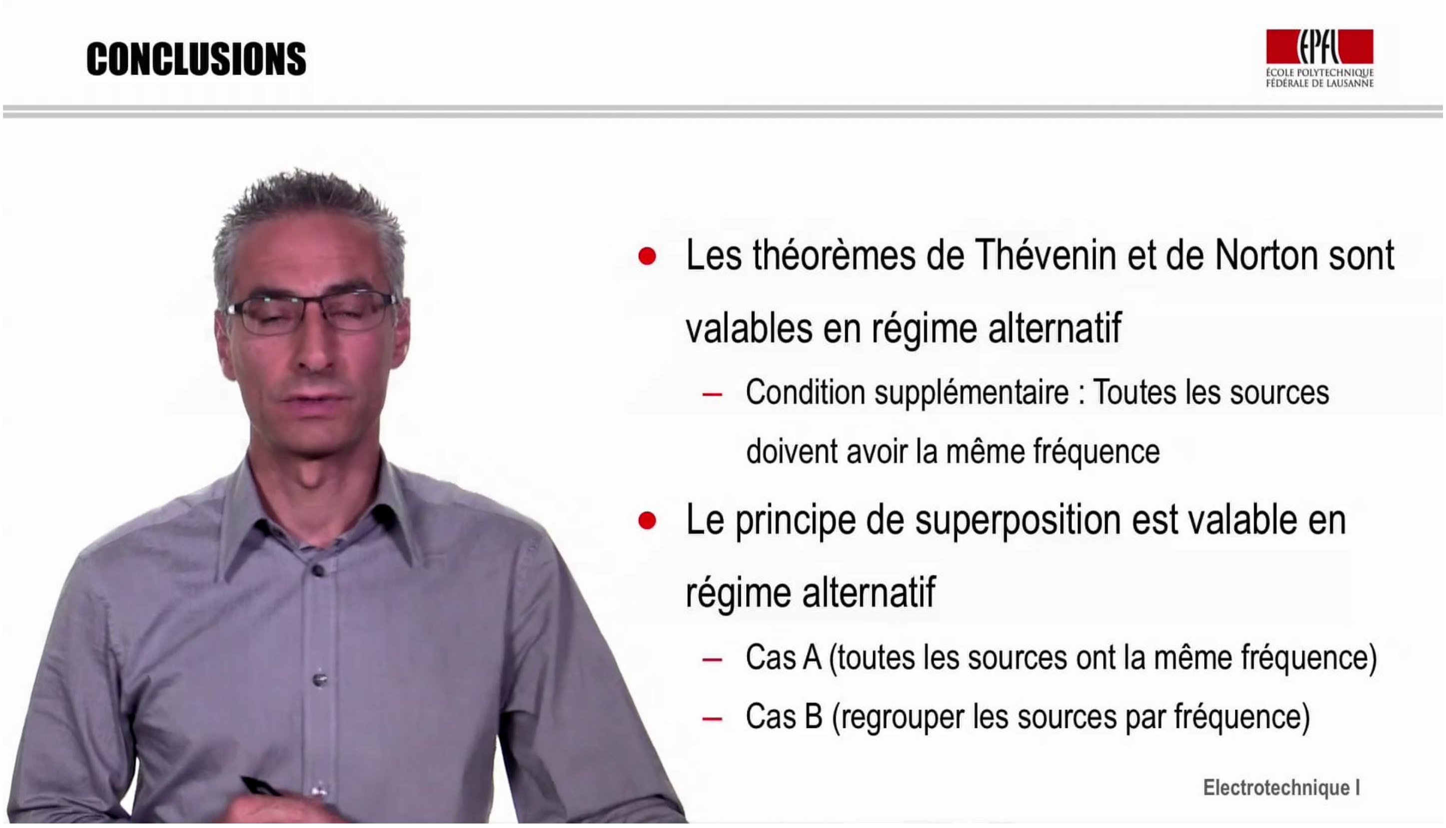


We assumed that U1 is equal to U1*exp(j0), no phase shift with respect to time gives 0, and then we have here the exponential of -jφ1, φ1 being the arguent here of the impedance Z. φ1 is equal to the arc tangente of the imaginary part ω1L/R. In instantaneous value, we obtain that the instantaneous complex current is equal to the peak value, that is to say the square root of 2 times the effective value I1 times exp(jω1t - φ1), that I express in the time domain, we have then I1 equals to the peak value Î1, this term, times sin (ω1t - φ1). The contribution current of the other supply is equal to I0 and we recall here that it is U0/R. The total current is therefore the sum of the two contributions... and it is equal to U0 over R plus square root of 2 times U1 divided by the norm of Z, this is the current, times sin (ω1t - φ1). This being the final result.

Notes

Summary

18m 53s



That's it, we have shown that the theorems of Thevenin and Norton are valid in the AC steady state but with the additional condition compared to the DC steady state, is that all supplies must have the same frequency. Concerning the principle of superposition, it is also valid under the AC steady state but we should distinguish two cases: The first case where all the supplies have the same frequency, and we can in that case make the addition in the complex domain with phasors, The second case where all the supplies don't have the same frequency, we should regroup in this case the supplies according to their frequency and then, in the end, go through the temporal domain to make the addition. Thank you for your attention.

Notes

Summary

21m 07s