ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

PUISSANCE MAXIMALE ET ADAPTATION

LEÇON 12

Électrotechnique I
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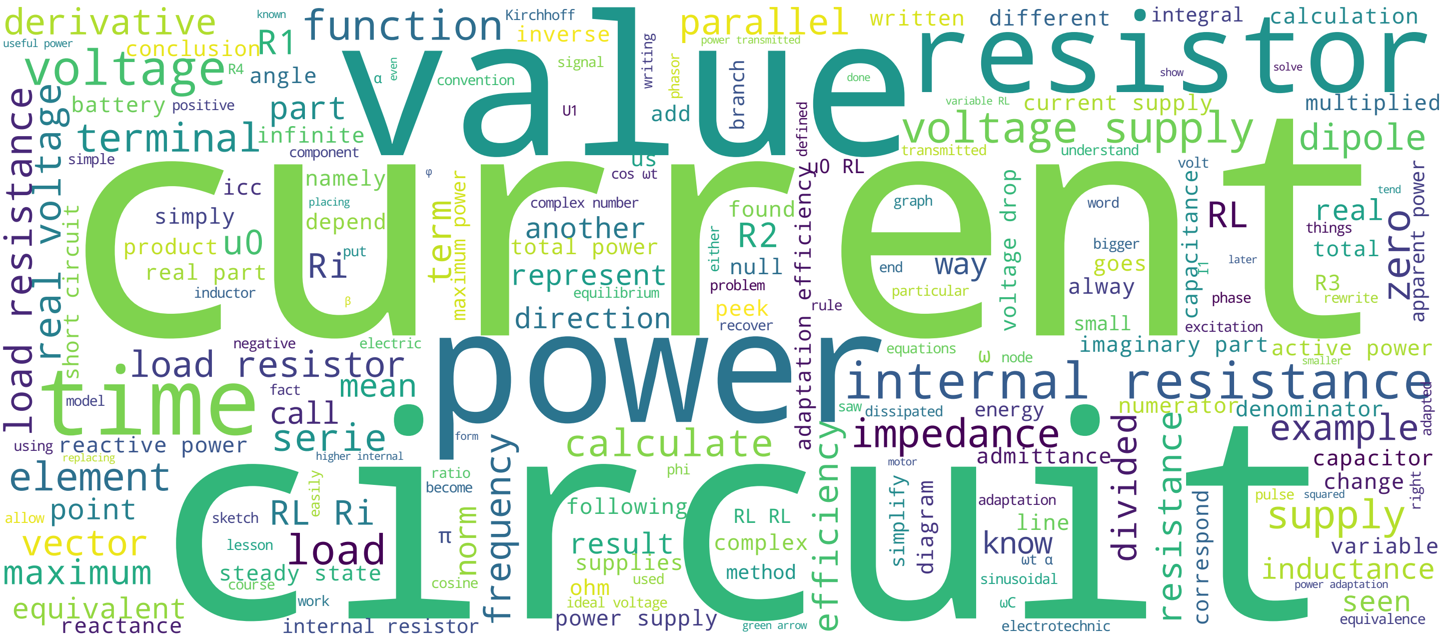
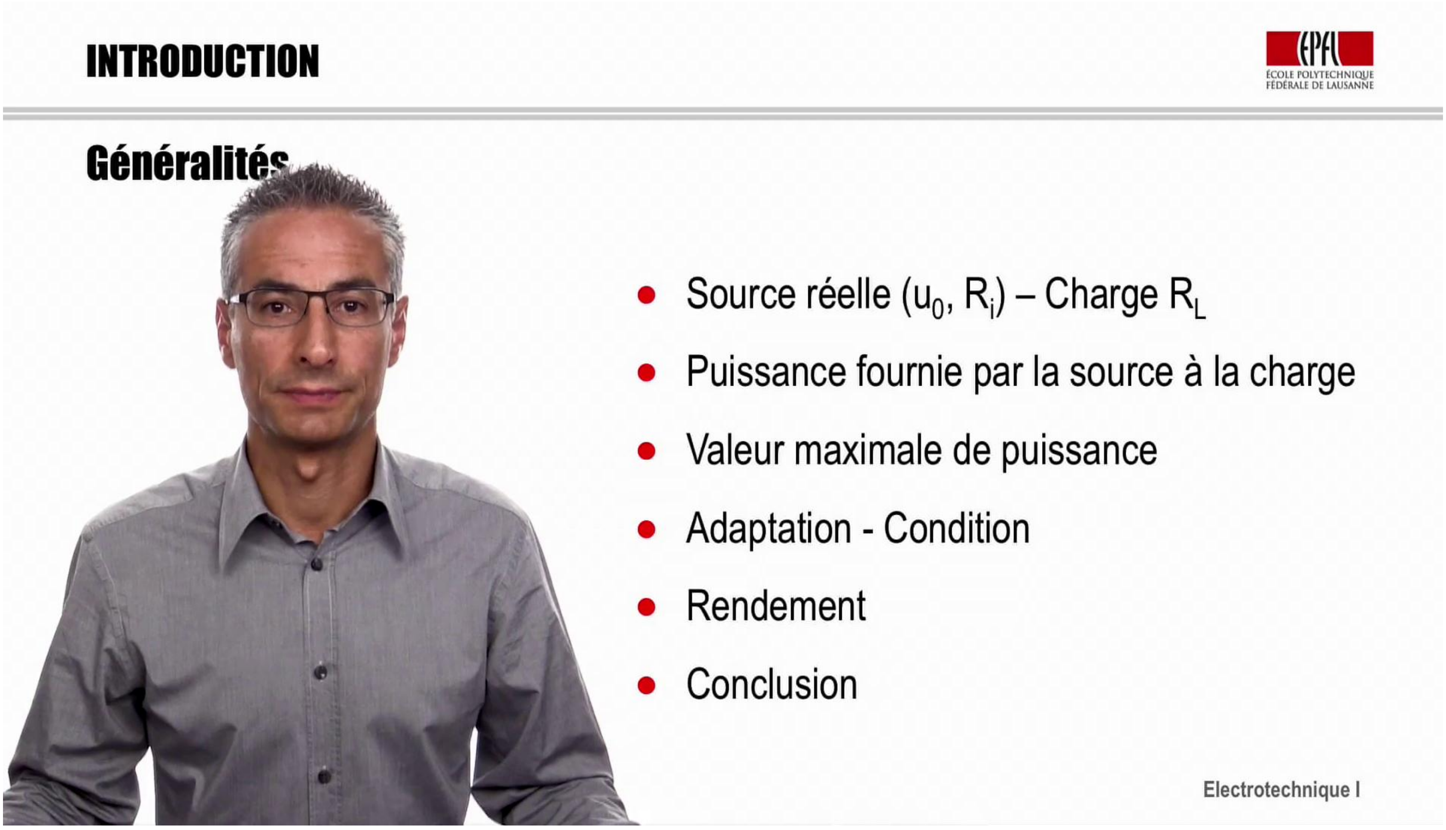
Laboratoire d'Actionneurs Intégrés



Video

EPFL



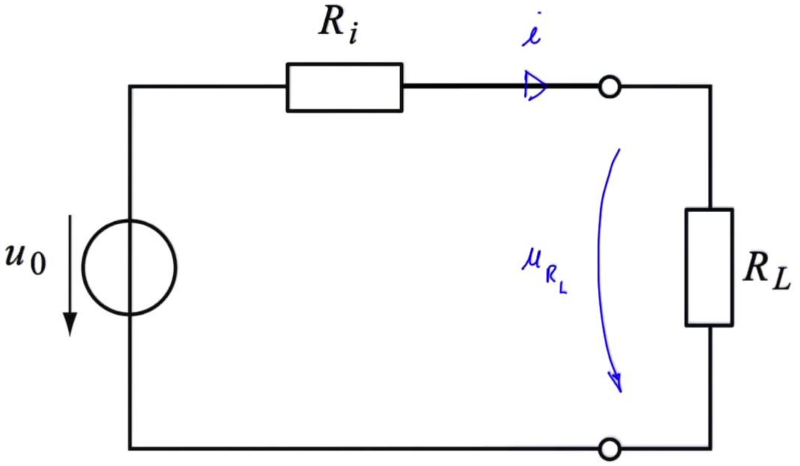
Hello. Today we will discuss about the power transmitted by a real voltage supply to a load. First of all, we will define the real voltage supply and the load. We will then calculate the power provided by the supply to the load. Then, we will calculate the maximal value of the power as function of the value of the load resistance. We will then define the condition of adaptation. And finally, we will calculate the efficiency of the system and we will and with a conclusion.

Notes

Summary

0m 04s

Source de tension réelle – Charge – Puissance



$$i = \frac{u_0}{R_i + R_L}$$

$$P_{R_L} = u_{R_L} \cdot i = R_L \cdot i^2$$

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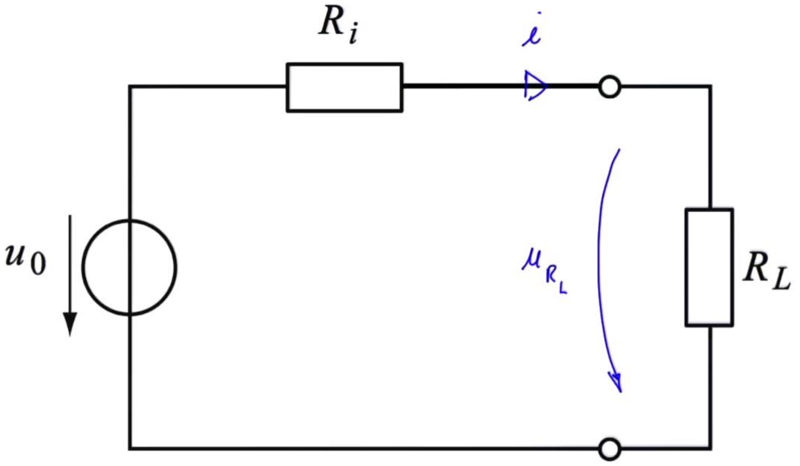
On this sketch, we see on the left part, the ideal voltage supply and the internal resistance that is both, the real voltage supply and on the right side, the load resistance. On this circuit we can define two values: the current that goes through the circuit, there is just one loop, there is just one current, and the voltage at the terminals of the load resistor that we call uR indication L. By Kirchhoff we find very easily, as long as the two resistors are in series, the current that goes through the circuit. We have that i is equal to u0 divided by Ri + RL. In terms of the power, power that is dissipated in the resistor RL, we can write that this power is the product of the voltage drop at the terminals of the resistor multiplied by the current, therefore it is uRL multiplied by the current and if we replace the voltage drop at the terminals of the resistor by the resistance times i, u is equal to Ri, we get that the power is equal to RL multiplied by i squared. By replacing the value of i by the expression found here above, we finally write that the power in the load resistor is equal to (u0^2 * RL)/(RL + Ri)^2 This is the first result.

Notes

Summary

0m 36s

Source de tension réelle – Charge – Puissance



$$i = \frac{u_0}{R_i + R_L}$$

$$P_{R_L} = u_{R_L} \cdot i = R_L \cdot i^2$$

$$P_{R_L} = \frac{u_0^2 R_L}{(R_L + R_i)^2}$$

si $R_L = 0 \rightarrow P_{R_L} = 0$

si $R_L = \infty \rightarrow P_{R_L} = \frac{u_0^2 R_L}{R_L^2} = \frac{u_0^2}{R_L} = 0$

Electrotechnique I

If we analyse this function, we see that in two extreme cases, when the load resistance is zero, then the power in this resistor is also zero. On the other extreme, if the load resistance is infinite, we have a power in this load that is equal to the term RL becoming much bigger than Ri we can express Ri, we have (u0^2 * RL)/RL^2, it is equal to u0^2/RL and it is also equal to zero. Therefore we see that for two extreme values the power is zero. And in between, there is necessarely a maximum, it is this maximum that we will calculate.

Notes

Summary

2m 29s

Puissance maximale

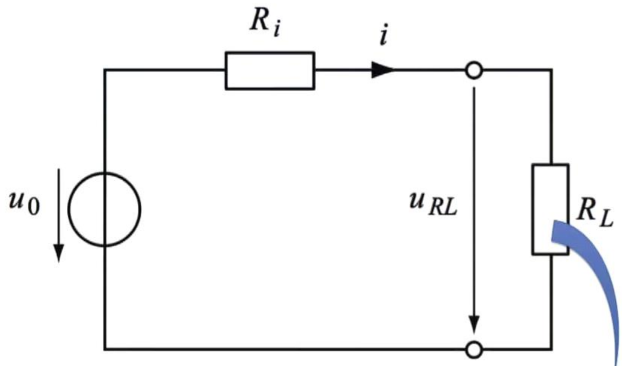
$$\max \rightarrow \frac{dP}{dR_L} = 0 \qquad f = R_L \cdot u_0^2$$

$$g = (R_L + R_i)^2$$

$$= \frac{f'g - g'f}{g^2} \qquad f' = u_0^2$$

$$g' = 2(R_L + R_i)$$

$$\frac{dP}{dR_L} = \frac{(R_L + R_i)^2 - 2(R_L + R_i) R_L}{\quad}$$



$$P_{RL} = \frac{{u_0}^2 R_L}{(R_L + R_i)^2} = \frac{f}{g}$$

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Is repeated on this slide the results obtained earlier. We have represented the same sketch and the power dissipated in the load resistor is given by this equation. To get the maximum of this function with the load resistor RL as a variable, we just have to derivate this function with respect to the variable RL. Therefore the maximum is found when the derivative of the power with respect to RL is equal to zero. To calculate this derivative, we will write that this power function is equal to a numerator that is a function f divided by the denominator of function g f is therefore equal to RL x u0^2, it is the numerator. The denominator g is equal to (RL + Ri)^2. And to calculate this derivative, we know that the derivative is equal to (f'g - g'f)/g^2. We now calculate the derivative of the function f, that we call f', derivative with respect to RL that is equal to u0^2, and the derivative of the denominator g' that is equal to 2 (RL + Ri). We can therefore replace these functions and those derivatives in this equation and is obtained for the derivative function power with respect to the variable RL that it is equal to... the numerator divided by the denominator squared which is equal to (RL + Ri)⇥^4.

Notes

Summary

3m 33s

PUISSANCE MAXIMALE FOURNIE PAR UNE SOURCE RÉELLE

Puissance maximale

$$\max \rightarrow \frac{dP}{dR_L} = 0 \qquad f = R_L \cdot u_0^2 \qquad g = (R_L + R_i)^2$$

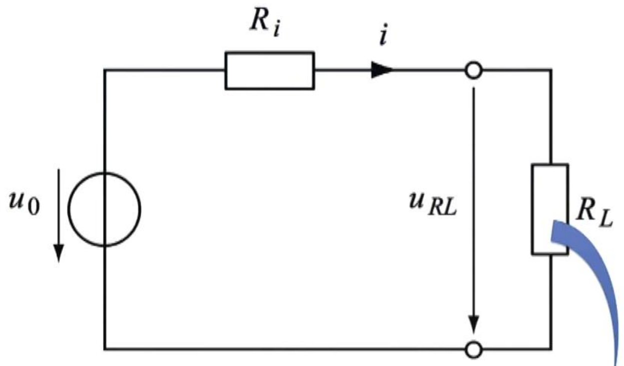
$$= \frac{f'g - g'f}{g^2} \qquad f' = u_0^2 \qquad g' = 2(R_L + R_i)$$

$$\frac{dP}{dR_L} = \frac{(R_L + R_i)^2 - 2(R_L + R_i) R_L}{(R_L + R_i)^4} = 0$$

$$R_L^2 + 2 R_i R_L + R_i^2 - 2 R_L R_i - 2 R_L^2 = 0$$

$$-R_L^2 + R_i^2 = 0$$

$$\underline{R_L = R_i}$$



$$P_{RL} = \frac{u_0{}^2 R_L}{(R_L + R_i)^2} = \frac{f}{g}$$

Electrotechnique I

And we will like to find a zero value of this derivative so we just have to say that the numerator is equal to zero, that we expand and rewrite here... we can simplify it by writing it like -RL^2 + Ri^2 = 0 and we then find the condition for what we call power adaptation which means that when RL is equal to Ri.

Notes

Summary

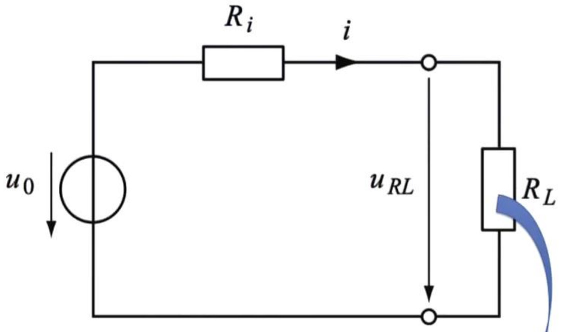
5m 55s

PUISSANCE MAXIMALE FOURNIE PAR UNE SOURCE RÉELLE

Puissance maximale

Condition : $R_L = R_i$

La condition d'adaptation de puissance est donc réalisée lorsque la valeur de la résistance de charge et celle de la résistance interne de la source sont égales



$$P_{RL} = \frac{{u_0}^2 R_L}{(R_L + R_i)^2}$$

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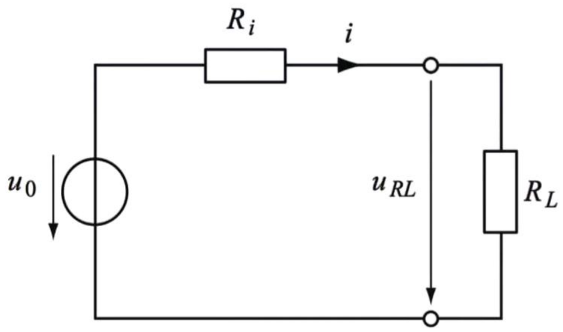
We have now found the power adaptation condition that is fulfilled when the value of the load resistor is equal to the value of the internal resistance. In this case, we say that the load is adapted to the power supply.

Notes

Summary

6m 40s

Rendement



$$\eta = \frac{P_{utile}}{P_{fournie\ source}} = \frac{R_L \cdot i^2}{(R_L + R_i) \cdot i^2}$$

$$\eta = \frac{R_L}{R_L + R_i}$$

À l'adaptation, donc pour $R_L = R_i$, on a :

$$\eta_{adaptation} = \frac{R_L}{2 R_L} = 0.5 \quad \text{pour } P_{max}$$

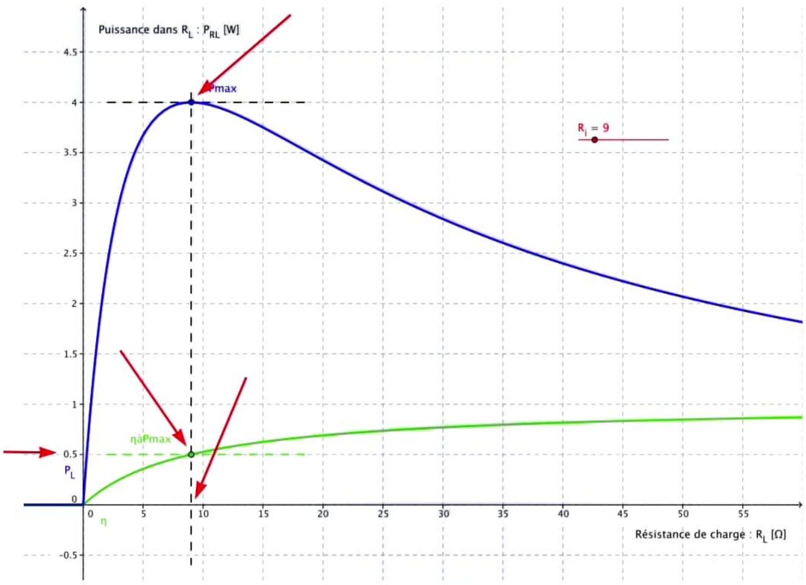
Now lets calculate the efficiency of the system. The efficiency is determinated by the ratio between the useful power, which means the power given to the load resistor, it is here represented by this little green arrow here, divided by the total power given by the power supply. This total power given by the power supply is represented here by the big green arrow. Therefore the useful power in the load resistor is RL x i^2. And the total power given by the power supply is the current squared times the sum of two resistors, which is (RL + Ri) x i^2. We can rewrite this formula in a simpler way. So the efficiency is given by the load resistance divided by the sum of the two resistors, load resistor and internal resistance. Let's look what is happening concerning the adaptation efficiency, so when the load resistance is equal to the internal resistance. We have the adaptation efficiency that is equal to RL/(RL + Ri) but these two values being equal, we simply have that the efficiency is equal to RL/2RL if we replace Ri by RL which are equal, so we have a 50% efficiency for Pmax and therefore 50% for the adaptation.

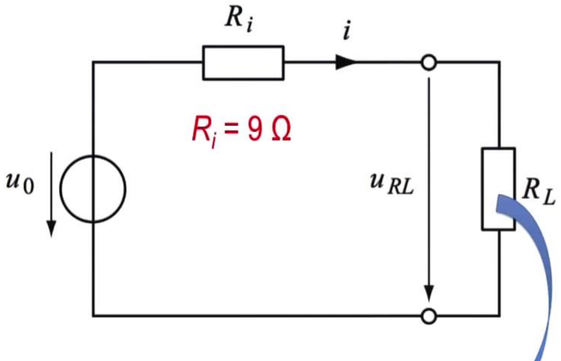
Notes

Summary

7m 04s

Courbe de puissance maximale $P_{RL} = f\,(R_L)$





$$P_{RL} = \frac{{u_0}^2 R_L}{(R_L + R_i)^2}$$

Electrotechnique I

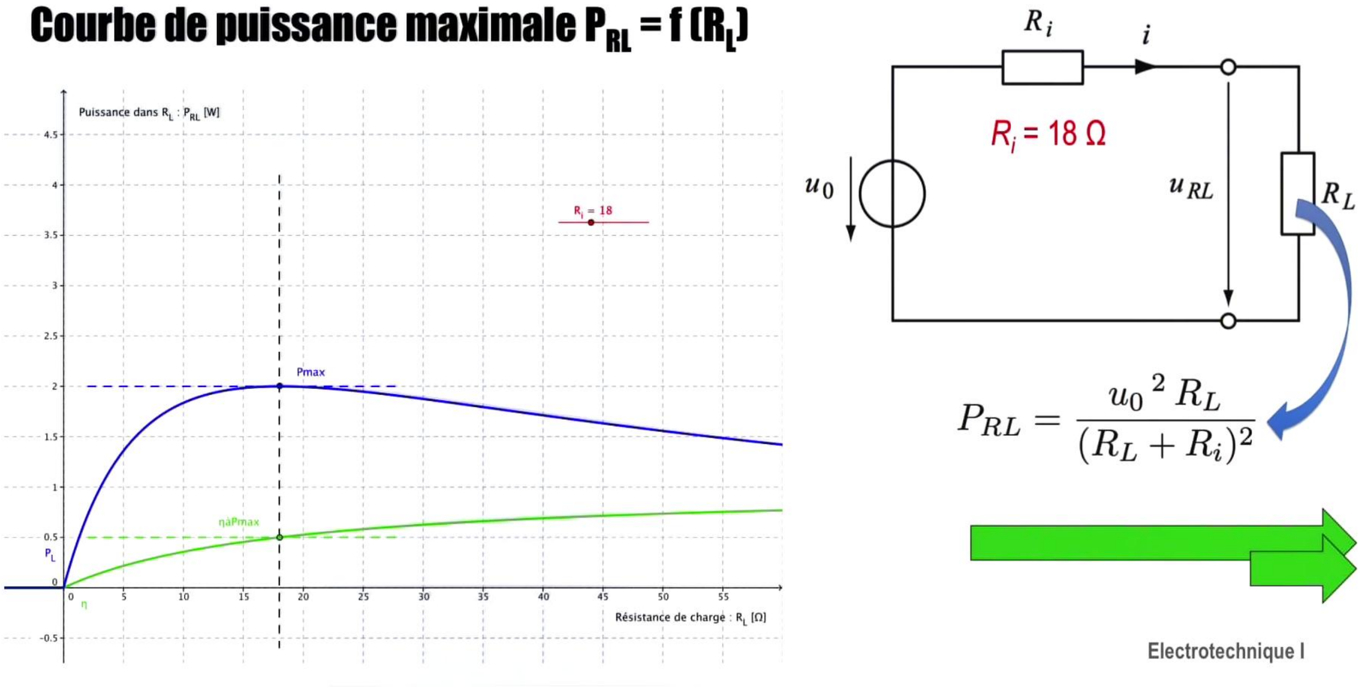
Let's take an example with numerical values. We recover here our real voltage supply with the load resistance, let's take the case of an internal resistance that is equal to 9 ohms. We can calculate the power as function of RL. We have traced it here, it is the blue curve. We see that the maximum power, here, is for a value RL, 9 ohms. We also see that for this load resistor value, the adaptation efficiency is 0.5 therefore 50%. So on this graph, we can see that the maximum power that can be transmitted to the load resistance RL is not only depending on RL but it is also very dependent on the internal resistance. We can see that there is a limit to the power given by the voltage supply. If we take another value with a higher internal resistance, therefore a less performant real voltage supply, we see that the maximal power decreases by this equation, but most of all the maximum is now again at a value that is equal to the value of the load resistance that matches the value of the internal resistance, so 12.5 ohms, and again the efficiency at this value is 50%.

Notes

Summary

9m 10s

Courbe de puissance maximale $P_{RL} = f(R_L)$

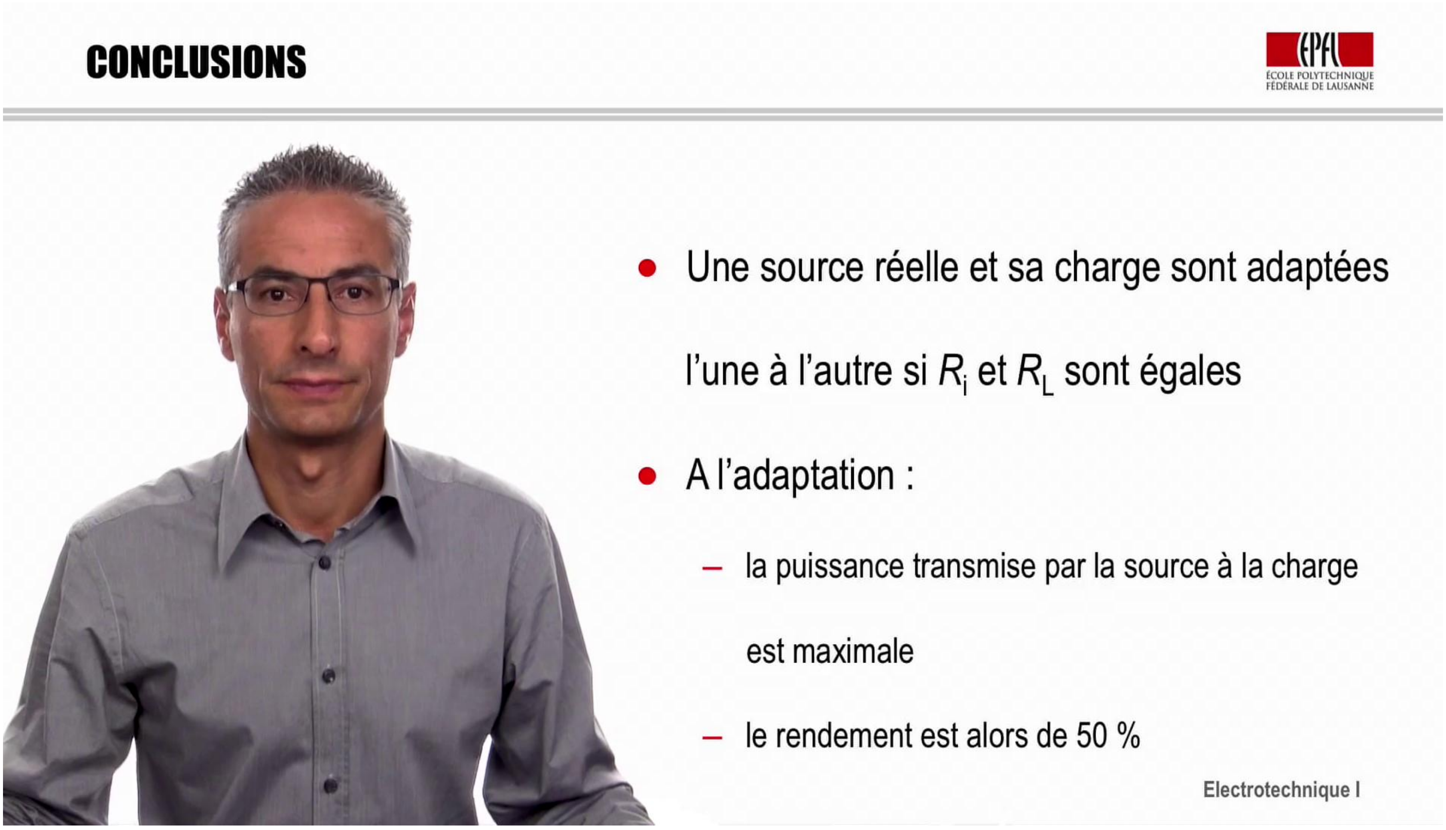


If we take a third internal resistance therefore a real voltage supply of lower quality with a higher internal resistance, the maximum power is again smaller but we recover the same adaptation conditions for the load resistor at the value of the internal resistance and the efficiency is 50% at the value of the internal resistance. If we consider a battery to power a car for example, electric, this battery can be modelled with a first approach by this real voltage supply with a no-load voltage and an internal resistance that represents the chemistry of the battery, the connectivity, the electrodes... worse the battery is lower the power thus bigger the internal resistance is and less power can be transmitted to the load therefore to the electric motor. We mostly see that we can not transmit an infinite power but a power that depends on the internal resistance.

Notes

Summary

10m 54s



In conclusion, we can say two things. First, a real voltage supply and its load are adapted in power to one another if the internal resistance of the real supply voltage is equal to the load resistance. And secondly, The power transmitted by the supply to the load is maximum and secondly, the adaptation efficiency is 50%. We understand it well because there is as much power that is dissipated in the internal resistance then in the load resistance because they are equal Thank you for your attention.

Notes

Summary

12m 05s