ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

CIRCUITS EN RÉGIME SINUSOÏDAL

RÉGIME PERMANENT SINUSOÏDAL

LEÇON 13

Électrotechnique I
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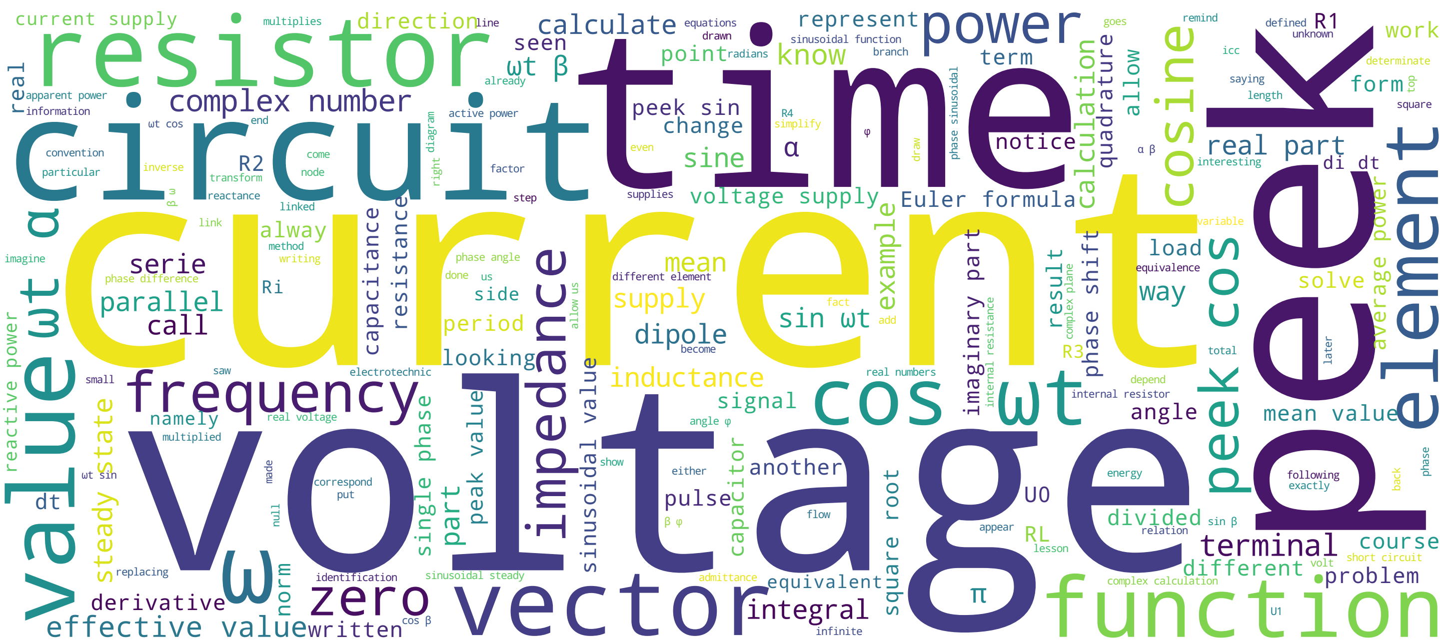


Video

Hello and welcome to this lesson dedicated to single phase sinusoidal steady state. We will see in this lecture what the single phase sinusoidal steady state is. and in which circumstances we use it and define its sinusoidal values at all time with a very specific way to write them. We will also see how the complex calculation will help us to solve some problems, it can seem paradoxicall to use vectors to solve temporal problems such as the single phase sinusoidal steady state.

Notes

Summary

0m 04s

Régime permanent : établie depuis $t = -\infty$

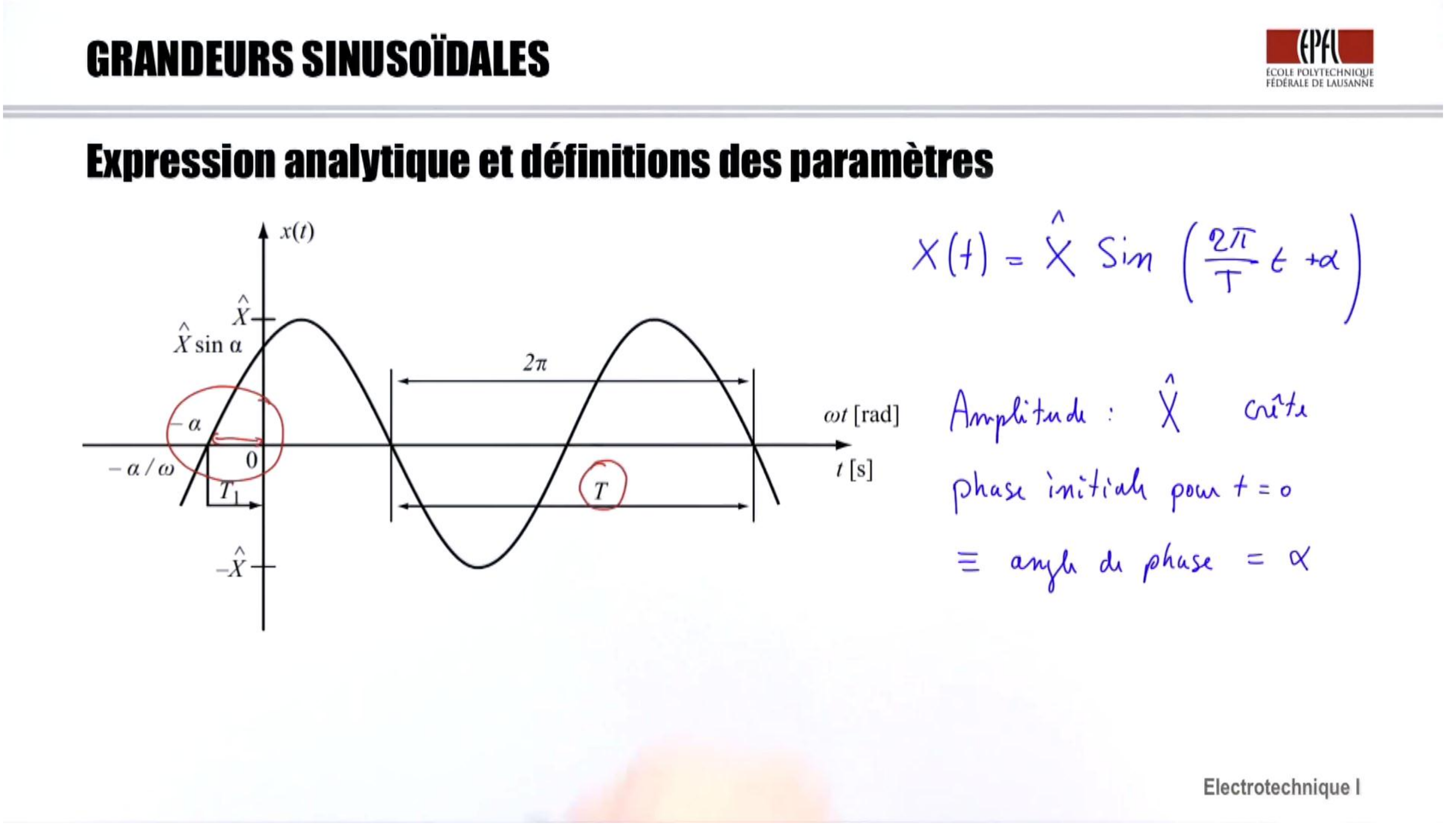
$\sin\alpha$, $\cos\alpha$

By definition, we can say that the single phase sinusoidal steady state, or the steady state, is when the external excitation, current, voltage, are sinusoidal functions that are supposed established in time from the infinite. We therefore have sinusoidal functions or in the same way, co-sinusoidal as it is the same function out of step by π/2. It is also interesting to note that the sine and the cosine are in reality the only periodic function that have a derivative or an analogue integral. It is the reason why this state is very particular. It has an important role in electricity and this prevalence is linked firstly to the fact that the industrial production of energy generally results from a mechanical/electrical conversion realised by the rotation of a winding placed in an magnetic induction field and the induced voltage is rightly sinusoidal. It's this characteristic that allows to assure an economical and efficient distribution. We will now see how to define this sinusoidal function.

Notes

Summary

0m 36s

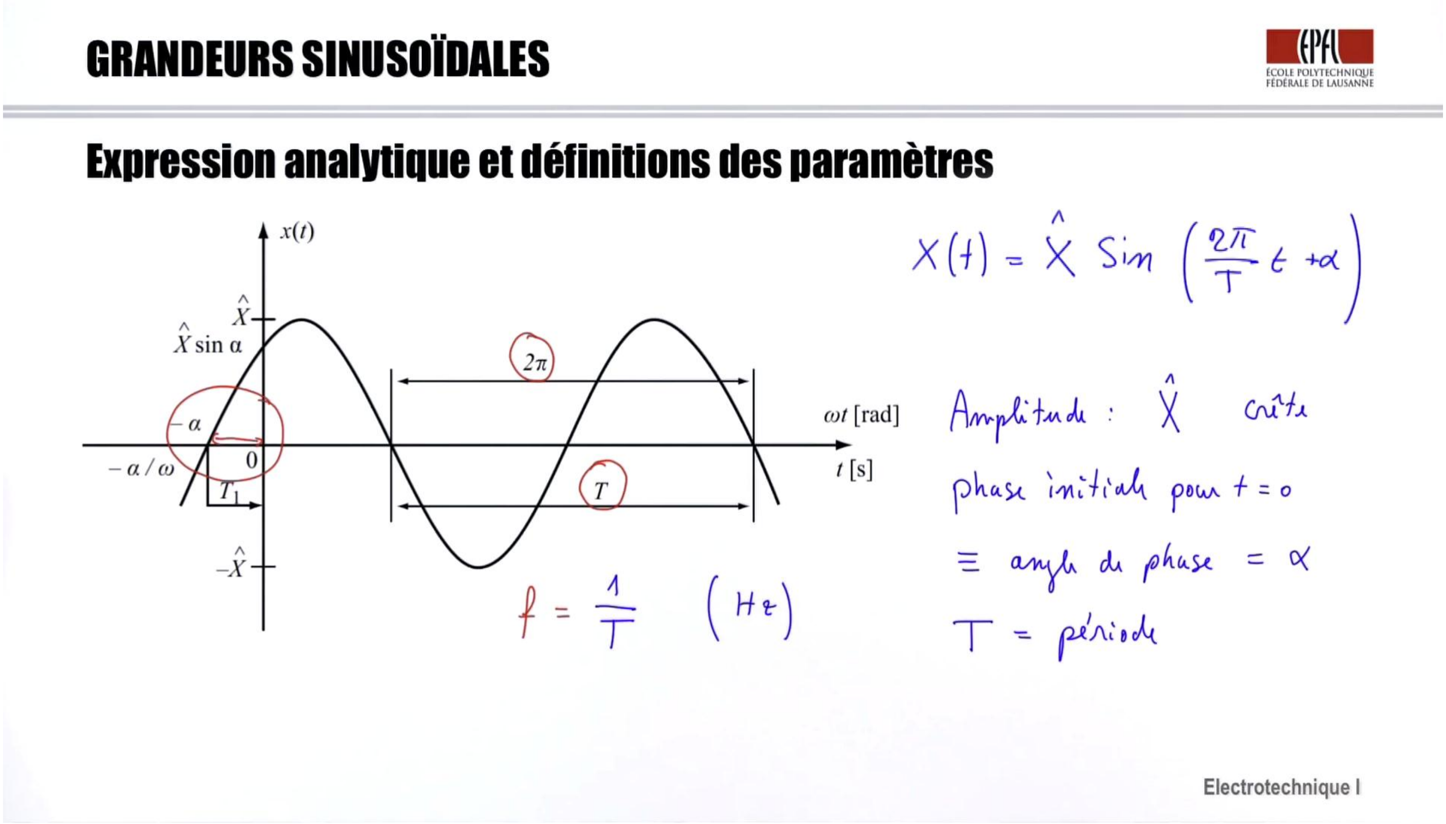


So here is a sine drawn as function of time, we took x(t) as function of t, here, X can symbolise any sinusoidal value. We have defined here all the elements that we will now have to understand to be able to use thereafter this sinusoidal function x(t). So if we write this function, we can write that x(t) is equal to x (peek)sin((2πT)xt + α). This is the equation of the function drawn on the graph. So we will be now able to define a certain number of things. First of all the amplitude. The amplitude of this sinusoidal signal is x (peek), or x said peak, that will be the maximal amplitude of this sine, the peak value. We can also define the initial phase. This initial phase for t which is equal to zero, we also call it the phase angle, and this phase angle is identical to the phase angle and this is equal to alpha (α). This is why the value that we see here that allows to define the shift that is between t=0 and - α. We have another fundamental element that is here: the period. This period T corresponds to an arithmetic sine to the function 2π therefore we go back to the beginning, and we start another cycle again, and this function T has for equation, or connection with the frequency, 1/T.

Notes

Summary

1m 56s

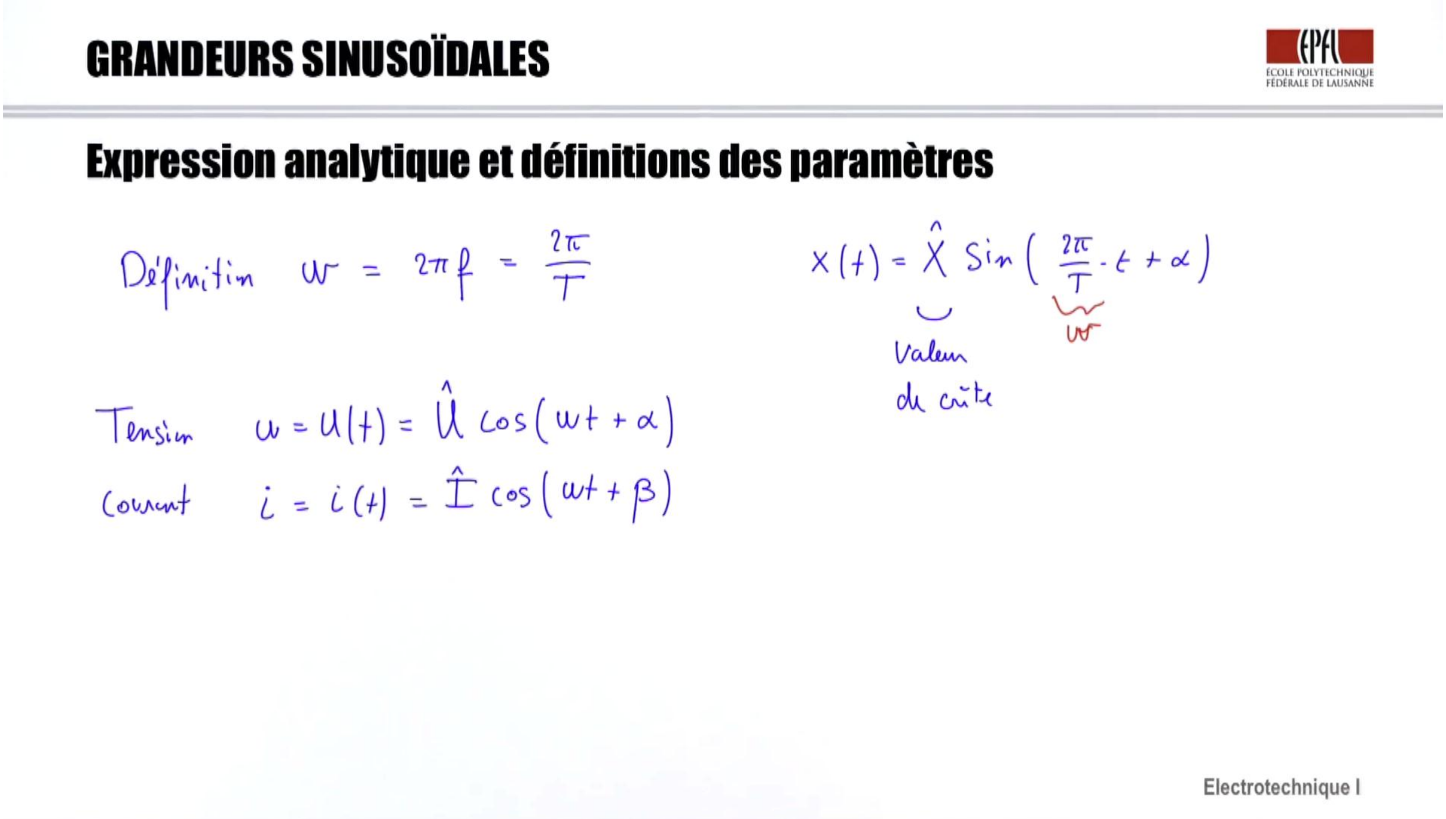


Therefore this frequency will be in hertz, of course, and is equal to 1 divided by the period T. So T will be by definition called the period. A few other elements can now be calculated from this.

Notes

Summary

3m 59s



So we can now define the pulse, that we will write down small omega ω, and this pulse is defined as being 2π times the frequency (f) which is 2π divided by the period (T). This is why we have previously seen in the equation that I remind you x(t) = x (peek) sin ((2π / T) x t + α) Therefore we have a pulse that appears and that allows to define this value as an angle in radians. Therefore this allows to make the transfer, or the transformation, between time and radians, between an angle and time. So as we have said before, this is the peak value by definition. And this now allows us to explain, or to write, for the first time a voltage equation of a current voltage. So for the voltage, we will write u(t) is equal to U (peek), the maximum value of the voltage, times cos(ωt + α). So here is a typical equation of a sinusoidal voltage. And, by convention in electrotechnics, we will simply note it u. So we will no longer write T in brackets, we will now see that when we write a lowercase letter, this means that we are looking, or that we are writing, the function of time (t). Therefore here for the current, Î cos(ω * t + β). We will add another angle. So you see that all the functions finally have the same form.

Notes

Summary

4m 18s

Expression analytique et définitions des paramètres

Définition $\omega = 2\pi f = \frac{2\pi}{T}$

$$x(t) = \underbrace{\hat{X}}_{\text{Valeur de crête}} \sin\left(\underbrace{\frac{2\pi}{T}}_{\omega} \cdot t + \alpha\right)$$

Tension $u = u(t) = \hat{U} \cos(\omega t + \alpha)$

Courant $i = i(t) = \hat{I} \cos(\omega t + \beta)$

Définition : Déphasage entre u et i : $\varphi = \alpha - \beta$

En phase : Toutes les grandeurs ont même fréquence, différence de phase est nulle

En quadrature : " " " " " est $\pm\frac{\pi}{2}$

Electrotechnique I

It is always a cosine, peak value, and a value as function of time, and an angle. And another definition that we will be able to write here, is when we have a voltage and a current in the circuit, we will define phase shift between these two values. You can see these two values are, here, at the same frequency. Therefore all the values in the circuit have the same frequency, but the phase which is here, so alpha (α) and beta (β) can be different for the voltage or for the current. Therefore, by definition, we will define the phase shift between the voltage and the current as being the angle φ that is equal to α - β. Another definition is when we say the word "phase". The word "phase", or" in phase", which is a more accurate word, means that all the values have the same frequency, it doesn't necessarily mean that the phase difference is zero. If the phase difference is equal to π/2, then we will say that these phases are "in quadrature". So I repeat, all the values have the same frequency, and the phase difference is zero. At this moment, we will say that the system is in phase, we will say "in quadrature", and all the values have the same frequency but the phase difference is equal to more or less π/2.

Notes

Summary

6m 13s

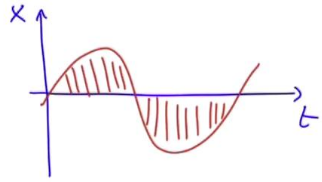
Expression analytique et définitions des paramètres

$x = \hat{X} \cos(\omega t + \alpha)$ → Valeur moyenne :

Définition : $\bar{X} = \frac{1}{T} \int_0^T x(t)\, dt$

pour un cos : → intégr. sur 2π

$\bar{X} = 0$



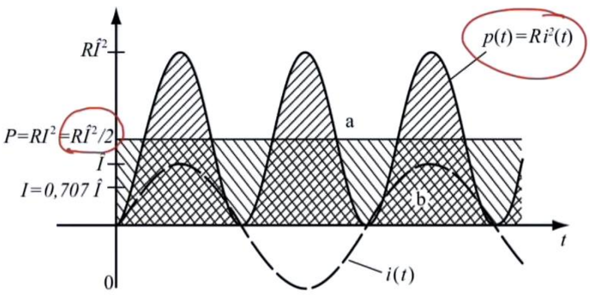
We will now define here a concept well known by students that is the mean value. Everybody knows how to calculate the average of their grades to know if they get to pass or not their school-year. We will here calculate the mean of a signal with its own definition of the mean. So when we have a... I repeat the equation x that is equal to x (peek) cos (ωt + α) if I am looking for the mean value of this signal. By definition, The mean value of a signal that we will write X uppercase underline is equal to 1/T integral from 0 to T of x(t) dt So for a cosine, if we integrate on 2/π, so if we integrate on 2π, we will get the mean value X(mean) equal to zero. We can also realize it. So if we have here X(mean) and the time t and that we have the sinusoidal or co-sinusoidal signal that is like this. We can clearly see that the integral so the part here under the curve will completely cancel itself so a cosine or a sine always has a mean value on its complete period that is zero. We therefore see that the information of a mean value isn't very interesting in electrotechnics because it will always be zero for periodic functions. We will then imagine another way to allow us to have an information especially an information that will be linked, you will see, to power.

Notes

Summary

8m 14s

Valeurs efficaces de grandeurs sinusoïdales



$p = u \cdot i \rightarrow$ Résistance

$u = R \cdot i$

$$p = \frac{u^2}{R} = \frac{\hat{U}^2 \cos^2(\omega t + \alpha)}{R}$$

$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{U^2 \cos^2(\omega t + \alpha)}{R} \, dt$$

Electrotechnique I

This will be the definition of the effective value of sinusoidal values. We have drawn here the power, so you can see the current i(t), that is drawn here. Then we did calculate Ri^2. We know that the power is the voltage (u) times the current (i) If we are in a resistor, and therefore that u is equal to R * i, we can write that this same power is equal to u^2/R. And if P is equal to u^2/R, we then have u (peek) cosine squared of ω * t + α divide by R. We also have u^2. So this the instantaneous power that is represented here with values that you have here, so Ri^2, or u^2/R : it's the same thing. And you have this that is strictly positive. We can clearly see it here, the mean of this function is not zero. It is determinate by Ri^2 divided by 2. We can calculate this to convince ourselves. Let's calculate the average power. So the average power of a resistor 1/T integral from zero to T of the average power so let's write u^2 cosine squared ω* t + α divided by R dt. Imagine that we don't know how to calculate the integral of cos^2, we don't have a numerical index on hand we will say that we shift this signal of π/2, we will get exactly the same form, but instead of having a cosine, we will have a sine, but i'ts the same thing.

Notes

Summary

10m 10s

GRANDEURS SINUSOÏDALES

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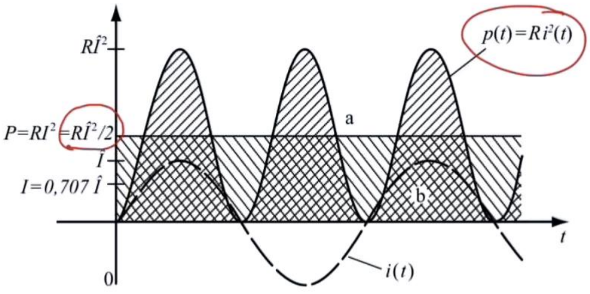
Valeurs efficaces de grandeurs sinusoïdales



$p = u \cdot i \quad \rightarrow$ Résistance

$u = R \cdot i$

$$p = \frac{u^2}{R} = \frac{\hat{u}^2 \cos^2(\omega t + \alpha)}{R}$$

$$\bar{P}_R = \frac{1}{T}\int_0^T \frac{\hat{u}^2 \cos^2(\omega t + \alpha)}{R}\, dt$$

$$\bar{P}_R = \frac{1}{T}\int_0^T \frac{\hat{u}^2 \sin^2(\omega t + \alpha)}{R} \cdot dt$$

$$2\bar{P}_R = \frac{1}{T}\int_0^T \frac{\hat{u}^2}{R}\Big(\underbrace{\sin^2(\omega t + \alpha) + \cos^2(\omega t + \alpha)}_{1}\Big)dt$$

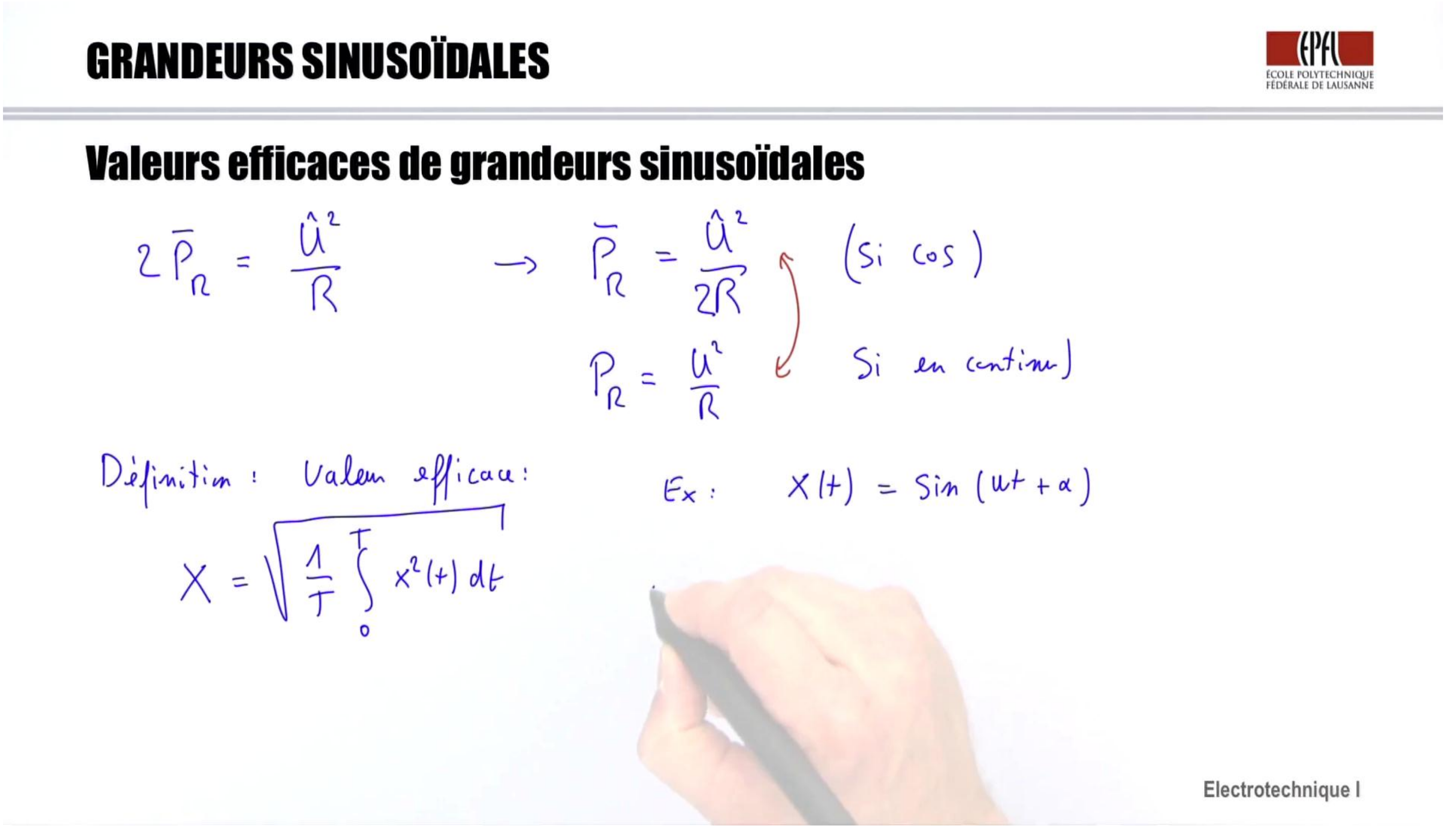
Electrotechnique I

Therefore sin^2 of ω * t + α divided by R dt. Then we add both because we want to get a well known trigonometric connection. So 2 times the average power gives us 1/T integral from 0 to T of u^2 (peek) I have not written it, sorry it's a mistake divided by R times, first of all, sin^2 of ω * t + α + cos^2 of ω * t + α times dt. What do we notice? We take notice that this relation sin^2 + cos^2 is equal to 1. And therefore simplifies our calculations.

Notes

Summary

12m 21s



So then we continue, we get 2 times the average power in a resistor it is U (peek)^2 divided by R. And then we find what we are looking for: the average that is equal to U(peek)^2/2R. So we see here that, contrary to the steady state, so when we where in cosine or in single phase cosinusoidal or sinusoidal but if we are in continuous, we know that the power of a resistor is U^2/R. We see here this difference of notions between U^2 and U. Finally, this doesn't give the same value, or the same concept of information on the power inside of the circuit. And this will lead us to define a new relation, that we will call the effective value. By definition, the effective value is equal to this: X square root of 1/T integral from 0 to T of x^2 dt. What have we done? We calculate the mean of the signal squared and we take the square root. So here is the definition of the effective value. So let's know see for a sine or a cosine what it can give. An example: for the sinusoidal value, I take an X(t) that is equal to sin (ω * t + α). And I look for the effective value. So X is equal to the square root of X (peek)^2/T -there is X (peek) missing here of course- X (peek)^2/T integral from 0 to T of sin^2 (ωt + α) dt.

Notes

Summary

13m 15s

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Valeurs efficaces de grandeurs sinusoïdales

$$2\bar{P}_R = \frac{\hat{U}^2}{R} \longrightarrow \bar{P}_R = \frac{\hat{U}^2}{2R}$$

(si cos) $= \frac{U^2}{R}$ ← valeur efficace

$$P_R = \frac{U^2}{R}$$

Si en continu)

Définition : Valeur efficace:

$$X = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

Ex : $x(t) = \hat{X}\sin(\omega t + \alpha)$

$$X = \sqrt{\frac{\hat{X}^2}{T}\int_0^T \sin^2(\omega t + \alpha)\,dt} = \frac{\hat{X}}{\sqrt{2}}$$

Valeur efficace de X

$$\hat{U} = U \cdot \sqrt{2}$$

Electrotechnique I

This when we do the entire calculation, will give to us X (peek)/√2. And by definition, we will therefore say that this is the effective value of X. We will now take back the example that is here, on top, and calculate, but with the effective value, this new average power. If I replace now my U (peek), I know that it is U(effective) times √2 by replacing here by the effective value, what do I notice? I notice that by putting the effective value instead of the peak value the 2 disappears thanks to the √2 and finally this effective value represents an identical energetic concept as the one that we had in continuous current. It is the value that will be systematically used in the calculation of powers, but not powers only, also general markings, we will generally write down that the effective value of the voltage is equal to U, the value of the current is equal to I.

Notes

Summary

15m 41s

GRANDEURS SINUSOÏDALES

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Valeurs efficaces de grandeurs sinusoïdales

$u = \sqrt{2}\, U \cos(\omega t + \alpha)$ $\quad \hat{U} = U \cdot \sqrt{2}$ ← facteur pour les grandeurs sinusoïdales

$i = \sqrt{2}\, I \cos(\omega t + \beta)$ $\quad \hat{I} = I \cdot \sqrt{2}$

$U = 230\ V$ est valeur efficace.

$\hat{U} = 230 \cdot \sqrt{2}$

Electrotechnique I

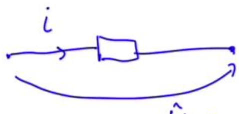
We therefore have for the voltage: √2 x U cos (ωt + α) with U (peek) that is equal to U x √2. We have for the current, √2 x I cos (ωt + β) with I (peek) equal to: I x √2. This √2 is the factor to transform the effective value into a peak value, but the factor for sinusoidal values. Warning, if we have a signal that is square or triangle or another, but not sinusoidal, well this factor won't be equal to √2 anymore, but to another value. In a conventional network that we have in a household or in an industry, we say that generally the value of the grid is equal to 230 volts. What does this U = 230 volts mean ? This is the efficient value. This means that the maximum signal that goes in the phase that we connect tot the network is not 230 volts peak voltage, but it is 230 * √2 so almost 300 volts in peak value. So the U (peek) is equal to: 230 * √2 in a conventional network.

Notes

Summary

17m 13s

Cas de la résistance



$u = \hat{U} \cos(\omega t + \alpha)$

$u = R \cdot i$

$\hat{U} \cos(\omega t + \alpha) = R\, \hat{I} \cos(\omega t + \beta)$

il résulte :

$\hat{U} = R \cdot \hat{I}$

$\alpha = \beta$ Tension et courant sont en phase

Electrotechnique I

We will now take every known element, the resistance, the inductance, the capacitance, to determinate what is happening when a resistor is crossed by a sinusoidal current-voltage; same for the inductance, same for the capacitor. So first of all, for the resistor, let's imagine a resistor where we will connect to a voltage U = U(peek) cos (ωt + α) Between those two terminals, will flow a current inside the resistor. Ohms law: U = R * i is always true. Therefore we can write this Ohms law by replacing now the different elements. First, we have U(peek) that is equal to cos (ωt + α) = R *.... The current must be the same kind and the same frequency, so we can already write I (peek) cos (ωt + β) There are unknowns here, these are β and I (peek), if we know the U (peek) and the α. We get that, U (peek) is equal to R * I(peek) by doing the identification, we see that it's the only way to be able to solve this problem And also that α is equal to β. This means that the resistor doesn't change the phase angle of the voltage. In other words, voltage and current are in phase in this configuration of the resistor. And this will be very handy in the future to always remember that the resistor doesn't change the angle phase.

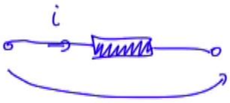
Notes

Summary

19m 02s

Cas de l'inductance

$$u = L \cdot \frac{di}{dt}$$



$$u = \hat{U}\cos(\omega t + \alpha)$$

$$\hat{U}\cos(\omega t + \alpha) = -\omega L \hat{I} \sin(\omega t + \beta)$$
$$= \omega L \hat{I} \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

$$\left.\right\} \quad \hat{U} = \omega L \hat{I} \qquad \alpha = \beta + \frac{\pi}{2}$$

Tension et courant en quadrature.

Electrotechnique I

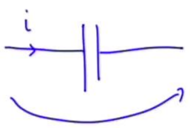
Now in the case of the inductance. Therefore we know that the law that links the voltage and the current in an inductance, is the fluctuation of the current as function of time multiplied by L, which gives the voltage at the terminals. So if we draw this sketch again. We have here an inductance. Here at the terminals we will put U = U (peek) cos (ωt + α) What about the current that goes through this inductance here ? We can write this equation again. First, the voltage equation: U (peek) cos (ωt + α) = -so here we have a derivative, the derivative of the cosine which gives a sine with the internal derivative, ω that goes up-front, and a minus sign- so we have - ω * L * I (peek) sin (ωt + β) Now, we will put back the sine and the cosine to have an equality on both sides, and therefore we transform this - ω sin (ωt) by a + ω cosine phase shifted of π/2 So we have an equal sign, here, at ω * L I (peek) cos (ωt + β + π/2) By doing this identification again on both sides, we get to the following conclusions. First, U (peek) is equal to ω * L * I (peek). We have a ratio that depends on the pulse. Bigger the pulse is, more this voltage will fluctuate as function of the current. Finally, α is equal to β + π/2. In short, the current and the voltage will be in quadrature. So a delay of the current of an angle π/2 compared to the voltage. So voltage and current in quadrature.

Notes

Summary

21m 05s

Cas du condensateur



$$u = \hat{U}\cos(\omega t + \alpha)$$

$$i = C\,\frac{du}{dt}$$

$$\hat{I}\cos(\omega t + \beta) = -\omega C \hat{U} \sin(\omega t + \alpha)$$
$$= \omega C \hat{U} \cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$$

$$\hat{U} = \frac{\hat{I}}{(\omega C)}$$

$$\alpha = \beta - \frac{\pi}{2}$$ Tension et courant en quadrature.

Electrotechnique I

Finally, the case of the capacitor. Here, we draw again the sketch. We have here at the terminals, a voltage U (peek) cos (ωt + α). We are looking for the current that will flow through this capacitance, the capacitor. The link that we know, is the fluctuation of the voltage as function of time multiplied by he capacitance will give us the current. So we write here, this equation as function of the current. First I (peek), that will necessarily be a cosine, ω * t + β is equal to the derivative this time, of the voltage. So we get the same elements as before so finally ω * C * U (peek) sin (ωt + α). We convert this sine into a cosine and we get ω * C * U (peek) cos (ωt + α + π/2) The result now is that U (peek) is in the end equal to I (peek)/ω * C and that α is equal to β minus π/2. We therefore have here the voltage and the current in quadrature. Here the current is ahead of π/2 on the voltage.

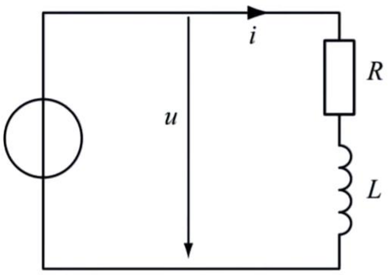
Notes

Summary

23m 19s

CALCUL COMPLEXE ASSOCIÉ

Cas d'une inductance et d'un résistance en série



$$u = R \cdot i + L \frac{di}{dt}$$

$$\hat{U} \cos(\omega t + \alpha) = R\hat{I} \cos(\omega t + \beta) - \omega L \hat{I} \sin(\omega t + \beta)$$

Electrotechnique I

So now let's imagine, the case of a more complex problem where we have R and L at the same time on a same circuit. What can we write down? How can we pull through in this way ? We will realize that if we stay with our simple Kirchhoff calculation like we did until now, the system will become very complex. So rather see. If we want to write here the Kirchhoff equation, of elements linked to this voltage, we can write that U is equal first of all to R * i + - at the crossing in the inductance- L di/dt By replacing now the different elements, we get U (peek) cos (ωt + α) = here we already have a simple connection, R * I (peek) * cos (ωt + β). So we always assume that when we are in the same circuit, the pulses have to be all the same. So our unknown, their are two for the current, it's the magnitude and the phase β plus here L di/dt that becomes minus, because the cosine becomes sine: ω * L * I (peek) sin (ωt + β). So you see here, that it becomes pretty complicated. We can not make an identification anymore by saying to the left or to the right of the equation, because we have here three terms and no longer two terms. We must then simplify this equation to be able to succeed, we must break down the functions sine and cosine into sums.

Notes

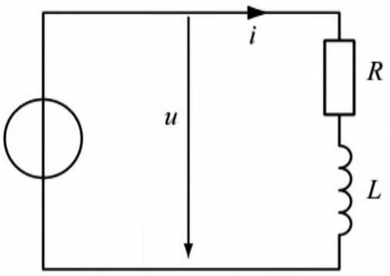
Summary

24m 59s

CALCUL COMPLEXE ASSOCIÉ

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Cas d'une inductance et d'un résistance en série



$$u = R \cdot i + L\frac{di}{dt}$$

$$\hat{U}\cos(\omega t+\alpha) = R\hat{I}\cos(\omega t+\beta) - \omega L\hat{I}\sin(\omega t+\beta)$$

$$\hat{U}\left(\cos\omega t \cdot \cos\alpha - \sin\omega t \sin\alpha\right) =$$

$$\hat{I}\left(R\cos\omega t\cos\beta - R\sin\omega t\sin\beta - \omega L\sin\omega t\cos\beta - \omega L\cos\omega t\sin\beta\right)$$

$$\left.\begin{aligned}\hat{U}\cos\alpha &= R\hat{I}\cos\beta - \omega L\hat{I}\sin\beta\\ \hat{U}\sin\alpha &= R\hat{I}\sin\beta + \omega L\hat{I}\cos\beta\end{aligned}\right\}$$

Electrotechnique I

Here come the following thing: we have U (peek) that multiplies the cos (ωt) *cos (α) - sin (ωt) * sin (α) and this is equal to I (peek) that multiplies R * cos (ωt) * cos (β) - R * sin (ωt) * sin (β) - ω * L*sin (ωt) * cos (β) - ω * L * cos (ωt) * sin (β) You can see that we get to very simple equations, also called transcendental, and that are not very easy to use, even if our circuit is "only" an RL circuit, so relatively simple. If we must do all this work for just RL, we will see that this will really complicate everything. So now we can do an identification so we can write that U (peek) cos(α) is equal to R * I (peek) * cos (β) - ω * L * I (peek) sin (β) And the we have the other one: U (peek) * sin (α) is equal to R * I (peek) * sin (β) + ω * L * I (peek) cos (β). So these are the two equations that we can write by identification and now by summing the squares of the equations that are here. we will be able, to find the following terms by gathering them, but after a lot of calculations.

Notes

Summary

26m 44s

CALCUL COMPLEXE ASSOCIÉ

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Cas d'une inductance et d'un résistance en série

$$\hat{U} = \hat{I}\sqrt{R^2+\omega^2L^2} \qquad \alpha = \beta + \varphi$$

La formule d'Euler :

$$\underline{X} = \hat{X} e^{j\theta} = \hat{X}\left(\underbrace{\cos\theta}_{\text{Partie réelle}} + j\,\underbrace{\sin\theta}_{\text{Partie imaginaire}}\right)$$

Electrotechnique I

We get: U (peek) = I (peek) √(R^2 + ω^2 L^2 So you can see that it is relatively complex to get to such a result. And we will finally get to α = β + φ. But I will skip these calculations. So the complexity of the result, will lead us now to another method. We will now involve complex calculation In our different resolutions of the circuit. So I remind you the formulation, or Euler's formula, to write any number, so I will write here X that is equal to X(peek) e^j θ and this is what Euler's formula tells us, is equal to X (peek) * cos (θ) + j sin (θ). So you can see that in our different sinusoidal connections, whether the current or the voltage, we have made it this form. We have a part of this vector, here it's a vector because we have a real part and an imaginary part. So this is the real part, and this is the imaginary part. And so, in our everyday lives their are sines and cosines, it is either the real part or the imaginary part of this vector or this relation, that I write X underline to show you here that it's a vector or a complex number. The big advantage for such a thing, is that by noting Euler's formula, we have here some derivatives - if we derive this complex number - that takes the same form as the first element.

Notes

Summary

28m 37s

CALCUL COMPLEXE ASSOCIÉ

Cas d'une inductance et d'un résistance en série

$$\hat{U} = \hat{I}\sqrt{R^2+\omega^2L^2} \qquad \alpha = \beta + \varphi$$

La formule d'Euler : $\underline{X} = \hat{X} e^{j\theta} = \hat{X}\,(\underbrace{\cos\theta}_{\text{Partie réelle}} + \underbrace{j\sin\theta}_{\text{Partie imaginaire}})$

Electrotechnique I

And same for the primitive of an integral, we get the same form. It's always an exponential. And this will paradoxically simplify our calculations when we will try to solve an alternative single phased circuit, is to use the complex notations.

Notes

Summary

30m 58s

CALCUL COMPLEXE ASSOCIÉ

Cas d'une inductance et d'un résistance en série

$$u = \hat{U}\cos(\omega t + \alpha) \longrightarrow \underline{u} = \hat{U} e^{j(\omega t + \alpha)}$$

Avec : $u = \mathrm{Re}\{\underline{u}\}$

calcul

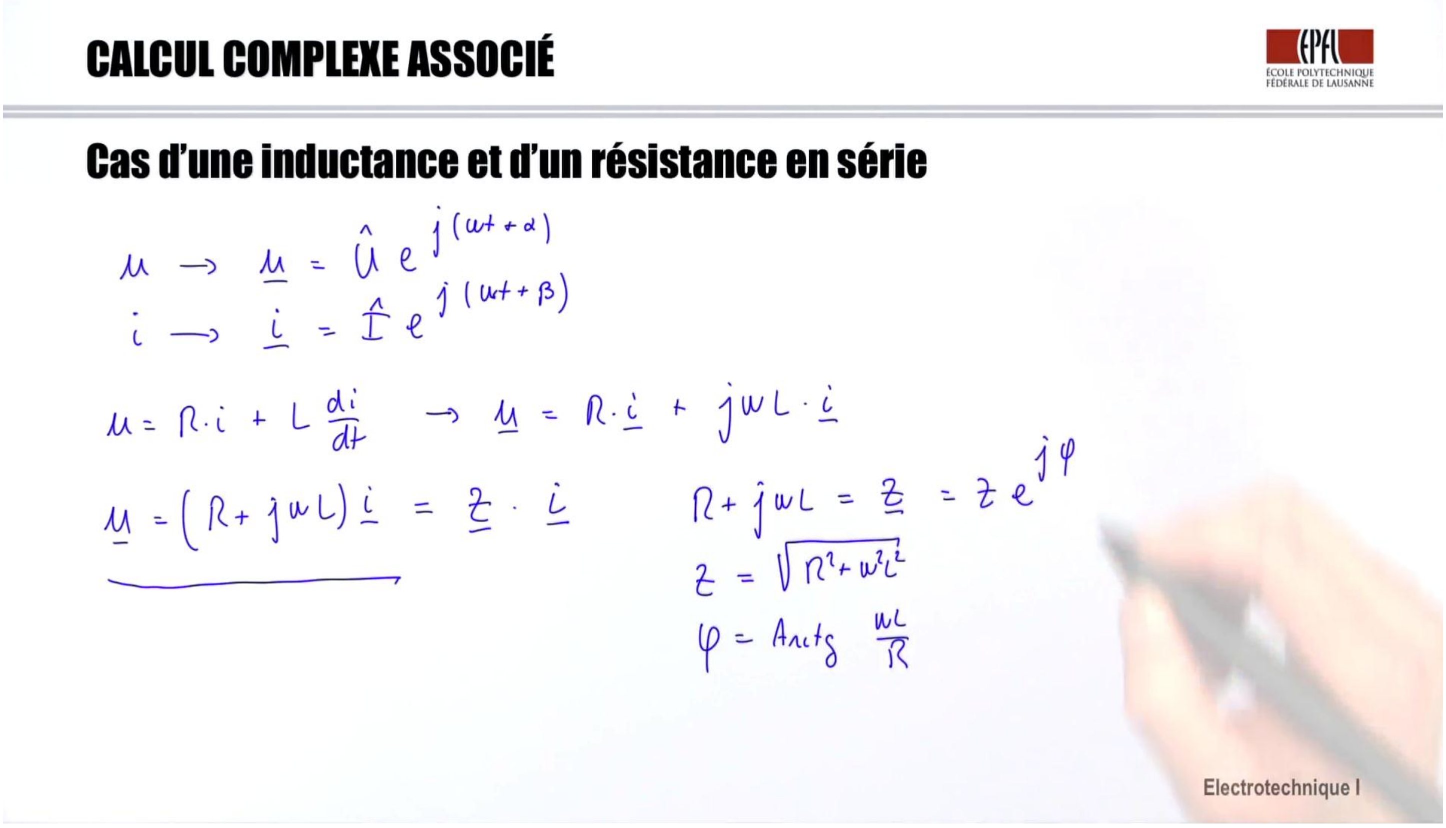
Electrotechnique I

So now, the idea is to do this: our voltage U written U (peek) * cos (ωt + α). This voltage that we will decide to write differently. We will write it U(vector). We will make a complex number with this relation. What we are doing is then purely conceptual, and will allow us to solve the problems of complicated calculations that we had previously. So by writing Euler's formula U = U (peek) e^j (ωt + α). After all, we define our small u, the real world as function of time, it is the real part of this complex number that we just made, because it is only the part of the cosine and not the sine. So right now, as an example, let's try to do exactly the same thing for our example RL. Since once we will finish all our calculations, but with complex numbers, we will be able to come back to the real numbers, by saying that U or I are the real parts of complex numbers.

Notes

Summary

31m 20s

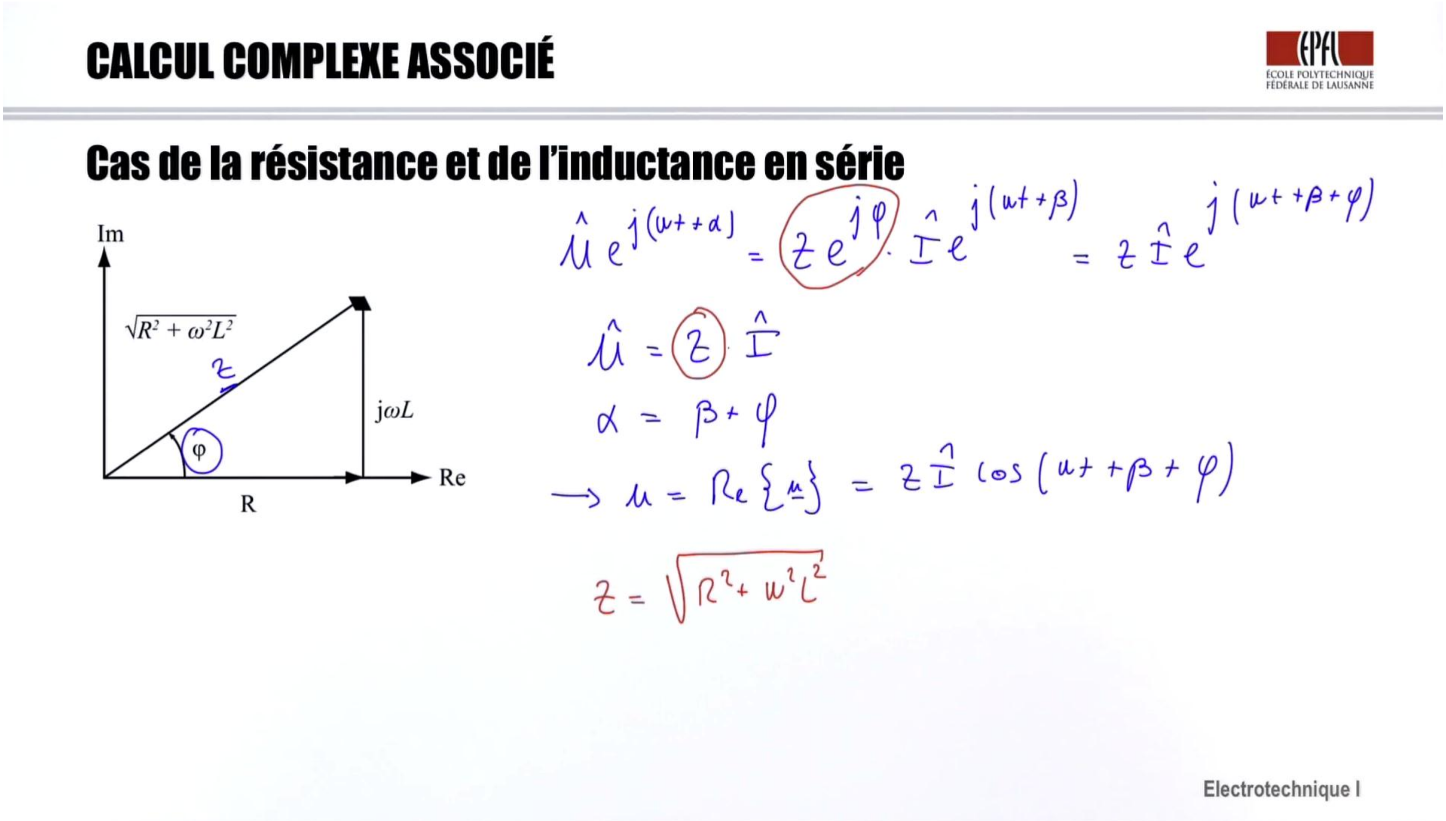


So we will now see U that becomes U (vector) = U (peek) * e^j (ωt + α). The current will also have a complex form : i (peek) * e^j (ωt + β) So we make complex numbers, we work with complex numbers. Our equation U = R * i + L di/dt becomes U (vector) = R * i (vector) + L di/dt + j ω L i (vector) You see, we have here on top the current i that is written with Euler's formula. I (peek) e^j (ωt + β). When we derive, there is the j ω that comes up front, and that simply multiplies i again. Therefore the derivation of a complex number is the same thing as multiplying it by j ω. We will write it later, to sum up. So we can now write this U (vector), by putting everything together, to R + j ω L times the current. And we will call this new complex number R + j ω L, we will call it Z if you wish. So we will have U = Z * i. In this Z, we can say that R + j ω L is therefore this Z that i just made with Euler's formula, it is Z e^j and we will define an angle : φ. So Z, the norm, will be the square root of R^2 + ω^2 L^2 because here we have the form a + bj. To look for the norm it's the square root of a^2 + b^2. And the angle φ is the tangent arc of ωL/R. So we can now write what we have here as a summary, to be able to replace in the equation of the beginning here, and also in the equation that I just wrote. Here, we will replace U and I by their functions.

Notes

Summary

32m 45s

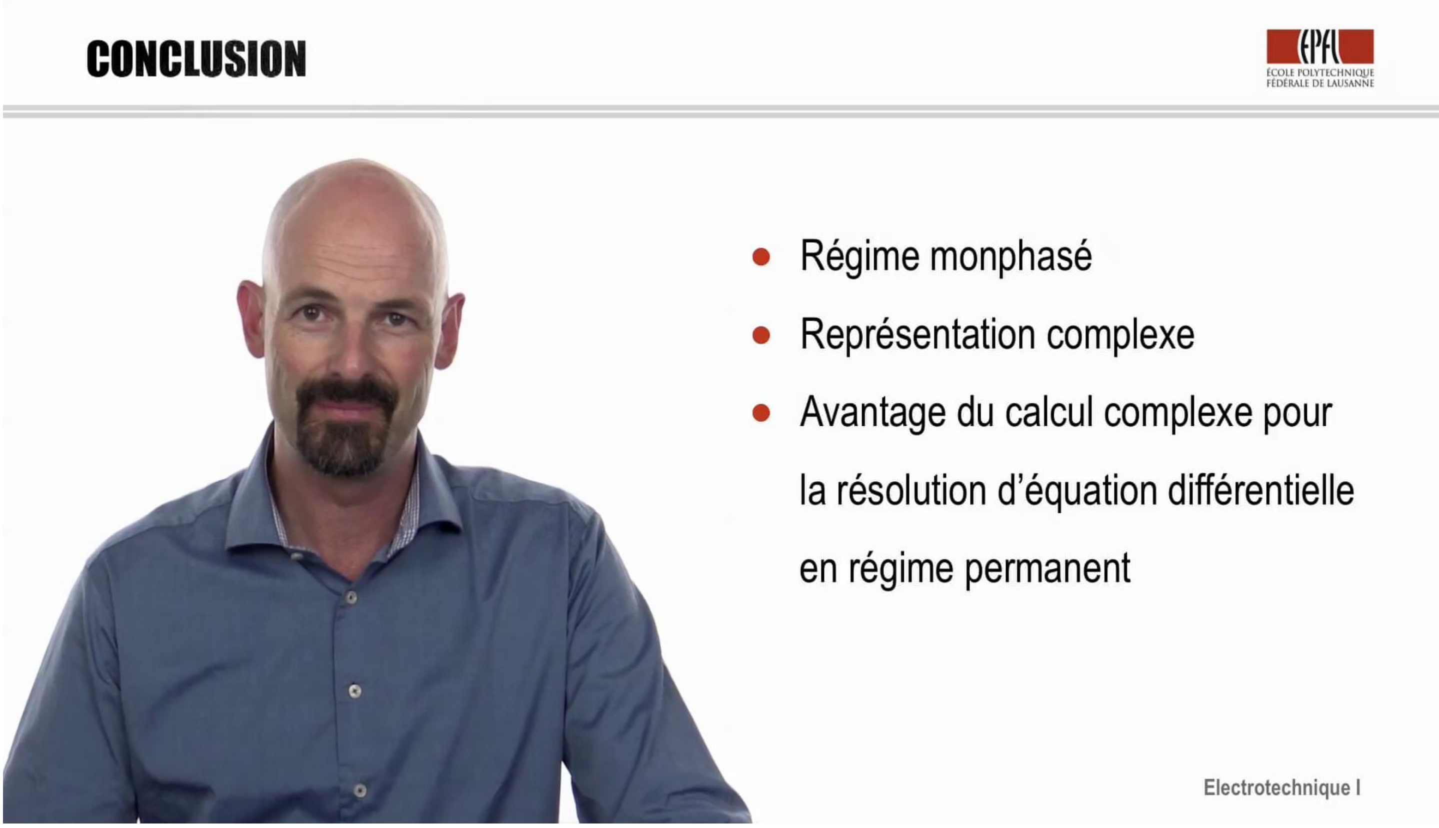


This becomes: U (peek) e^j (ωt + α) on one side, Z e^j (φ) that multiplies the current I (peek) e^j (ωt + β) and this could be simplified into Z I (peek) e^j (ωt + β + φ) since the multiplication of the two complex numbers here will make the sum of the arguments. We get to the following conclusion: U (peek) = Z I (peek) we have found in our I if we had the U as data, α = β + φ. Finally, what interest us, because they are complex numbers, we can come back to real numbers by saying that U is the real part of U (vector), so it's Z I (peek) cos (ωt + β + φ). And you see that we have drastically simplified the calculation of RL, by finding here directly, the answer of what we were looking for. We can also represent, it's interesting, the impedance Z. This Z here, or written also here with its complex form. This Z that i remind you, has for value R^2 + ω^2 L^2, we can represent it in the complex plane, and the real or imaginary components make one vector which length is equal to the modulus of the impedance Z. So we will write here that this length is Z. Here it's the vector Z. The length is therefore the modulus of the impedance and the angle φ, that is here, is the tangent arc of ω L/R, that will be also be the phase shift definition between voltage and current, as we see here. This φ is the phase shift between the voltage and the current.

Notes

Summary

35m 18s



So to conclude, we saw the single phased state, its definition, how to represent these vectors with complex numbers that will help us paradoxicaly to simplify calculations. Therefore the advantages of complex calculations for this differential equation in steady-state resolution which otherwise becomes pretty difficult if we use real numbers. Thank you.

Notes

Summary

37m 46s