TRANSFORMATION Π-T

LEÇON 10

Électrotechnique I
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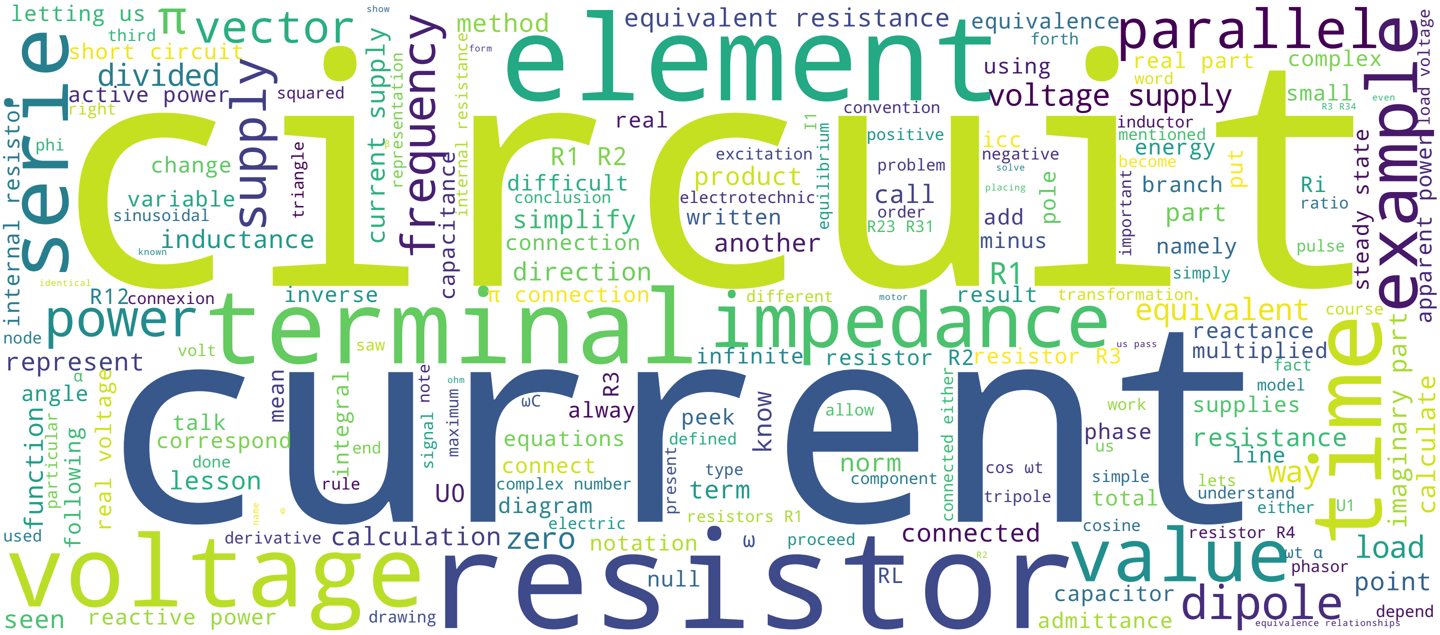
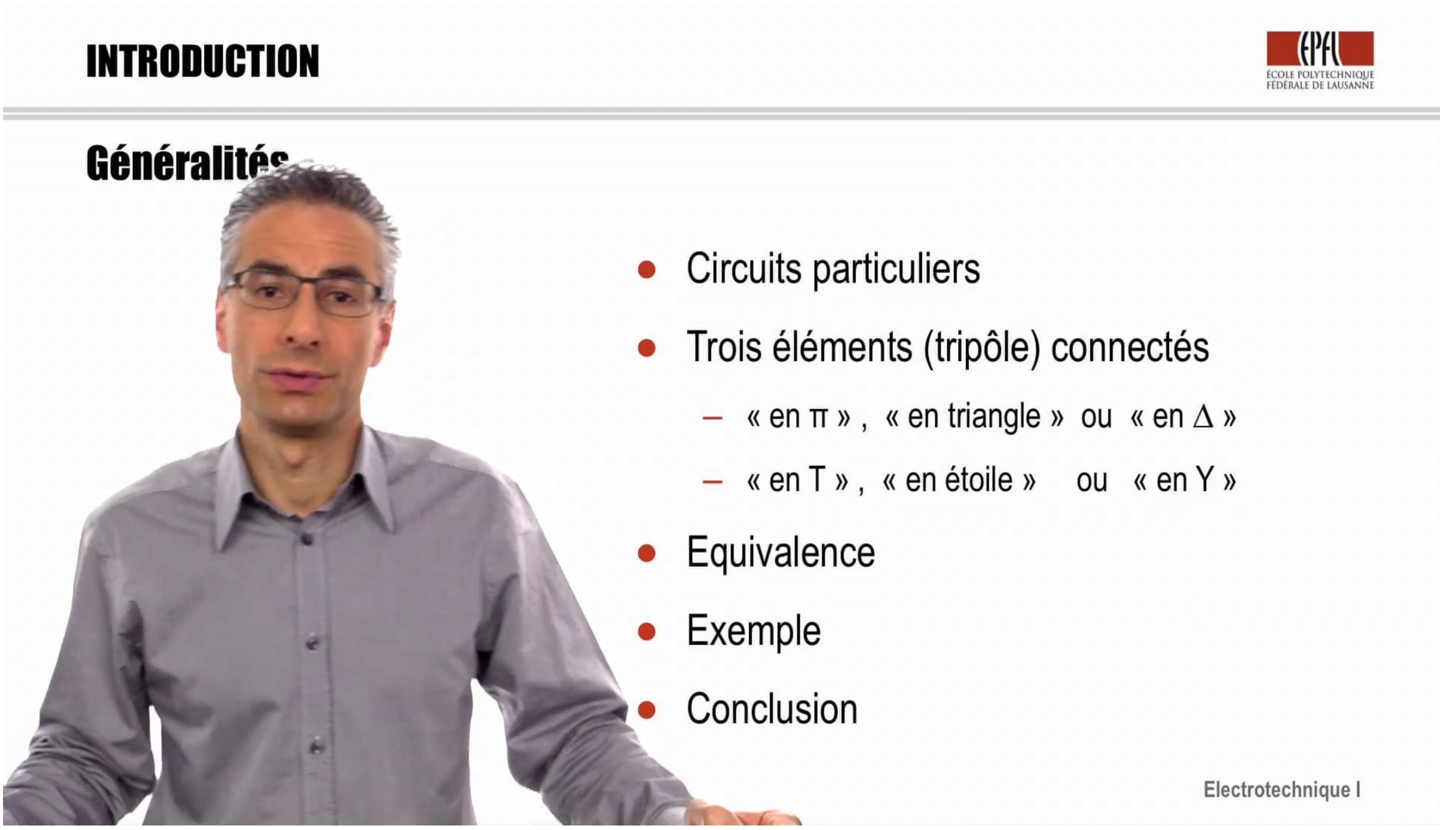


Video



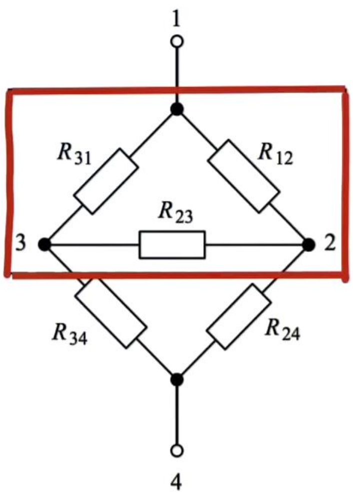
Hello. During this lesson, we will present a method that lets us handle tripoles. We will consider partial circuits composed of three elements. We will first see particular circuits whose element are difficult to combine or to simplify two by two. We will then consider three elements that can be connected either in "pi" or in "T". Then, we will see the equivalence between these two types of circuits in order to easily pass from one to the other and to then simplify the rest of the circuit. We will treat an example and then formulate a conclusion.

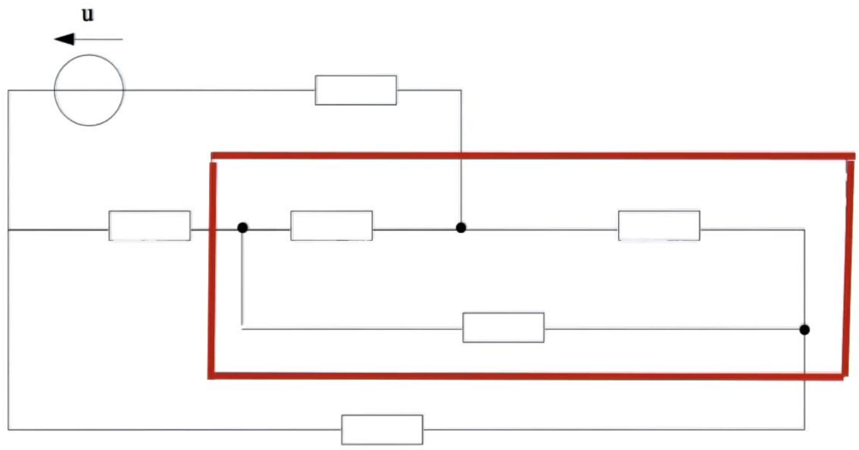
Notes

Summary

0m 04s

Exemples de circuits difficiles à simplifier





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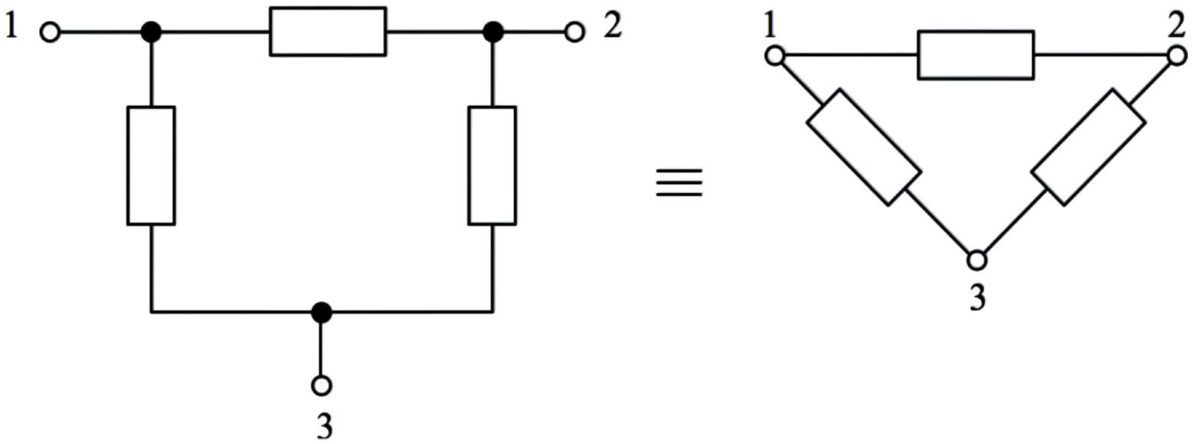
If we consider these two circuit examples, we can see that they are difficult to simplify. Why? Because if we consider this first circuit, we see that the elements are not in series for R31 and R32. Why? Because there is an additional current coming here, and so it is more difficult to simplify the circuit. Ideally, for this circuit, that presents about the same topology. We will see a method letting us transform these circuits for simplification later on. We can evidently resolve these circuits by applying Kirchoff's law at the knots and loops, but we will see a much simpler method.

Notes

Summary

0m 52s

Tripôles « en π » (ou « en triangle », « en Δ » ou « *Delta connection* »)



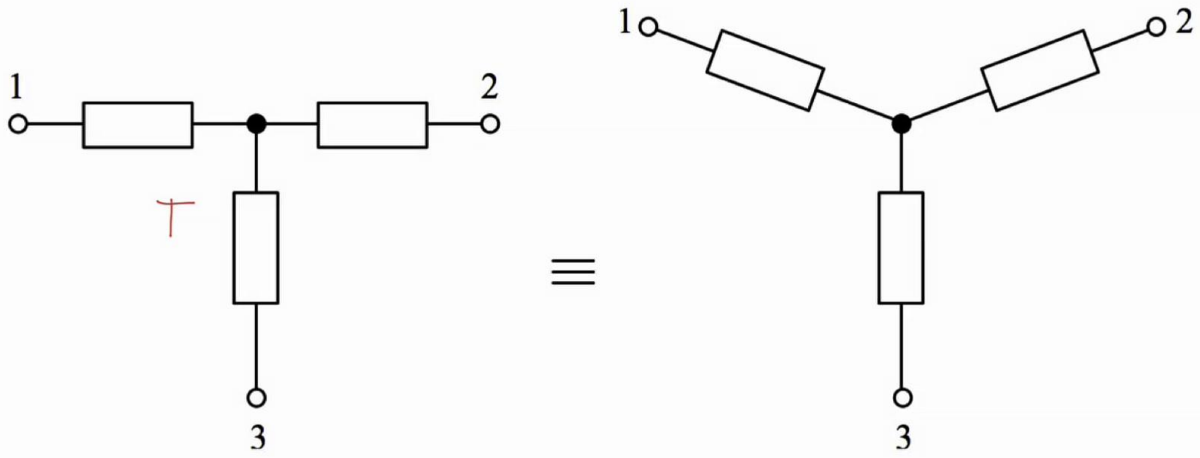
Three elements can be connected between them in two ways. First way, is a "π" connection such as represented in the first figure. We understand this notation because the circuit looks like a capital π. If we represent this same connection, drawn a little differently, we see that it resembles a triangle, which is why we also talk of "delta" connection. We also use this notation, the greek delta Δ and in French literature, we rather talk of a connexion en triangle.

Notes

Summary

1m 40s

Tripôles « en T » (ou « en étoile », « en Y » ou « *Star connection* »)



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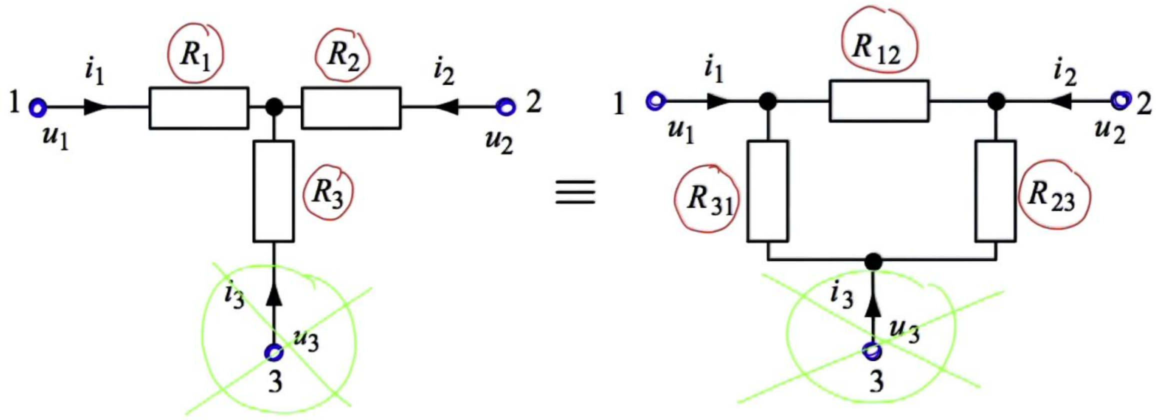
The second method to connect three elements between them is the "T" connection. On this drawing, we easily understand why we give it such a name, because the connection resembles the letter T. We also call it a "Y" connection or a "star" connection. On this drawing, we see that the same equivalent drawing resembles a Y shape. In French, we talk about "connexion en étoile".

Notes

Summary

2m 19s

Équivalence de tripôles « en T » et « en π »



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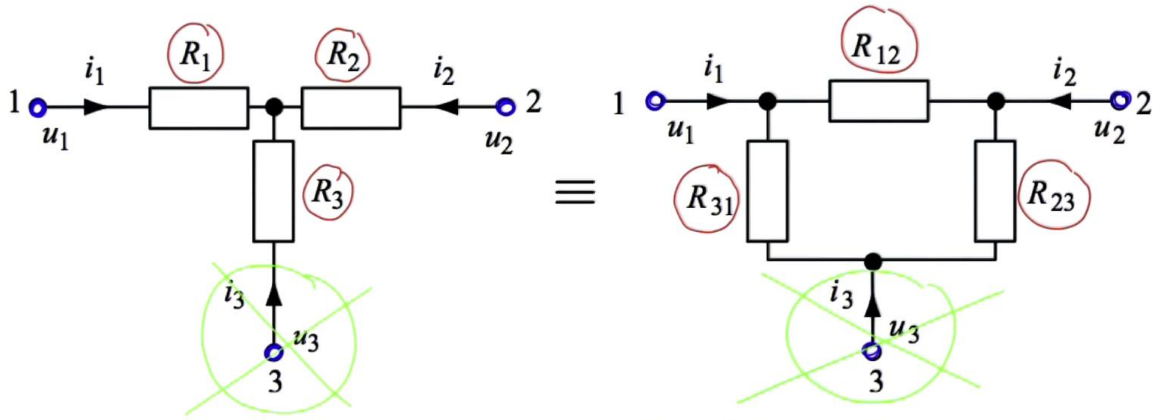
On this drawing, we represent the two connections: either in "T", or in "π". We see that they are tripoles with a first pole here, a second pole and a third pole. On the second representation, we find these same poles. Between these tripoles, we connect the three elements, three resistors in this example as it happens. In T, we have a resistor R1 connected to the terminal 1; a resistor R2 connected to the terminal 2; a resistor R3 connected to the terminal 3. In delta or in π, we once again have three elements: R1-2 which is connected at the terminals 1 and 2, and so forth, the resistor 2-3 is connected at the terminals 2 and 3 and the resistor 31 is connected at the terminals 3 and 1. What we would like to do here, is find an equivalence between the resistors R1, R2, R3 and the resistors R12, R23, R31 so that both circuits are identical. To do so, we will proceed in the following manner: lets first consider that this terminal 3 doesn't exist, and we will write the equation of the resistance seen at the terminals 1 and 2 so that they are equivalent. We can write that, here, the equivalent resistance, they are in series since we deleted the terminal number 3, R1 + R2 must be equal to this equivalent resistance in the π connection at the terminals 1 and 2.

Notes

Summary

2m 48s

Équivalence de tripôles « en T » et « en π »



$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

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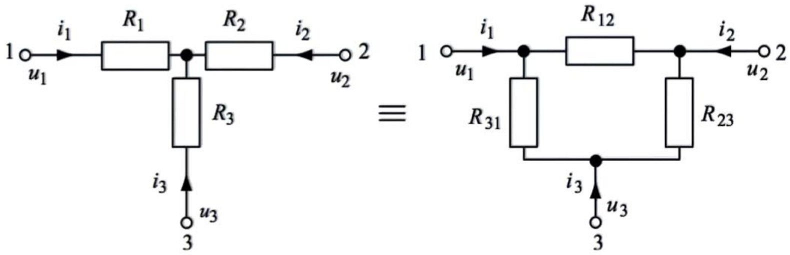
And we see that these two resistors are in series, since this terminal is cancelled out, this equivalent resistance is in parallel with the resistor R12. So we can write that the equivalent resistance is: the product of this resistor over the sum of these two divided by the sum of this resistor over the sum of these ones. We then write that R12 multiplied by R23 + R31 divided by the sum of these three resistors needs to be equal to R1 + R2.

Notes

Summary

4m 35s



$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

$$R_2 + R_3 = \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

$$\boxed{\begin{aligned} R_1 &= \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \\ R_2 &= \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} \\ R_3 &= \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} \end{aligned}}$$

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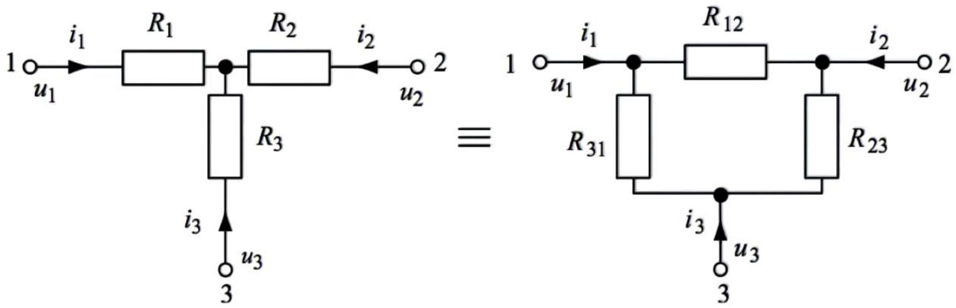
We repeat, here, the result obtained for the first equation and if we proceed in an identical way for the boundary pairs 2 and 3, and 3 and 1, we get two extra equations. We are not making the development, but the method is the same. Two supplementary equations, that give a system of three independent equations with three unknowns, from which we easily extract the equivalence relationships that let us replace a circuit in π with a circuit in T. We just need to do the sum of these three equations. So, lets take 0.5 times the first equation, minus equation 2, plus equation 3. Having done all calculations, we fall on this system of three equations letting us pass from one circuit to the other. The resistor linked to the terminal 1 of the circuit in T is the product of two resistors linked to the terminal 1 of the circuit in π, so R12 and R31 here, divided by the sum of the three resistors. And so forth for the resistors R2 and R3.

Notes

Summary

5m 15s

Équivalence de tripôles « en T » et « en π » - Opération inverse



$$\boxed{\begin{aligned} R_{12} &= R_1 + R_2 + R_1\,R_2/R_3 \\ R_{23} &= R_2 + R_3 + R_2\,R_3/R_1 \\ R_{31} &= R_3 + R_1 + R_3\,R_1/R_2 \end{aligned}}$$

$$= \frac{R_1 R_3 + R_2 R_3 + R_1 \cdot R_2}{R_3}$$

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By doing a similar calculation at the terminals pair when the third one is disconnected, we get the equivalence relationships that let us replace the elements of a circuit in T by a circuit in π. These equations written in a slightly different manner give the ratio of T double vector product of two resistors taken two by two R1 x R3 + R2 x R3 + R1 x R2 divided by the resistor R3.

Notes

Summary

6m 35s

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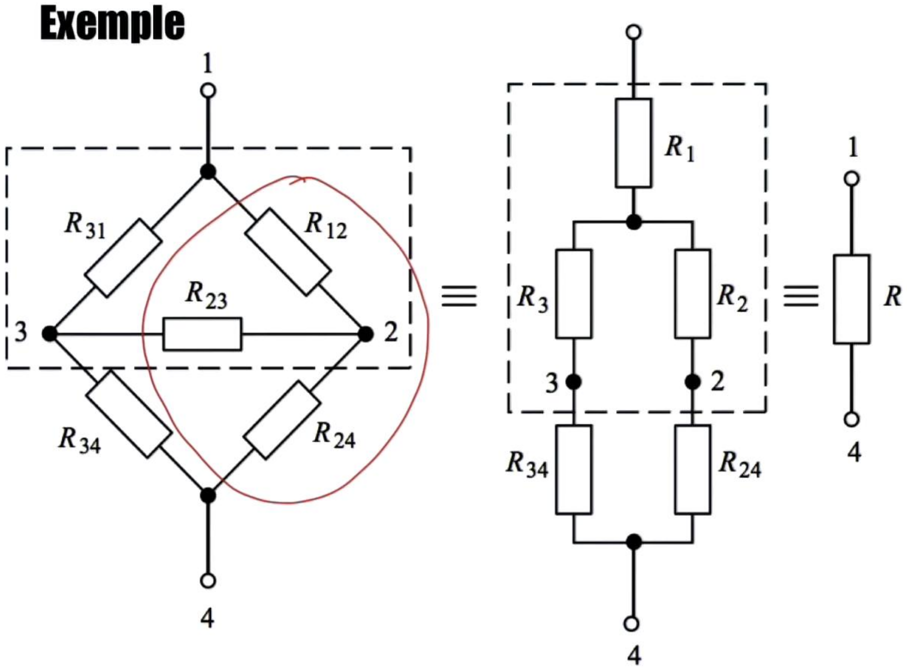
Exemple



$$R = R_1 + \frac{(R_2 + R_{24})(R_3 + R_{34})}{R_2 + R_{24} + R_3 + R_{34}}$$

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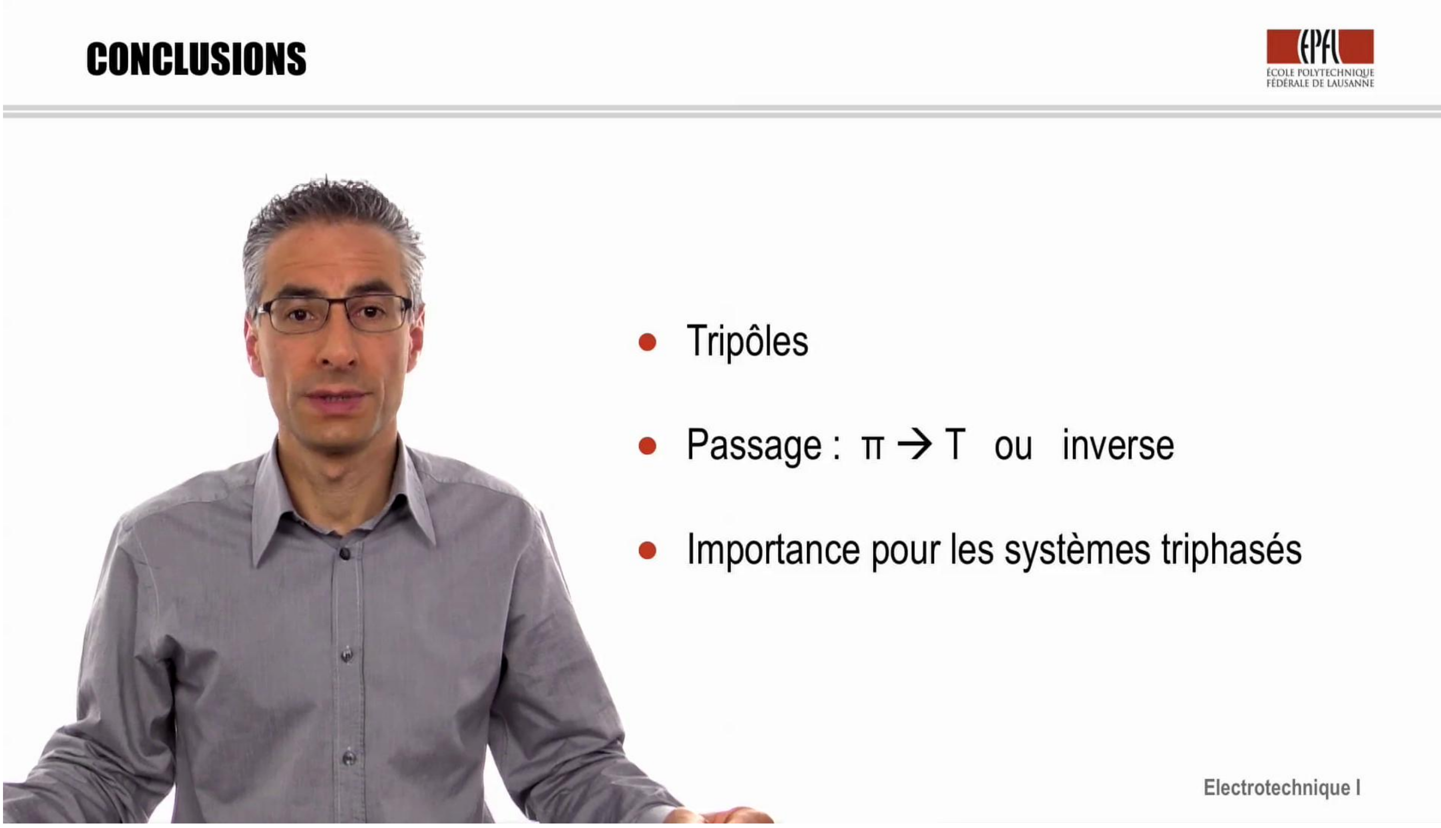
Let take the example that we mentioned at the very beginning of the lesson. If we consider this circuit, with this tripole composed of three resistors, well we can favourably replace it by this equivalent circuit from which we can calculate the resistors R1, R2 and R3, by using the equations that we developed, namely the transformation from T to π. It then becomes easy to calculate the equivalent resistors since the resistor R3 and the resistor R4 are in series. The resistor R2 and the resistor R4 too, that are in parallel, and that we can put in series with R1. We obtain the result for the equivalent resistance of the whole circuit that is equal to R = R1 + these two equivalent resistances in parallel, so their product over their sum. The product of these two resistors in series gives R2 + R24 multiplied by R3 + R34 divided by the sum of these two equivalent resistances, namely R2 + R4 + R3 + R34. This is the final result. Lets also note that we could have seen the problem differently by considering this time these three elements, here, which are three resistors connected in T and that we could have transformed in π and also easily simplify the circuit. We won't do the calculation here.

Notes

Summary

7m 17s



There, we have considered the case of three elements that are connected either in π, either in T at its three boundaries. We have the equivalence letting us pass from a π connection to a T connection. All this in the aim of simplifying this circuit and continuing to reduce it. We will see that this method is very important for three-phase systems that we will cover in the next semester.

Notes

Summary

9m 28s