

LECON 14

Yves PERRIARD & Paolo GERMANO
Laboratoire d'Actionneurs Intégrés





- Introduction
- Calcul complexe associé
- Représentation de Fresnel
- Impédances et admittances
- Source avec impédance interne
- Conclusion

Electrotechnique I

Hello and welcome to this lesson dedicated to the second part of sinusoidal single phase state. During this lesson, we will first resume an example allowing us to better understand the transit in complex calculations to, paradoxically, simplify the calculations seen during the first lesson, then we will discover what the admittance and the impedance are and also Fresnel's representation that will allow us to visualize these elctrotechnic values, as the voltage and the current, in a complex plane, that is to say vectors.

Notes

Summary



0m 04s

Exemple et rappel

$$u(t) = \hat{U} \cos(\omega t + \alpha) \longrightarrow \underline{u} = \hat{U} [\cos(\omega t + \alpha) + j \sin(\omega t + \alpha)]$$

$$= \hat{U} e^{j(\omega t + \alpha)}$$

$$\leftarrow \underline{\text{Re}}[\underline{u}]$$

Electrotechnique I

First, a small example and a reminder to tell you what we saw during this first lesson dedicated to sinusoidal single phased state. We then saw that a function of time, such as the voltage at the terminals of any element in electrotechnics, that is thus written... and that represents in real time the value of the voltage at any time at the terminals of an element; we will be able to transform this variable in another variable completely conceptual that we will indicate by the \underline{u} (vector), underlined means that we have a vector, of an element in the complex plane, that then takes the following form: First of all, a real part and an imaginary part. This can also be written in another form according to Euler. Thus, we can work in this plan said complex, with real parts and imaginary parts. This element underlined represents then the voltage with real numbers and when we finish the calculations with complex numbers, we can then come back to real numbers by taking the real part of this element underlined, of this complex voltage. By taking the real part, we come back to $\hat{U} \cos(\omega t + \alpha)$ that matches with the real value of the searched voltage.

Notes

Summary



Exemple et rappel

$$u(t) = \hat{u} \cos(\omega t + \alpha) \longrightarrow \underline{u} = \hat{u} [\cos(\omega t + \alpha) + j \sin(\omega t + \alpha)]$$

$$= \hat{u} e^{j(\omega t + \alpha)}$$

$$\leftarrow \text{Re}[\underline{u}]$$

Electrotechnique I

We can of course, it is just a suggestion to take the cosine, if we are more comfortable, we can also do it with sinusoidal variables and the transit between complex numbers and imaginary numbers will be done, not by taking the real part, but the imaginary part of this complex number.

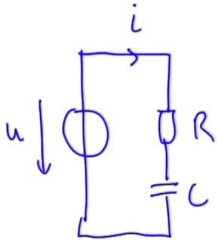
Notes

Summary



2m 35s

Exemple et rappel



$$u = \hat{U} \cos(\omega t + \alpha)$$

$$\hat{I} ? \quad \varphi = \alpha - \beta$$

$$i = \hat{I} \cos(\omega t + \beta)$$

1) Schéma à dessiner

Electrotechnique I

I suggest a small simple exercise to now give the working method and the different approaches to use the complex calculations during the resolution of a simple circuit like here with an RC circuit. We then have a voltage u that will have the following form: at the terminals of our R and our C and therefore a current will be established in the circuit, this current will also have a sinusoidal form, or co-sinusoidal, with simply another phase shift and we are looking here the value of the current that flows in the circuit and we are looking for an angle φ , that is to say a phase shift between α and β . How do we do? In every circuit to analyse there is often three first very simple steps to take, that seem sometimes to easy and that we don't do, but I suggest you to do them systematically to help you to understand the method right. The first step, is to redraw the sketch. Of course here the sketch is very simple, we will not draw it again, but sometime, redrawing makes the comprehension of the sketch much better for the person who draws it his way, with the connections at the right places, therefore it is often a step that we avoid, but i recommend you to do it each time.

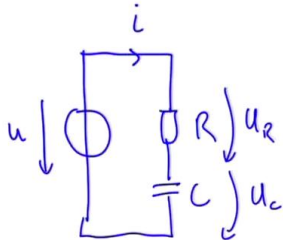
Notes

Summary



2m 59s

Exemple et rappel



$$u = \hat{U} \cos(\omega t + \alpha) \quad \hat{I} ? \quad \varphi = \alpha - \beta$$

$$i = \hat{I} \cos(\omega t + \beta)$$

- 1) Schéma à dessiner
- 2) Sens des tensions et des courants
- 3) Kirchhoff

$$u = u_R + u_C$$

$$u = R \cdot i + \frac{1}{C} \int i dt$$

$$u_R = R \cdot i$$

$$u_C = \frac{1}{C} \int i dt$$

Electrotechnique I

The second important step, is to establish the direction of the voltages and the currents. So if I already have the arrow for u , the arrow for i , We can also say that here there is a voltage at the terminals of R and here a voltage at the terminals of the capacitance and finally we can generally apply Kirchhoff's law to allow us to better understand the circuit that establishes the equations of the circuit. For this last step, we can then write that u at the terminals of our circuit is equal to U at the terminals of the resistor plus U at the terminals of the capacitance. We can also indicate what is equal U of R , it will be equal to R times i , and U and C , $1/C$ integral of i dt. We can therefore, to sum up these three steps, write that the voltage u is equal to $Ri + (1/C)$ integral of i dt. As we see, we have a differential equation here. Since we have an integral inside, we come back to the same problems that we have already met during the first part of this topic on the single phased state, with a differential equation and the complications linked to the resolution of the equations if we remain in the time mode like here.

Notes

Summary



4m 39s

Exemple et rappel

4) Passage au calcul complexe :

$$u = \hat{U} \cos(\omega t + \alpha) \rightarrow \underline{u} = \hat{U} e^{j(\omega t + \alpha)}$$

$$\underline{u} = R \cdot \underline{i} + \frac{1}{C} \int \underline{i} dt$$

5) Dérivée, Intégrer

$$\frac{d\underline{i}}{dt} = j\omega \hat{I} e^{j(\omega t + \beta)}$$

Electrotechnique I

Then we now transit to complex numbers, it is the step number four, we transit to complex calculations. What does it involve? I remind you that u is equal to $\hat{U} \cos(\omega t + \alpha)$ will therefore become another \underline{u} (vector) that is equal to $\hat{U} e^{j(\omega t + \alpha)}$. All of my equation found at the third step with Kirchhoff's law transforms itself into three elements that are now all complex. \underline{u} (vector), complex voltage, is equal to $R \cdot \underline{i}$ (vector), complex current, + $(1/C)$ integral of \underline{i} (vector), complex, dt . The fifth step now is to derive or to integrate. It is this part here, of the integral or in the case of the inductance, we would have a derivative. So let's work a little bit on this derivative and integrate to see what happens when we have complex numbers. What do we note? We note that when we have a derivative, for example the derivative of a complex number like this one, if one has something of the form $\hat{I} e^{j(\omega t + \beta)}$, by deriving as the derivative of an exponential always gives an exponential but with the internal derivative, we get $j\omega \hat{I} e^{j(\omega t + \beta)}$ by taking the equation of the current that we wrote before. We notice that if it is truly our \underline{i} (vector), we simply have $j\omega \underline{i}$ (vector) here.

Notes

Summary



6m 07s

Exemple et rappel

4) Passage au calcul Complexe :

$$u = \hat{U} \cos(\omega t + \alpha) \rightarrow \underline{u} = \hat{U} e^{j(\omega t + \alpha)}$$

$$\underline{u} = R \cdot \underline{i} + \frac{1}{C} \int \underline{i} dt$$

5) Dérivée, Intégrer

$$\frac{d\underline{i}}{dt} = j\omega \underbrace{\hat{I} e^{j(\omega t + \beta)}}_{\underline{i}} = j\omega \underline{i} \quad \left| \quad \int \underline{i} dt = \frac{1}{j\omega} \underbrace{\hat{I} e^{j(\omega t + \beta)}}_{\underline{i}} = \frac{\underline{i}}{j\omega}$$

Electrotechnique I

So deriving a complex number comes back to multiplying it by $j\omega$. We can do the same for the integral. So now by integrating a complex number like here the integral of $i dt$, we will have the integral of an exponential that is always an exponential and by considering the integral calculation we get $(1/j\omega) \hat{I} e^{j(\omega t + \beta)}$ and we notice, again, that here, if this is also our i (vector), integrating a complex number comes back to dividing this complex number by $j\omega$. And now you see why it becomes easy to transform our equations in complex form, because the integral that you see here, in the complete Kirchhoff equation, this integral solves itself very easily by dividing i (vector) by $j\omega$.

Notes

Summary



Exemple et rappel

$$\begin{aligned} \underline{u} &= R \cdot \underline{i} + \frac{1}{j\omega C} \underline{i} \\ \hat{u} e^{j(\omega t + \alpha)} &= \left(R + \frac{1}{j\omega C} \right) \hat{i} e^{j(\omega t + \beta)} \\ \cancel{\hat{u} e^{j\omega t}} \cdot e^{j\alpha} &= \underbrace{\left(R + \frac{1}{j\omega C} \right) \hat{i}}_{\underline{Z}} \cancel{e^{j\omega t}} \cdot e^{j\beta} \end{aligned}$$

Electrotechnique I

We therefore have our equation that becomes: \underline{u} (vector) = $R * \underline{i}$ (vector) and we have $(1/j\omega C) \underline{i}$ (vector). And you see as things get easier. We will now replace the different suspected elements \underline{u} and \underline{i} , therefore unknown. First $\hat{U} e^{j(\omega t + \alpha)}$ is the voltage of the beginning, it is what is given, it is equal to, we highlight the \underline{i} , we then have $[R + (1/j\omega C)] \hat{i} e^{j(\omega t + \beta)}$. We will here separate things to understand better. In the $e^{j(\omega t + \alpha)}$, here as elsewhere in the β , we have a part that depends on the time and a part that is a fixed angle. When we have a sum here, we can replace it by $e^{j\omega t} \cdot e^{j\alpha}$, the rest doesn't change. And we have the same thing for the current, one part fluctuates with time and another part is fixed. Then we see that there is here a left part and a right part that is identical, it is this part that fluctuates with time. So we can simplify this equation and solve an equation that doesn't depend on time. We may also be interested for a moment at this part $R + (1/j\omega C)$, that I will call \underline{Z} , it's a complex number, we will then see what it represents. And if we draw this element in the complex plane, so here imaginary and real we see that we have a real part, it's R , the resistance that is here, and we have an imaginary part, $(1/j\omega C)$.

Notes

Summary



9m 15s

Exemple et rappel

$$\underline{u} = R \cdot \underline{i} + \frac{1}{j\omega C} \underline{i}$$

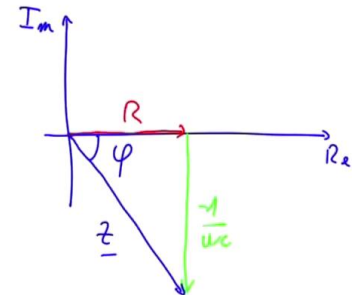
$$\hat{u} e^{j(\omega t + \alpha)} = \left(R + \frac{1}{j\omega C} \right) \hat{i} e^{j(\omega t + \beta)}$$

$$\cancel{\hat{u} e^{j\omega t}} \cdot \cancel{e^{j\alpha}} = \underbrace{\left(R + \frac{1}{j\omega C} \right) \hat{i}}_{\underline{z}} \cdot \cancel{e^{j\omega t}} \cdot e^{j\beta}$$

$$\varphi = \text{Arctg} \left(\frac{-1}{R\omega C} \right)$$

$$|\underline{z}| = \sqrt{R^2 + \left(\frac{1}{\omega C} \right)^2}$$

$$\frac{1}{j\omega C} \cdot \frac{j}{j} = \frac{j}{-\omega C}$$



Electrotechnique I

Maybe to make things more precise, when we have $1/j\omega C$, we can multiply by j and j at the top and at the bottom and we get $j/\omega C$ and j times j makes -1 , so $-j/\omega C$ is something negative, we will therefore have here a part that depends on ωC but that will be negative and finally, we have here this vector that I will call note Z , represented here. The angle that is called here ϕ , φ . ϕ is therefore the tangent arc of this imaginary part on the real part, therefore $-1/R\omega C$. and the norm of Z , it's the square root of the real part squared plus the imaginary part squared, $\sqrt{R^2 + (1/\omega C)^2}$. That is, we have all the elements now to solve and find what we are looking for, that is to say the phase shift between voltage and current, and the value of this current that flows through the circuit.

Notes

Summary



Exemple et rappel

6) Résoudre et identifier :

$$\hat{u} \cos \alpha + j \hat{u} \sin \alpha = Z e^{j\varphi} \cdot \hat{I} e^{j\beta}$$

$$\underbrace{\hat{u} \cos \alpha + j \hat{u} \sin \alpha}_{\hat{u} e^{j\alpha}}$$

7) Solutions complexe et réelle :

$$\underline{i} = \frac{\hat{I}}{Z} e^{j(\omega t + \alpha - \varphi)} = \frac{\hat{u}}{Z} e^{j(\omega t + \alpha - \varphi)}$$

$$i = \operatorname{Re}\{\underline{i}\} = \frac{\hat{u}}{Z} \cos(\omega t + \alpha - \varphi)$$

Electrotechnique I

We get to the sixth step of our method, that is to say solve and identify. We have our $\hat{U} \cos \alpha + j \hat{U} \sin \alpha$ on one side, that is equal to our $Z e^{j\varphi}$, I write this Z that I just mentioned like this, multiplied by the vector $\hat{I} e^{j\beta}$. So this, it is also what we wrote $\hat{U} e^{j\alpha}$. So there is our equation and we can now determinate very simply the solution first complex, then the real solution, Then the complex solution is real. But first complex. We are looking for the i here, therefore this small i (vector) that is equal to $\hat{I} e^{j(\omega t + \beta)}$ is equal to $(\hat{U}/Z) e^{j(\omega t + \alpha - \varphi)}$. The final solution that we are looking for is of course the real i , this i here. we know that to find the real i we must take the real part of my complex i and therefore we will get: $(\hat{U}/Z) \cos(\omega t + \alpha - \varphi)$. It is the final answer to this problem and you know the different steps, that you will do faster once you will be used to it, and that allows you to finally solve very easily an electrotechnical problem with R,L or C, or the three at the same time.

Notes

Summary



Valeur instantanée complexe et phaseurs complexes

$$\underline{u} = \hat{u} e^{j(\omega t + \alpha)} \rightarrow \text{Valeur instantanée complexe}$$

$$\underline{u} = \hat{u} e^{j\alpha} \rightarrow \text{Phaseur de crête}$$

$$\underline{u} = u e^{j\alpha} \rightarrow \text{Phaseur}$$

Electrotechnique I

Now let's see by definition what is the instantaneous complex value and what are complex phasors. We saw previously in our small example, a \underline{u} (vector) $\hat{u} e^{j(\omega t + \alpha)}$, is by definition the instantaneous complex value. Instantaneous because it depends on time. We also saw that it wasn't very important because we can eliminate this temporal component. We will therefore come up with a new element that we will call a phasor and that doesn't depend on time anymore. This phasor that we will write note by \underline{U} capital. Then there are two phasors: the peak phasor, with the peak value as modulus of this vector, or simply, the phasor by taking its effective value of this variable. We will therefore work with these three variables, primarily the last two, or even the last one, of these phasors. We saw before, this instantaneous complex value vector is a vector that will rotate in the complex plane because of the ωt . How fast will it rotate ? Well, it's the ω that determinates how fast this vector rotates. Then, we will decide to eliminate the $e^{j\omega t}$ on both sides of the equations, since we have something that doesn't depend on time.

Notes

Summary



15m 07s

Valeur instantanée complexe et phaseurs complexes

$$\underline{u} = \hat{u} e^{j(\omega t + \alpha)} \rightarrow \text{Valeur instantanée complexe}$$

$$\underline{u} = \hat{u} e^{j\alpha} \rightarrow \text{Phaseur de crête}$$

$$\underline{u} = u e^{j\alpha} \rightarrow \text{Phaseur}$$

Electrotechnique I

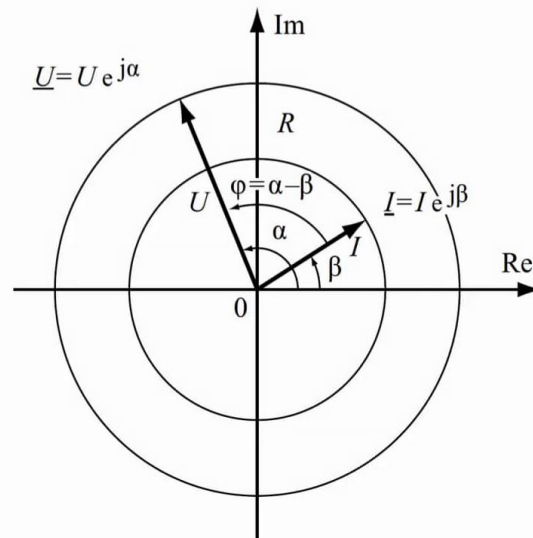
By doing this, it's like if we had photographed at an instant t and we just look at the angle of the voltage at this moment in the complex plane. And just, we stopped the current and what interest us is the angle difference between this voltage or this current, that allows us to determinate different elements that we are looking for and that we would like to know in the circuit.

Notes

Summary



Diagramme des phaseurs



Electrotechnique I

This is what we can show in the complex plane. This current phasor here, you have two of them that are marked here- We did this photograph or this snapshot. Our voltage and our current don't depend on time, we don't need $\omega t + \alpha$ or $\omega t + \beta$ anymore, we just have α or β , and therefore we have a snapshot, a vector at on given moment, same for U with an angle β , that we recover here, another angle, α , here. So we have a voltage and a current and what we notice is that this angle difference, between voltage and current, is what we are generally looking for, that will depend on the elements R, L, C in the circuit and that are equal to $\alpha - \beta$, therefore $\varphi = \alpha - \beta$, we have this angle difference. Now, we can have not only a voltage but several voltages, we also made earlier a sum of voltages. So we must be very careful compared to what we saw in the steady state where the voltages and the currents are constant, we can do the arithmetical sum of voltages and currents. When we work with phasors, when we work with complex calculations, we work with vectors. The sum of two vectors is not just an arithmetical addition. So, if this vector U represents, let's say 50 volts, and that there is another U at another moment, 50 volts plus 50 volts do not necessarily make 100 volts when we add them, unless these two vectors are collinear.

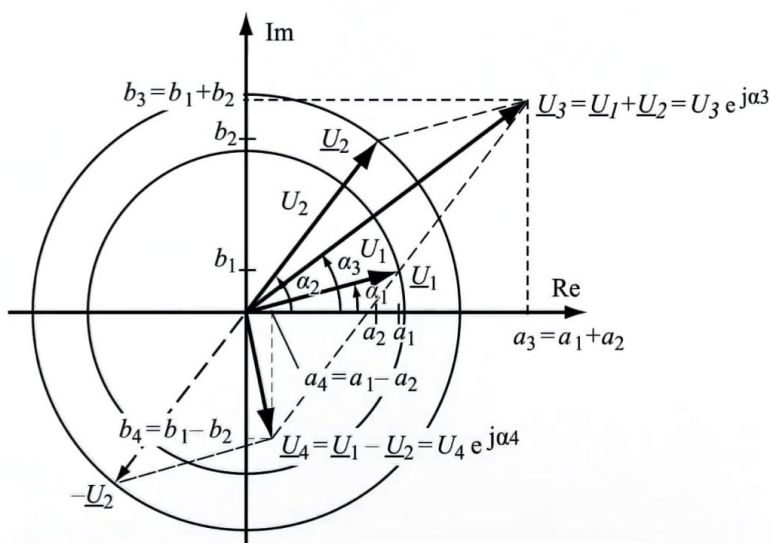
Notes

Summary



17m 07s

Diagramme des phaseurs



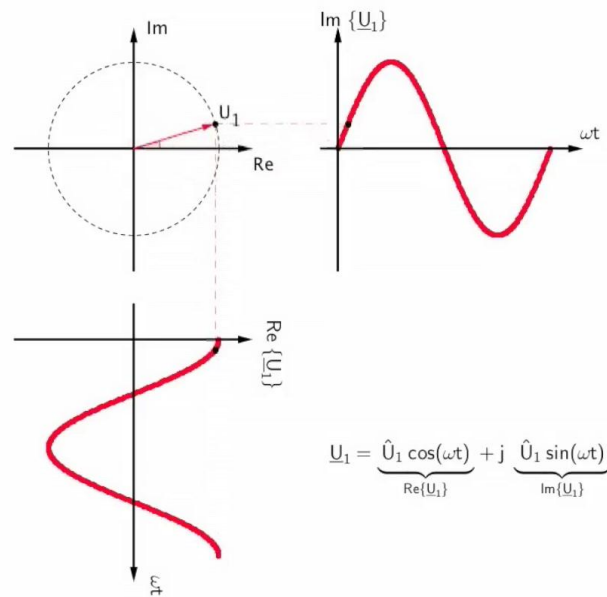
Electrotechnique I

Here's an example where you can see different voltages that we have, for example here, added, \underline{U}_1 and \underline{U}_2 that will give us \underline{U}_3 . Therefore you have the first \underline{U}_1 that is here with a certain angle, a \underline{U}_2 that is here with another angle. And you see that when we do $\underline{U}_1 + \underline{U}_2$, we do a vector sum to get \underline{U}_3 that is not equal to the sum of \underline{U}_2 and \underline{U}_1 but that must be calculated by the geometrical elements. We will then see that it's pretty simple in the case where it's orthogonal because we can use the well known Pythagorean relation, but here it is not orthogonal, we must calculate very precisely in complex numbers, the sum of \underline{U}_1 and \underline{U}_2 , which will add a little bit of additional work but not to forget to be very careful.

Notes

Summary





Electrotechnique I

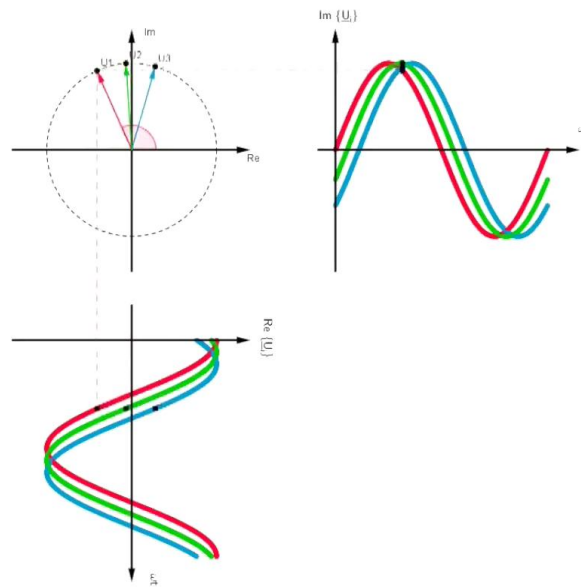
So you see here on this image, or on this animation, on the right the imaginary part of the voltage and on the left at the bottom, the real part of this same voltage, and on the top left, well, this vector that changes as function of time. You can see the equation at the bottom right, this vector representation of the voltage, $\hat{U}_1 \cos \omega t + j \hat{U}_1 \sin \omega t$. So, at the top right, the sine, at the bottom left, the cosine. Then we can choose one or another of these representations. What will interest us is now to always talk about a cosine, it's the part on the bottom left, but that will be written in the complex plane by a vector, a vector that rotate as you can see up here. We can also decide to stop it, this is what we do when we talk about phasors. Now we will see a second video that will allow us to see what is happening when i put another signal.

Notes

Summary



19m 46s



Electrotechnique I

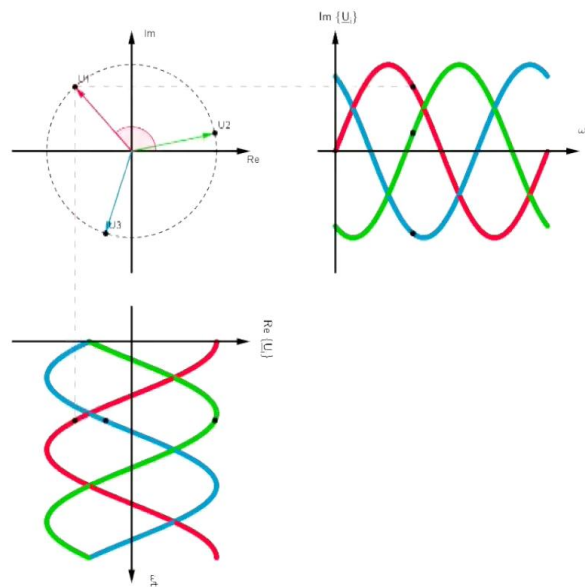
So on this video, we put three voltages that are slightly phase shifted one another. You see in the real world, that is to say always the drawing at the bottom left, these are cosines, three cosines slightly phase shifted of 20 degrees. It is what gives us in the complex domain these three vectors that rotate, U_1 , U_2 , U_3 , at the same speed but slightly shifted and that follow each other. If we prefer the sine representation, that is to say the imaginary part, we then take the top right representation and then we will always work with the imaginary part of the vector.

Notes

Summary



20m 50s



Electrotechnique I

Then on this last video I will show here three vectors phased shifted by 120 degrees from each other. Why? Well, because the single phased sinusoidal steady state will follow after in the three phase to allow us to transmit energy. And so, we will have these kind of signals very well known and frequent since it's the power supply of most countries throughout the world to use a network and a symmetrical three-phase system. You can now see the three vectors, U_1 , U_2 , U_3 , that spin at the same speed in a symmetrical way and whose sum is always zero. We call this representation of sines, cosines and vectors, Fresnel's representation and is what we are going to talk about when we also want to say in the complex plane, we can talk of Fresnel's representation.

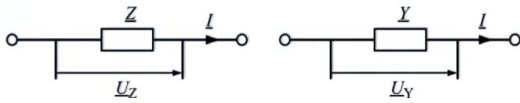
Notes

Summary



21m 28s

Définitions



$$\underline{z} = \frac{\underline{u}}{\underline{i}} = \frac{\underline{U}}{\underline{I}} = \frac{\hat{U}}{\hat{I}}$$

$$\underline{y} = \frac{1}{\underline{z}} = \frac{\underline{I}}{\underline{U}}$$

Electrotechnique I

We will now see what are the impedance and the admittance; impedance and admittance are foundations of electrotechnics that allows us to model the different elements R, L, C and also that leads us to the famous Ohm's law equation but here with complex numbers as we have seen before when we talked about constant current and voltage. This Z underlined, therefore the impedance by definition, is equal to the instantaneous voltage divided by the instantaneous current, but what is also great about it, is that it's also the voltage phasor divided by the current phasor, or the peak voltage phasor divided by the peak current phasor. These three relations are strictly identical. in the same way we define an admittance that we will define as the inverse of the impedance and that also has some particular properties that can sometime be used easier than the impedance, it's the reason for which we will also define it here.

Notes

Summary



22m 22s

Propriétés

$$\underline{z} = \frac{\underline{u}}{\underline{i}} = \frac{u e^{j\alpha}}{I e^{j\beta}} = \frac{u}{I} e^{j(\alpha-\beta)} = \frac{u}{I} e^{j\varphi}$$

$$\underline{z} = \frac{u}{I}$$

$$\varphi = \alpha - \beta$$

$$\underline{y} = \frac{1}{\underline{z}} = \frac{I}{u} = \frac{1}{z} e^{-j\varphi} = y e^{-j\varphi}$$

Electrotechnique I

Notes

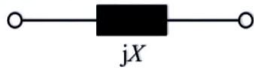
The properties of this impedance are the following: first, when we write our Z , that is equal to the voltage phasor divided by the current phasor, we therefore have $U e^{j\alpha}$ for the voltage phasor, $I e^{j\beta}$ for the current phasor and we have, by using the mathematical property of exponential, a new exponential that is equal to $\alpha - \beta$. And we define this as being the angle φ . φ is therefore equal to $\alpha - \beta$ as we saw before. By identification, we have the norm of this vector Z that is here, is simply equal to U over I . We are not surprised because it's the already known Ohms law and this angle φ is $\alpha - \beta$. We can write the same way for the admittance this time, since it's the inverse of Z , that it's a current over a voltage and that it's equal to $(1/Z) e^{-j\varphi}$ or then $Y e^{-j\varphi}$.

Summary



23m 26s

Résistance et réactance



$$Z \cos \varphi = R = \operatorname{Re}[Z]$$

$$Z \sin \varphi = X = \operatorname{Im}[Z]$$

$$\underline{Z} = R + jX \quad |\underline{Z}| = \sqrt{R^2 + X^2}$$

$$\varphi = \arctan \frac{X}{R}$$

$$\underline{Z} = Z e^{j\varphi} = Z \cos \varphi + j Z \sin \varphi$$

$$= \underbrace{a}_{\text{Résistance}} + j \underbrace{b}_{\text{Réactance}}$$

Electrotechnique I

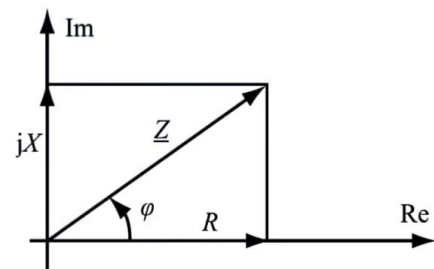
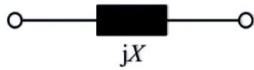
Now let's see what happens inside of this Z. We say that this impedance Z is equal to $Z e^{j\varphi}$ therefore $Z \cos \varphi + j Z \sin \varphi$. It's a complex number but we can also write it, as we see here $a + j b$. We therefore have a real part, it's this $Z \cos \varphi$, an imaginary part, it's this $Z \sin \varphi$. We will call the part "a", the resistance, and we will call the "b" part, the reactance. The resistance is well known by everybody, therefore this $Z \cos \varphi$ is necessarily real, there is just one element that can represent this reality it's the resistor R, it is therefore the real part of Z. We have this $Z \sin \varphi$ that can be represented either by a capacitance or an inductance. We will give it the symbol X and call this the reactance and this reactance would be equal to the imaginary part of the impedance. We symbolize this X, or this imaginary part, by, you have it here on the symbol, a black square that represents the fact that this element here is imaginary or has an imaginary component. We therefore have, our impedance that will always be written as $R + jX$, with the norm of this vector that will be equal to $\sqrt{R^2 + X^2}$ with a φ angle that will be equal to the tangent arc of X over R.

Notes

Summary



Résistance et réactance



Electrotechnique I

We have a very clear representation, so you have the impedance Z , its real part R , here, drawn, a part of a positive reactance, this also means that finally this reactance behaves as an inductance, since I remind you that the inductance will have a positive part, it's the $j\omega L$. So we will have ωL and we will have $-1/\omega C$ for a capacitance.

Notes

Summary



27m 02s

Cas de la résistance

$$\underline{u} = \underline{z} \underline{i}$$

$$\underline{z}_R = R = \frac{\underline{u}}{\underline{i}} = \frac{u}{i}$$

$$\varphi_R = 0$$

$$Y_R = \frac{1}{R} = G = \text{conductance}$$

Electrotechnique I

Let's now take case-by-case R, L and C to determine how they behave as function of the impedance and the admittance and that it will give us Ohms law for each element. The first case, very simple, the resistor. Let's admit that we know that $U = Z I$, is Ohms law but here with the complex case. Our impedance for a resistor, is simply R, and is equal to U (vector) divided by I (vector) written as previously, or U divided by I. Why? because U and I are in phase and that this phi of the resistor, namely $\alpha - \beta$, the phase shift between voltage and current, are equal to zero for the resistance. And in the case of the admittance, the Y of R that is equal to $1/R$, we sometimes call it G uppercase, or conductance.

Notes

Summary



27m 32s

Cas de l'inductance

$$\underline{u} = j\omega L \underline{i}$$

$$u = L \frac{di}{dt}$$

$$\underline{u} = L \frac{d\underline{i}}{dt} = j\omega L \underline{i}$$

$$\underline{z}_L = \frac{\underline{u}}{\underline{i}} = j\omega L$$

$$Z_L = \omega L$$

$$\varphi_L = \frac{\pi}{2}$$

$$R_L = 0$$

Electrotechnique I

There is now the case of the inductance. In the case of the inductance, we have an equation U that links the current I with a relation that we have already seen before and that is equal to $j\omega L$. We therefore have $U = j\omega L * I$. We can maybe explain for a reminder, the equation in time is $U = L (di/dt)$. If written in vector, $U = L (di(\text{vector}) / dt)$. We saw that to derive it was simply the current times $j\omega$. So we have $j\omega$ and the L that stays and finally the small i . We therefore have this U equal to $j\omega L i$. And now if we remove, or if these are two vectors that spin as function of time, of ωt , if we divide by ωt on both sides, so if we do as I said before a photograph, well we get phasors again, it's the equation that I wrote here. My impedance Z_L is equal to U (vector) divided by I (vector) and therefore is equal to $j\omega L$. It's the impedance of an inductance. We therefore have Z_L , the norm, that is simply equal to ωL . We have the angle φ_L , as it is equal to the tangent arc of $\omega L/R$, this is just in the RL case, but here in this case of the inductance we have a j , therefore one is perfectly vertical, this angle is equal to $\pi/2$. And for the impedance, the resistor is zero.

Notes

Summary



Cas de condensateur

$$i = C \frac{du}{dt}$$

$$\underline{i} = j\omega C \underline{u}$$

$$\underline{I} = j\omega C \underline{U}$$

$$\underline{Z}_C = \frac{\underline{U}}{\underline{I}} = \frac{1}{j\omega C} = -j \frac{1}{\omega C}$$

$$Z_C = \frac{1}{\omega C}$$

$$X_C = -\frac{1}{\omega C}$$

$$\varphi_C = -\frac{\pi}{2}$$

$$R_C = 0$$

Electrotechnique I

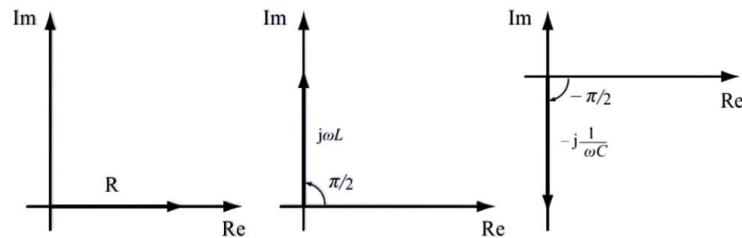
Finally the case of the capacitor. We have in the temporal case $i = C (du/dt)$. We have the i (vector) that is then equal to $j \omega C \times u$ (vector). Or then, I (vector), we are now in phasors, equal to $j \omega C \times U$ (vector). Said otherwise, the impedance Z_C , that is by definition, equal to U/I for phasors, will be equal to $1/j\omega C$ or $-j (1/\omega C)$. We can write that the norm of the Z of the capacitor is $1/\omega C$, that the reactance of the capacitor is $-1/\omega C$, the angle φ_C , is negative, so it will be equal to $-\pi/2$, and again the resistance of C is zero.

Notes

Summary



Résumé



	$\text{Re}[\underline{Z}]$	$\text{Im}[\underline{Z}]$	Z	φ	\underline{Z}
R	R	0	R	0	R
L	0	ωL	ωL	$\pi/2$	$j\omega L$
C	0	$1/\omega C$	$1/\omega C$	$-\pi/2$	$-j/\omega C$

Electrotechnique I

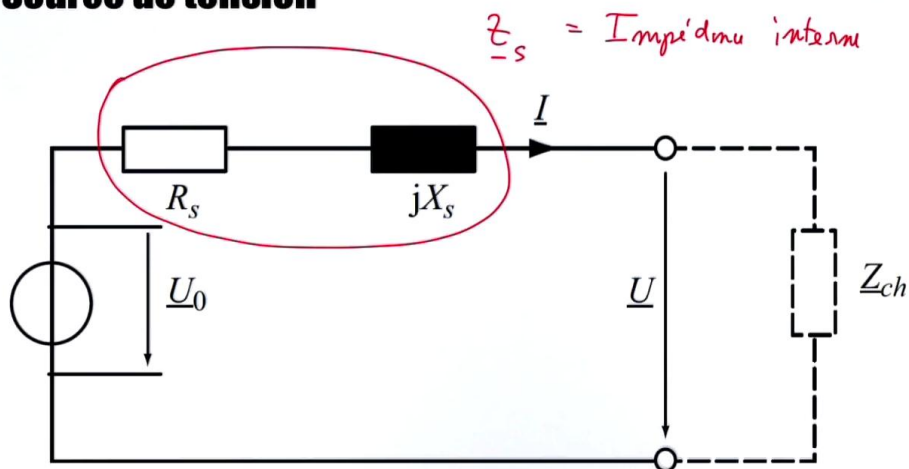
To sum up, we sketched a graph in the complex plane of the three impedances R , L and C that we just wrote. You have here, in the first column of this table, the real part of Z and the imaginary part. In other words, the resistance and the reactance. Then the norm, the angle and finally, the impedance written under the complex form. We therefore have our first vector R that is on the real part. If you take a second element L with a $j\omega L$, there is a vertical vector, the angle is equal to $\pi/2$. And finally, with the capacitance we saw that it was negative because of $1/j\omega C$ or $-j(1/\omega C)$. We have a perfectly vertical vector but negative so $-\pi/2$ for the angle.

Notes

Summary



Source de tension



Electrotechnique I

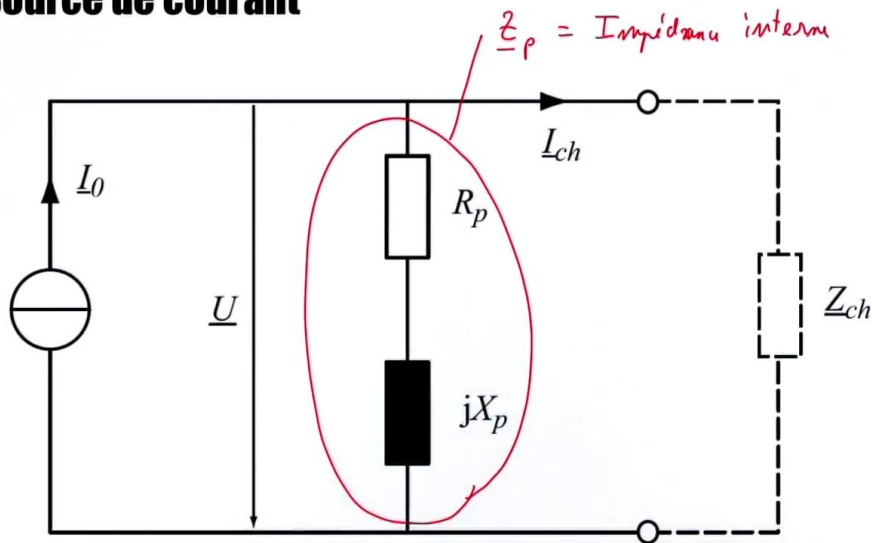
The last thing we have yet to see are the power supplies. When we have a voltage supply that works in alternative, in a sinusoidal single phased steady state we must ask what is happening, as for the steady state or the DC voltage. So in the same way as the DC steady state, an ideal supply doesn't work. Therefore we always have a voltage supply or a current supply that must be associated to an internal impedance that is put in parallel of serial. Therefore for the voltage supply, we have here this element here that is an internal impedance of the supply. This Z_s is the internal impedance. It is put in series with the supply allowing us to have, this time, a voltage supply said real.

Notes

Summary



Source de courant



Electrotechnique I

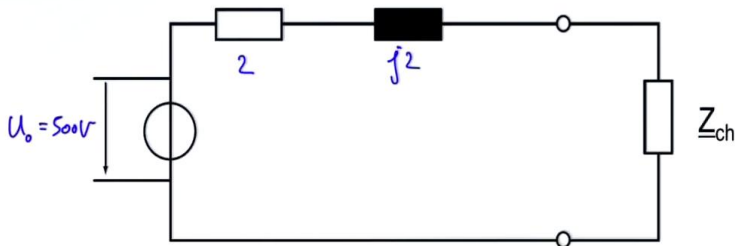
In the same way for the current, we must put a real impedance in order to have a real current supply, this impedance is said to be internal to the supply, and will allow us to define this real current supply and not ideal as before. Therefore this Z_p here is also the internal impedance.

Notes

Summary



Source réelle : Exemple



Calcul du courant I circulant dans le circuit si Z_{ch} prend les valeurs suivantes :

- a) $Z_{ch} = 48 \Omega$ (résistance)
- b) $Z_{ch} = j48 \Omega$ (inductance)
- c) $Z_{ch} = -j48 \Omega$ (capacité)

$$\begin{aligned}
 a) \quad Z_{ch} &= 48 \Omega \\
 Z_{tot} &= Z_i + Z_{ch} = 2 + j2 + 48 = 50\Omega + j2 \\
 |Z_{tot}| &= \sqrt{50^2 + 2^2} \approx 50 \Omega \\
 I &= \frac{U}{Z_{tot}} = 10 A \quad U_{chR} = Z_{chR} \cdot I = 480 V
 \end{aligned}$$

Electrotechnique I

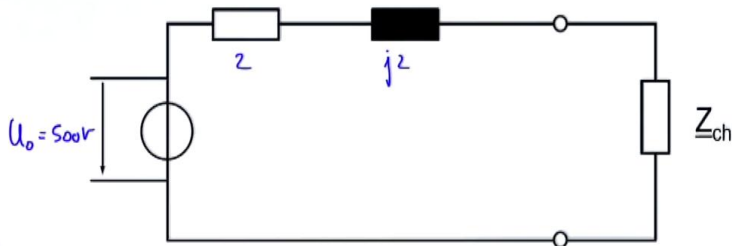
To finish, I suggest you this small example to picture what we just saw with the supplies and the calculations of impedances. We have here a voltage of 500 volts, a resistance of 2 ohms and a reactance of also 2 ohms, that I write here $j2$, and our load Z_{ch} that would take three different values, either a resistance of 48 ohms, or an inductance of 48 ohms, or either a capacitance of 48 ohms and you will see each time that the impedance is different. The first case, the case where the resistance by itself. We therefore have Z_{ch} that is only equal to 48. What is the total impedance? Well here, these three impedances are in series, we will then add the internal impedance plus the impedance load therefore $2 + j2 + 48$, be careful to not add apples and oranges, we have a real part, an imaginary part. Therefore the real part is equal to 50 and the imaginary is always 2. So my Z total, or the norm is $\sqrt{50^2 + 2^2}$ which is about 50 ohms. We can also calculate here the current. So the current I is U/Z_{tot} and is worth 10 amps. At the terminals of the load of my resistor, well, it's the Z of the load resistance times the current and we get 480 volts.

Notes

Summary



Source réelle : Exemple



Calcul du courant I circulant dans le circuit si Z_{ch} prend les valeurs suivantes :

- a) $Z_{ch} = 48 \Omega$ (résistance)
- b) $Z_{ch} = j48 \Omega$ (inductance)
- c) $Z_{ch} = -j48 \Omega$ (capacité)

b) $Z_{ch} = j48$
 $Z_{tot} = 2 + j50$
 $|Z_{tot}| \approx 50$
 $I = 10 \quad U_{chL} = 480V$

c) $Z_{ch} = -j48$
 $Z_{tot} = 2 + j2 - j48 = 2 - j46$
 $|Z_{tot}| \approx 46 \Omega$
 $I = 10,87 A$
 $U_{chC} = 521,7 V$

Electrotechnique I

Let's get to the b case. For the b case, we now have our Z_{ch} that is equal to $j48$, it's different, our Z total will then be added with the rest, but in the real part it's 2 only and the imaginary part is 50. So Z total but normed, the norm of this vector, square root of 2 squared plus 50 squared, will also give the same number as before, roughly 50. And with the current, we find the same numbers. The U at the terminals of this inductance is also 480 volts. Last case is, the c, with this time a capacitance, a capacitor. Our Z_{ch} is -48 ohms, our Z total will now slightly change compared to before, because we will have $2 + j2 - j48$ and we get $2 - j46$ and our norm will be different from the two previous cases, about 46 ohms, with a current that is then equal to 10.87 A and a voltage at the terminals of this capacitance that is 521,7 volts.

Notes

Summary





- Calcul complexe associé
- Représentation de Fresnel
- Impédances et admittances

Electrotechnique I

To sum up, we now saw all the chapter on sinusoidal single phased state. You saw the advantages of the transit to complex calculations, and finally the paradox of easier calculations when we write the elements with vectors where the different values, as the voltage, current or impedances, with vectors and this Fresnel representation that allows us to represent these vectors in the complex plane, that these vectors are fixed or move as function of time, allowing us to have a picture of what is happening in the complex world, but showing conceptually the reality. Thank you.

Notes

Summary



37m 07s