ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

IMPÉDANCES ET RÉSEAUX D'IMPÉDANCES

LEÇON 15

Électrotechnique I
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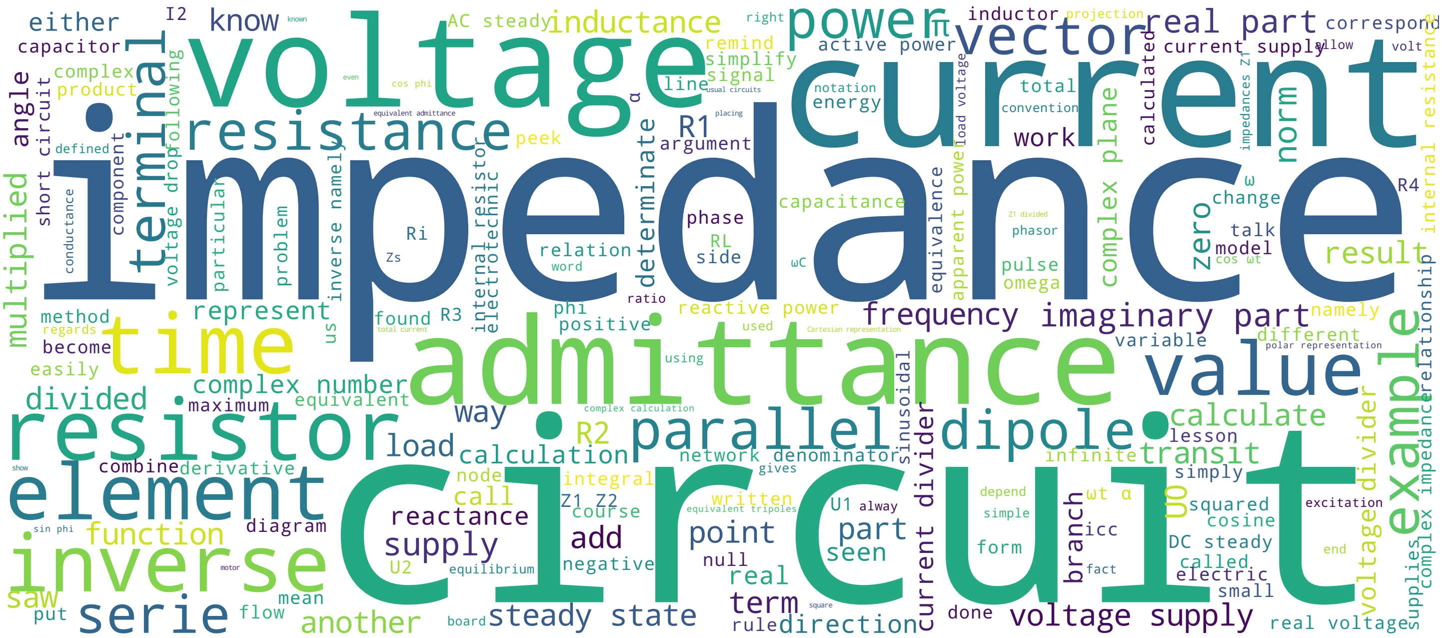
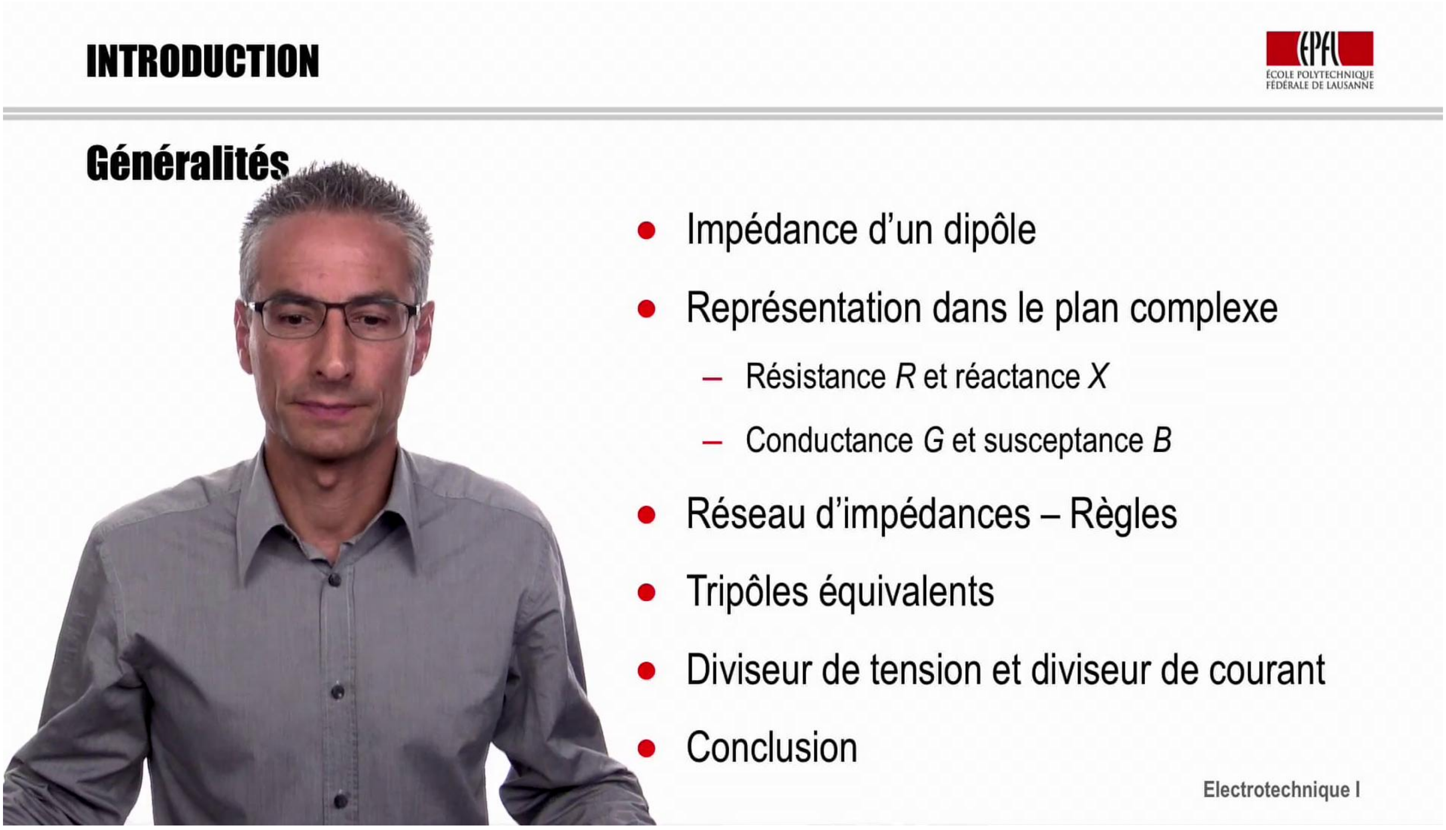
Laboratoire d'Actionneurs Intégrés



Video



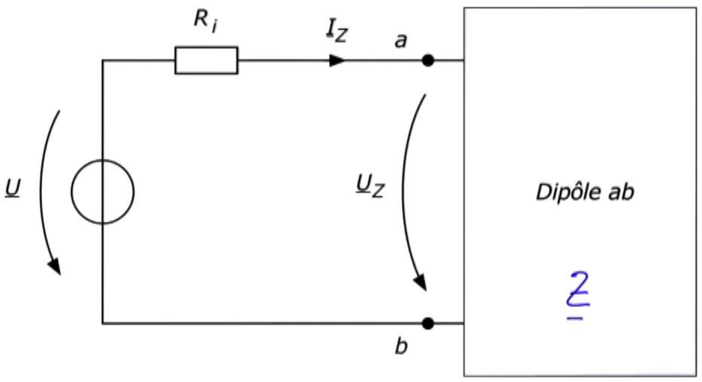
Hello! Today we will talk about the impedance of a dipole and its inverse the admittance, and also the way which can be carried to combine them when they are in a network. We will first remind the definition of the impedance and see its representation in the complex plane. We will then see how we will combine these impedances, We will finally see the equivalent tripoles in AC steady state also the voltage divider and the current divider, as we saw in DC steady state but this time in AC steady state.

Notes

Summary

0m 04s

Impédance d'un dipôle



$$\underline{Y} = \frac{1}{\underline{Z}}$$

$$R = Re\{\underline{Z}\} = Re\left\{\frac{\underline{U}}{\underline{I}}\right\} = \frac{U}{I}\cdot\cos\varphi$$

Résistance

$$X = Im\{\underline{Z}\} = \frac{U}{I}\cdot\sin\varphi$$

Réactance

$$G = Re\{\underline{Y}\} = \frac{I}{U}\cdot\cos\varphi$$

Conductance

$$B = Im\{\underline{Y}\} = -\frac{I}{U}\cdot\sin\varphi$$

Susceptance

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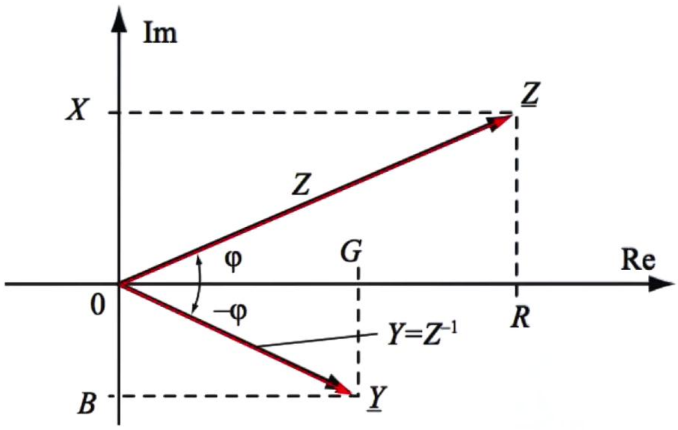
So either a dipole characterized by a complex impedance, its the real part of the complex impedance, called the resistance R of the dipole. The resistance R is therefore the real part of Z. I remind you that Z, is the ratio of the voltage over the current, and this is equal to U over I, multiplied by cos phi, it's the resistance. The imaginary part of the complex impedance is called the reactance, of the dipole written X. Therefore X is the imaginary part of Z, and it's equal to U over I times sin phi. Now lets considerate the inverse of the impedance, that is to say the admittance. The real part of the admittance is called the conductance. G is therefore the real part of the inverse of the impedance, therefore the admittance that we write Y, and is equal to I over U, multiplied by cos phi, it's the conductance. Finally the imaginary part of the complex admittance is called susceptance, and is written B, it is the imaginary part of Y, that is equal to I times U time sin phi, with the minus sign here because phi is negative. And this is the susceptance.

Notes

Summary

0m 42s

Représentation dans le plan complexe



$$\boxed{\underline{Z} = Ze^{j\varphi} = R + \mathrm{j}X}$$

Avec les équations de transformation :

$$R = Z\cos\varphi \qquad X = Z\sin\varphi$$

$$Z = \sqrt{R^2 + X^2} \qquad \varphi = \arctan\frac{X}{R}$$

$$\boxed{\underline{Y} = Y\mathrm{e}^{-\mathrm{j}\varphi} = G + \mathrm{j}B}$$

avec les équations de transformation :

$$G = Y\cos\varphi \qquad B = -Y\sin\varphi$$

$$Y = \sqrt{G^2 + B^2} \qquad \varphi = \arctan\left(-\frac{B}{G}\right)$$

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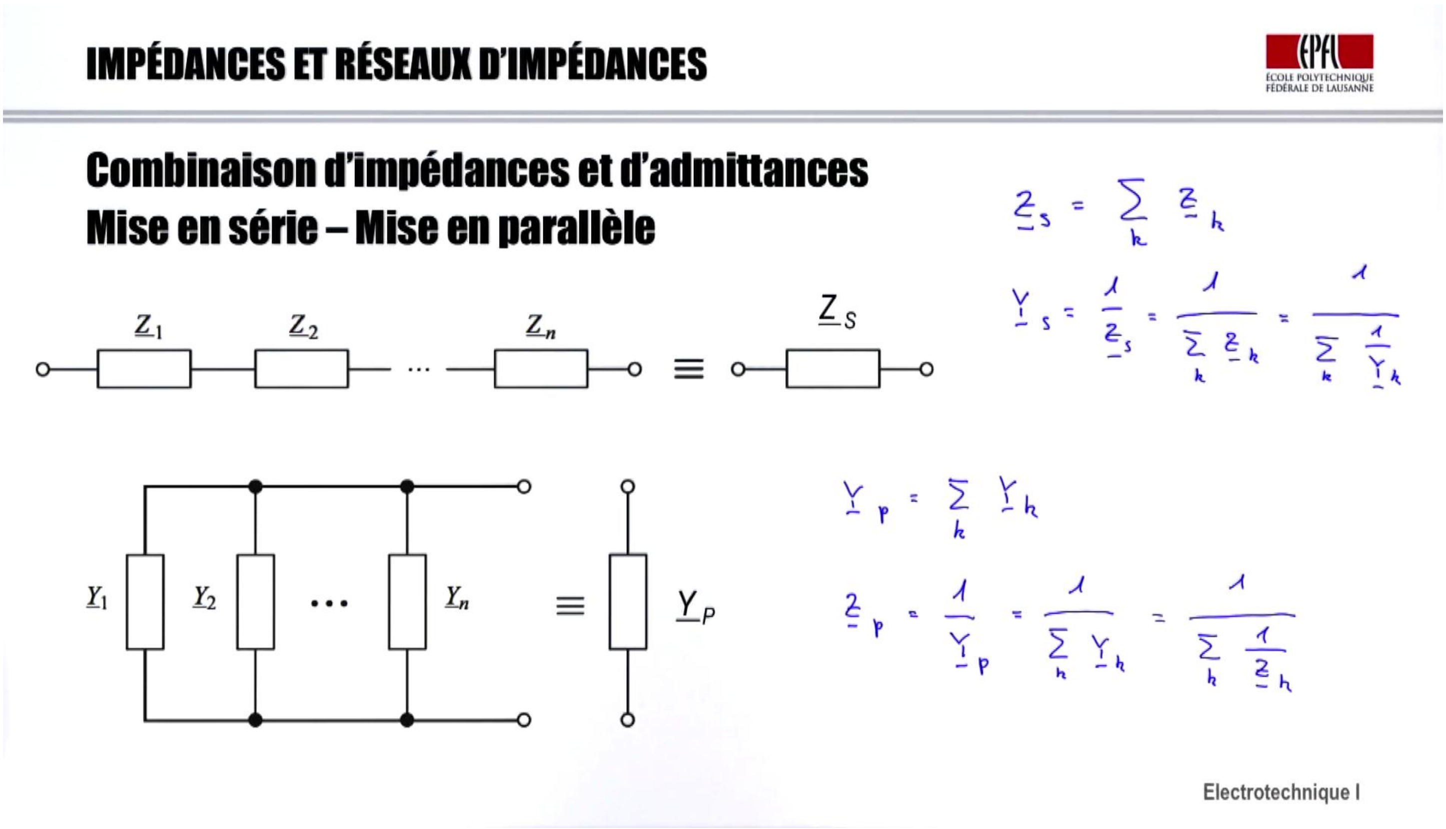
In the complex plane, we can represent the impedance and the admittance. The impedance Z is represented here. These transformation equations allow to transit from a polar representation to a Cartesian representation or rectangular. Knowing that R, the real part is the projection of the vector Z on the real axis, the imaginary part is the projection of this vector on the imaginary axis, and for the transit in the other way, the norm of the vector Z can be found by Pythagorean by calculating the square of each side of the right-angled triangle. And the angle is determinate by the tangent arc of the imaginary part over the real part. For the admittance, which is the inverse of the impedance, again, we have the polar representation with a norm and an argument, or the Cartesian representation, With the real coordinate and the imaginary coordinate. As the admittance Y is the inverse of the impedance Z, at the norm, the length of the vector, it will be the inverse, namely if Z has a norm of 1.25, Y will have a norm of 1 over 1.25, namely 0.8. For the argument, it will be the opposite, that is to say that if phi is positive for Z it will be negative for Y, we can easily see it here.

Notes

Summary

2m 57s



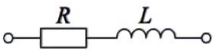
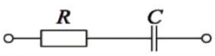
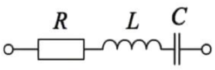
We will now combine the impedances in series or in parallel. In the first case, the serial case, we want to know the equivalent serial impedance Z, that is the sum of the impedances Z1, Z2, till Zn, we find that Zs is equal to the sum of k, d Zk. By against for the total equivalent admittance Ys, is the inverse of Zs, and this inverse of the sum of the impedances, or the inverse of the sum of the inverses, namely 1 divided by the sum over K of the inverses. As regards the admittances, when they are in parallel, they add themselves, that is to say that the equivalent admittance in parallel, is the sum over K of all the admittances K. Therefore we here see the advantage to use the admittances, because in parallel it's easy, we add them. By against, for the equivalent parallel impedance, again, by analogy, one is the inverse. Namely Yp is the inverse of Zp, and is also the inverse of the sum of the admittances, or the inverse of the sum of the inverses.

Notes

Summary

4m 40s

Quelques circuits usuels :

Circuit	Impédance $\underline{Z} = R_z + \mathrm{j}X$	Admittance $\underline{Y} = G_z + \mathrm{j}B$
	$R + \mathrm{j}\omega L$	$\dfrac{1}{R + \mathrm{j}\omega L} = \dfrac{R - \mathrm{j}\omega L}{R^2 + \omega^2 L^2}$
	$R + \dfrac{1}{\mathrm{j}\omega C} = R - \mathrm{j}\dfrac{1}{\omega C}$	$\dfrac{R\omega^2 C^2 + \mathrm{j}\omega C}{1 + \omega^2 R^2 C^2}$
	$\mathrm{j}\left(\omega L - \dfrac{1}{\omega C}\right) = \mathrm{j}\dfrac{\omega^2 LC - 1}{\omega C}$	$\mathrm{j}\dfrac{\omega C}{1 - \omega^2 LC}$
	$R + \mathrm{j}\left(\omega L - \dfrac{1}{\omega C}\right)$	$\dfrac{R - \mathrm{j}\left(\omega L - \dfrac{1}{\omega C}\right)}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

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Electrotechnique I

On the following board, we have a few examples of usual circuits, for which we have calculated the impedance and the admittance. We see that for two impedances in series, here a resistance that is purely resistive, we can add the impedance of the inductance, that is omega L, to get the equivalent total of this dipole, that is R + JωL, ωL being the part, the reactance of the inductance. We can calculate the inverse, namely the admittance of this circuit, of this dipole, and we find after a few calculations, this result here with a real part that is R divided by the denominator, that is a scalar, and the imaginary part omega L divided by this same denominator. We have here other examples.

Notes

Summary

6m 30s

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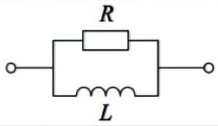
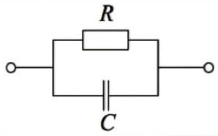
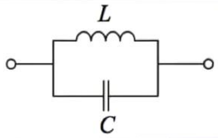
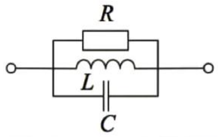
Circuit	Impédance $\underline{Z} = R_z + \mathrm{j}X$	Admittance $\underline{Y} = G_z + \mathrm{j}B$
	$\dfrac{R\omega^2L^2 + \mathrm{j}\omega LR^2}{R^2 + \omega^2L^2}$	$\dfrac{1}{R} - \mathrm{j}\dfrac{1}{\omega L}$
	$\dfrac{R - \mathrm{j}\omega CR^2}{1 + \omega^2R^2C^2}$	$\dfrac{1}{R} + \mathrm{j}\omega C$
	$\mathrm{j}\dfrac{\omega L}{1 - \omega^2LC}$	$\mathrm{j}\left(\omega C - \dfrac{1}{\omega L}\right)$
	$\dfrac{R - \mathrm{j}R^2\left(\omega C - \frac{1}{\omega L}\right)}{1 + R^2\left(\omega C - \frac{1}{\omega L}\right)^2}$	$\dfrac{1}{R} + \mathrm{j}\left(\omega C - \dfrac{1}{\omega L}\right)$

Electrotechnique I

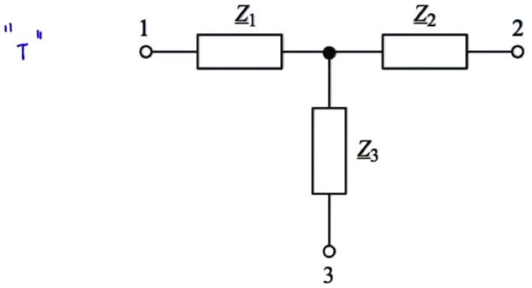
Here, few examples of circuits in parallel, we see the whole interest to work with admittances because the admittances in parallel add themselves and we find the admittance of R, it's 1/R, the admittance of the inductance, is 1/ωL, and we can do minus 1/ωL, and we then can do the sum very easily, and then return, at the impedance, with a notation of that form there. Here, a few other examples.

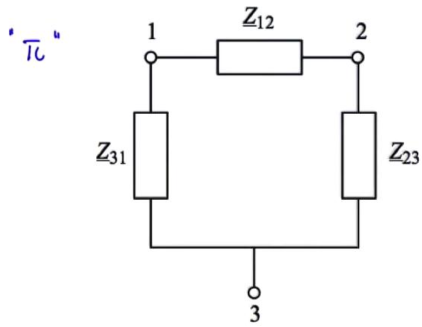
Notes

Summary

7m 35s

Tripôles équivalents





$$\underline{Z}_1 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{31}}{\underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31}}$$

$$\underline{Z}_{12} = \frac{\underline{Z}_1 \underline{Z}_2 + \underline{Z}_2 \underline{Z}_3 + \underline{Z}_3 \cdot \underline{Z}_1}{\underline{Z}_3}$$

In DC steady state, we saw the transformation πT. This is a circuit in π, and this one is a circuit in T, and we have calculated a relation to determinate, to make the transit from one another, namely to determinate the impedances, or the resistances in this case here, of Z12, Z31 and Z23, knowing Z1, Z2, Z3. These two circuits are perfectly identical, they behave in the same way. To make the transit between one another, by considering the impedances, we can do the same reasoning that we did with the resistances, but we will do it this time with the impedances, we directly give the result, for example, if we want to determinate the value of Z1 as function of the elements of the circuit in π, Z1 is equal to Z12 times Z31, namely the impedance from point 2 is from point 3, divided by the sum of three impedances of the circuit in π. Z23 + Z31. We write here that the additions and these multiplications, divisions, must be done with complex numbers. The transit of the circuit T into a circuit π is done by applying the following relation: we have that Z12, namely the impedance between point 1 and point 2, is equal to the sum of duplicates, Z1 times Z2, plus Z2 times Z3, plus Z3 times Z1 divided by the impedance of the opposite knot, namely Z23.

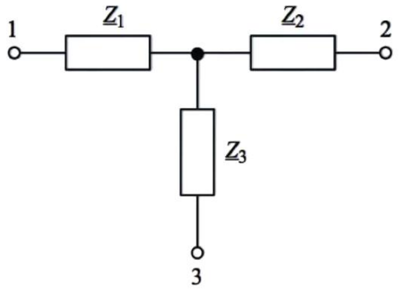
Notes

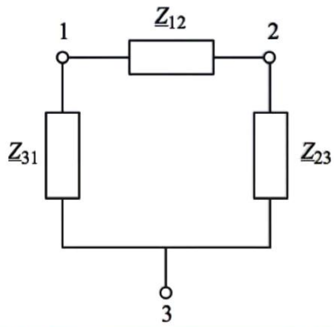
Summary

8m 08s

IMPÉDANCES ET RÉSEAUX D'IMPÉDANCES

Tripôles équivalents





$$\boxed{\underline{Z}_1 = \frac{\underline{Z}_{12}\underline{Z}_{31}}{\underline{Z}_{12}+\underline{Z}_{23}+\underline{Z}_{31}}} \quad \boxed{\underline{Z}_2 = \frac{\underline{Z}_{12}\underline{Z}_{23}}{\underline{Z}_{12}+\underline{Z}_{23}+\underline{Z}_{31}}} \quad \boxed{\underline{Z}_{12} = \frac{\underline{Z}_1\underline{Z}_2+\underline{Z}_2\underline{Z}_3+\underline{Z}_3\underline{Z}_1}{\underline{Z}_3}}$$

$$\boxed{\underline{Z}_3 = \frac{\underline{Z}_{23}\underline{Z}_{31}}{\underline{Z}_{12}+\underline{Z}_{23}+\underline{Z}_{31}}} \quad \boxed{\underline{Z}_{31} = \frac{\underline{Z}_1\underline{Z}_2+\underline{Z}_2\underline{Z}_3+\underline{Z}_3\underline{Z}_1}{\underline{Z}_2}} \quad \boxed{\underline{Z}_{23} = \frac{\underline{Z}_1\underline{Z}_2+\underline{Z}_2\underline{Z}_3+\underline{Z}_3\underline{Z}_1}{\underline{Z}_1}}$$

Electrotechnique I

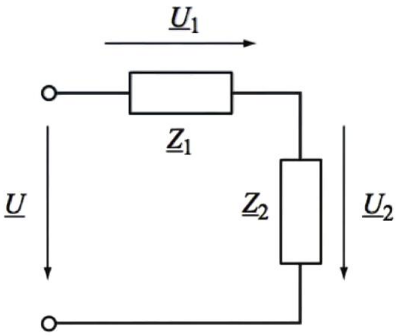
We give all these relations to calculate all the impedances, either the circuit in T, or either the circuit in π.

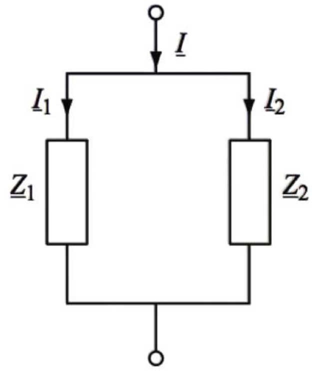
Notes

Summary

IMPÉDANCES ET RÉSEAUX D'IMPÉDANCES

Diviseur de tension et diviseur de courant





$$\underline{U}_1 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2}\underline{U}$$

$$\underline{U}_2 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\underline{U}$$

$$\underline{I}_1 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\underline{I}$$

$$\underline{I}_2 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2}\underline{I}$$

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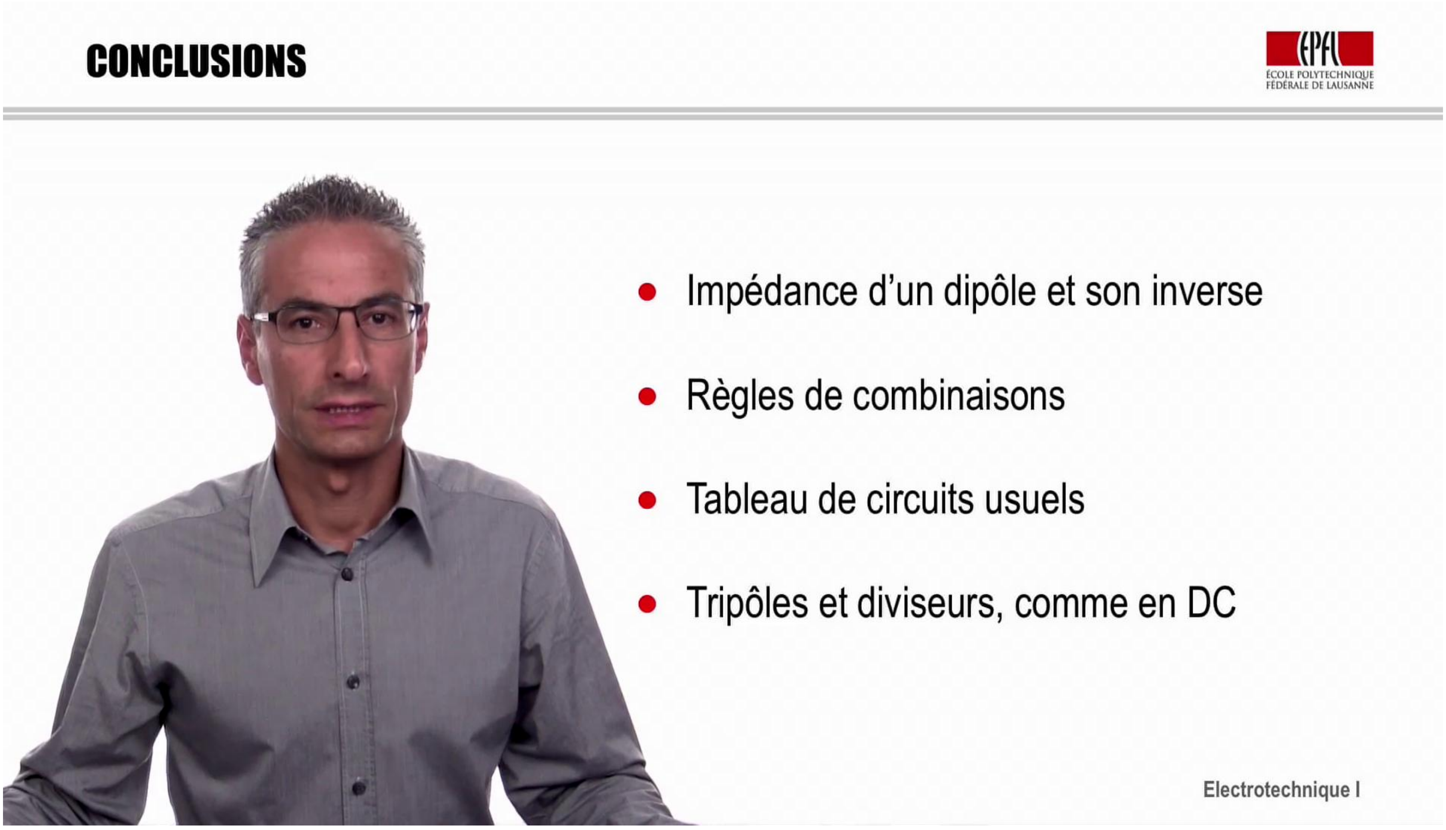
As regards the voltage divider or the current divider, that are represented here, voltage divider, current divider, we found a result with resistances, one can extrapolate the impedances, by knowing we must use complex numbers to treat these impedances, for example U1 which is the voltage drop at the terminals of this impedance Z1, is equal to Z1 divided by the sum of the two impedances, multiplied by U. For the current, we have found the result that was the following type. The current I1 that flows through a branch, is the Z2 impedance's quotient, opposed to the branch over the sum of the impedances Z1 + Z2, multiplied by the total current. It also gives the relationship for U2 and I2. Again, I remind you that these variables are complex numbers, and must be treated with the rules of the complex calculation.

Notes

Summary

10m 32s



We define the dipole's impedance and its inverse the admittance, and also their graphical representation in the complex plane. We saw the combination rules of these elements in a network, and we also saw that it was sometimes useful to work with admittances to simplify the calculations. We saw a board of usual circuits, this board is not exhaustive. As in the DC steady state, we saw the equivalent tripoles, and the voltage dividers, current dividers. Always remember to work with complex variables. Thank you for your attention.

Notes

Summary

11m 44s