

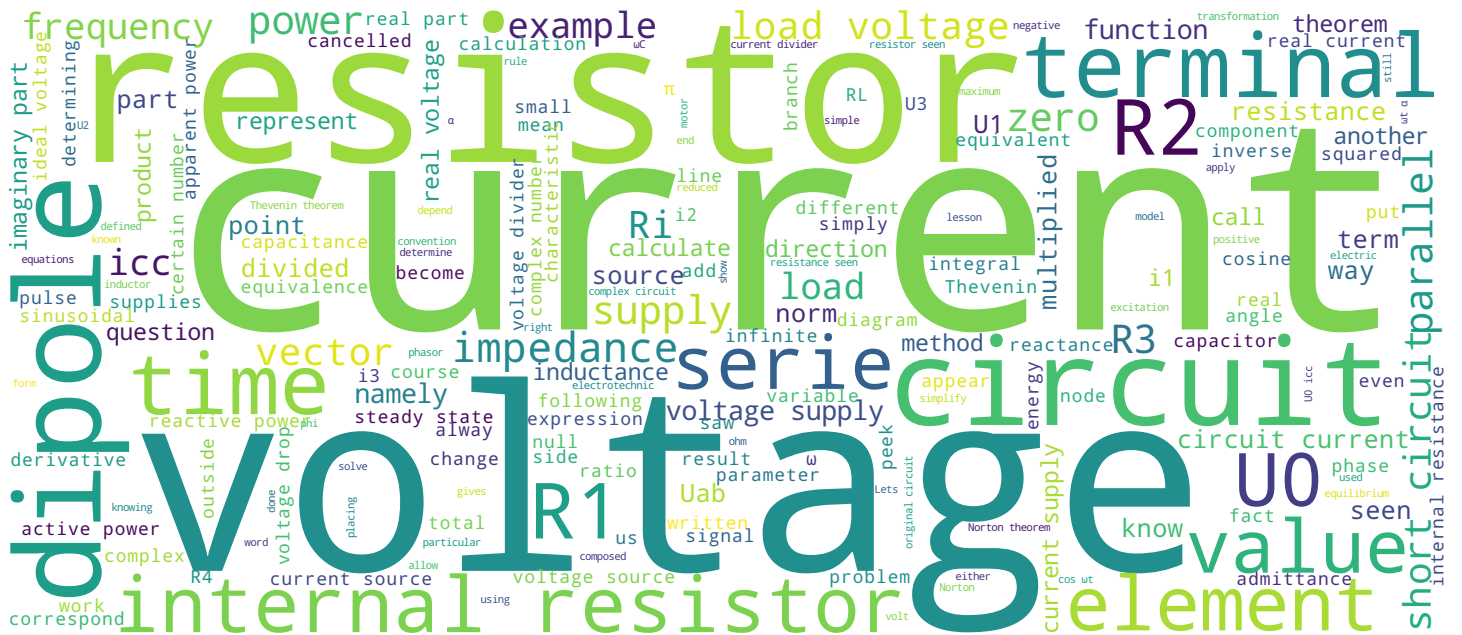
THÉORÈMES DE THÉVENIN ET DE NORTON

CIRCUITS ÉQUIVALENTS

LEÇON 8

Électrotechnique I

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Généralités



- Bipôles équivalents
- Réduction de circuits
 - Série – Parallèle 
 - Diviseur de tension ou diviseur de courant
 - Autres
- Théorèmes de Thévenin et de Norton
 - Circuits très complexes
 - Également pour tout circuit

Electrotechnique I

Hello, welcome to this new lesson of the Electrotechnique 1 course. In the framework of electrical circuits, it is rarely needed to know the evolution of the currents and voltages, in every location of the circuit. It is then possible to find a basic equivalent circuit which is simpler but which accurately translates the behaviour of the original circuit. We call equivalent bipoles or dipoles two dipoles that have the same current at all times when they are under the same voltage, or vice versa. A dipole can be reduced to dipole by way of different reductions, provided that they keep the same characteristics. We have seen different possible reductions, such as the Series or Parallel reductions, that apply to resistors, current sources, and voltage sources. We have seen the voltage divider, or the current divider, we will see even other transformations, like the Y- Δ transformation or other methods. In most cases, these reductions and transformations are easily applicable, but of, in particular, complex circuits, we will mostly apply the principle of Thevenin or the theorem of Norton. These two theorems and also valid for more simple circuits. We will now develop these two theorems.

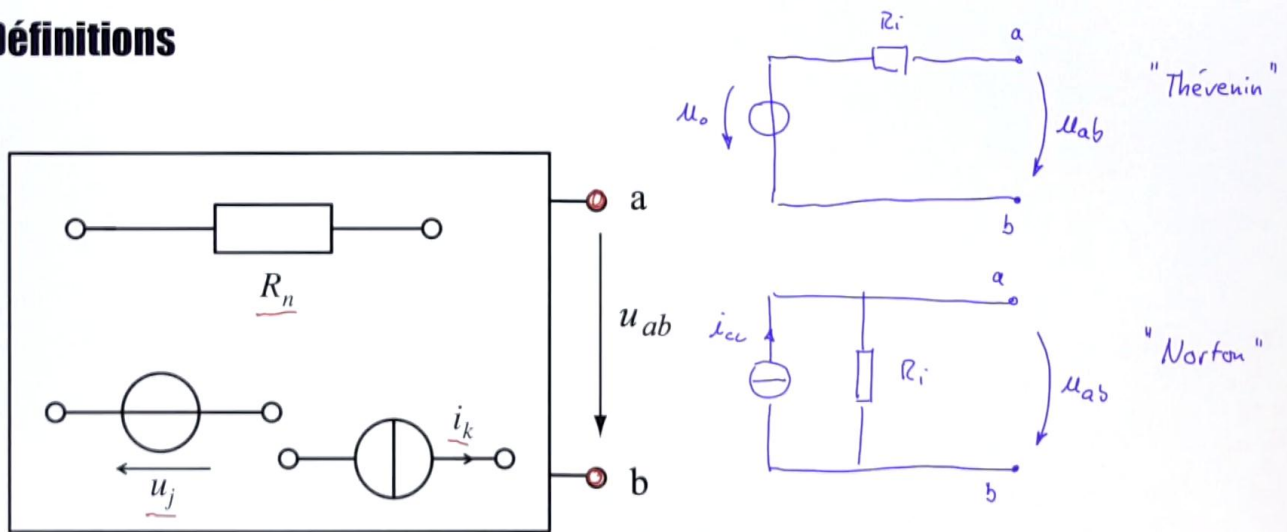
Notes

Summary



0m 04s

Définitions



Electrotechnique I

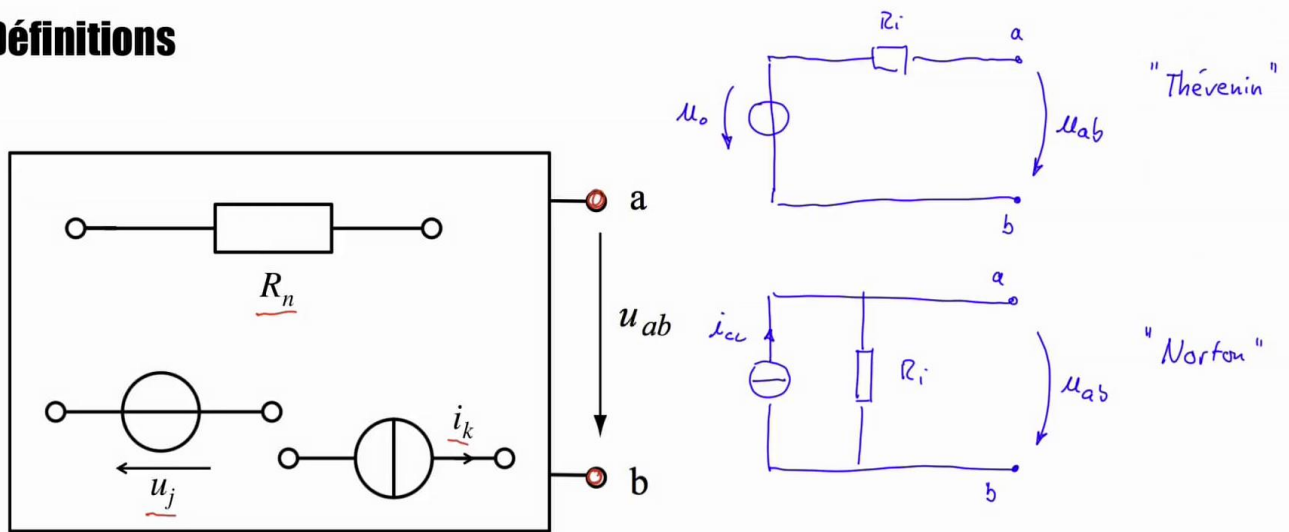
Lets consider a dipole that is characterised by a certain number of resistors, a certain number of sources, whether these are voltage sources or current sources, for which we have two terminals, two poles. The respective expression of the Thevenin or Norton theorems say that it is always possible to reduce such a dipole, containing any combination of independent sources and resistor to a dipole that finally contains only one ideal voltage source, U_0 , with, in series, an internal resistor R_i . So, this dipole - we put the terminals a and b - presents a voltage U_{ab} , and the extreme simplification of this original circuit, is the theorem of Thevenin. The theorem of Norton says that this dipole can be reduced to a current source, equal to i_{cc} , in parallel with an internal resistor, the same internal resistor, put in parallel. Once again, the terminals a and b and the voltage at the side of these terminals, is the expression of the Norton theorem. The equivalence of these dipoles is fulfilled if they have the same no-load voltage, and the same short-circuit current, and so fatally, the same internal resistor, is the equivalence between the real voltage source and the real current source, that we saw in the previous lesson.

Notes

Summary



Définitions



Electrotechnique I

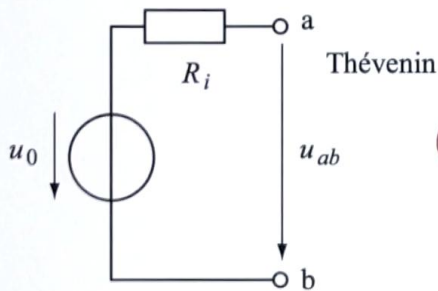
It is now a question of determining these three parameters, the no-load voltage, the short-circuit current and the internal resistor, that, in fact, represent this source, this more complex dipole.

Notes

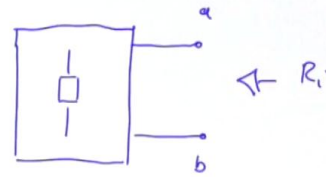
Summary



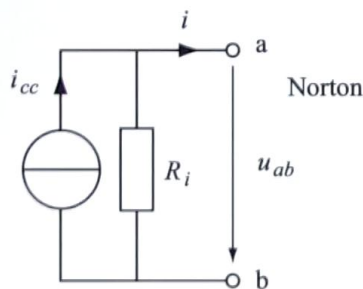
Tension à vide – Courant de court-circuit – Résistance interne



$$u_0 = u_{ab} \Big|_{i=0}$$



$$R_i = R_{ab} \Big|_{\substack{u_j=0 \\ i_j=0}} = \frac{u_0}{i_{cc}}$$



$$i_{cc} = i \Big|_{u_{ab}=0}$$

Electrotechnique I

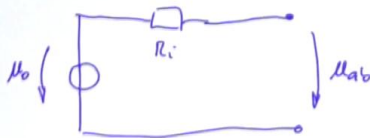
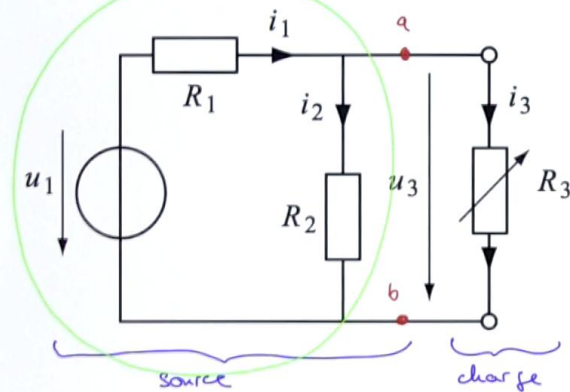
So, let's take these diagrams back, it is now a question of determining the no-load voltage U_0 , which is, in fact, the voltage U_{ab} , from the original voltage, when the current is equal to zero. It is now still a question of representing or determining the current i_{cc} , which is the current between the terminals a and b , when the dipole is short-circuited, namely when the voltage U_{ab} is equal to zero. The internal resistor is the resistance seen from outside the dipole, when all the sources are cancelled. I represent the dipole here, the terminals a and b , the internal resistor seen from outside the dipole, R_i , it is the resistance between the terminals ab , when all the sources are cancelled out, namely all the u_j are equal to zero, and all the i_j are also equal to zero, and we have seen that this internal resistor is the ratio of U_0 over i_{cc} . By knowing these three parameters between U_0 , i_{cc} , R_i , we completely determine the source, knowing that R_i is equal to the ratio of u_0 over i_{cc} .

Notes

Summary



Exemple



• tension "à vide"

$$i_1 = i_2 \quad i_2 = \frac{u_1}{R_1 + R_2}$$

Electrotechnique I

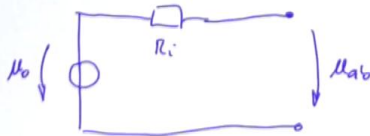
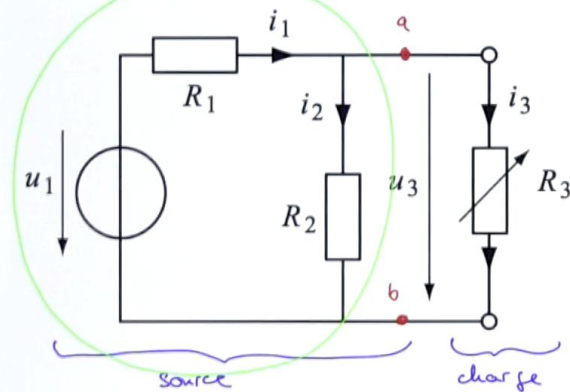
Lets now pass to a concrete example. We consider this circuit, which is composed of a source and a load, we define the dipole ab between these two terminals a and b . This is then the source, this is the charge, and this dipole here can be simplified into the following dipole, by the Thevenin theorem, a no-load voltage in series with an internal resistor. It now a question of calculating this no-load voltage U_{ab} , and to calculate the short-circuit current. For the no-load voltage... the no-load voltage is the dipole voltage when the resistor R_3 doesn't exist, the circuit is open. We can then write the following equation, firstly, the current i_1 is equal to the current i_2 , so there is only one loop in the circuit, and so i_1 or i_2 is equal to the source U_1 , divided by the sum of these two resistors R_1 and R_2 , since they are in series. Finally, the voltage that appears between the points a and b in no-load, also correspond to the voltage drop at the terminals of the resistor, namely that U_3 is equal to R_2 times i_2 and this is equal to R_2 divided by R_1 plus R_2 , multiplied by U_1 , and this is the no-load voltage.

Notes

Summary



Exemple



• tension "à vide"

$$i_1 = i_2 \quad i_2 = \frac{u_1}{R_1 + R_2} \quad u_3 = R_2 \cdot i_2$$

$$= \frac{R_2}{R_1 + R_2} \cdot u_1 = u_0$$

• courant "de court-circuit"

$$i_{cc} = \frac{u_1}{R_1}$$

$$R_i = \frac{u_0}{i_{cc}} = \frac{R_2}{R_1 + R_2} \cdot \frac{u_1}{u_1} \cdot \frac{R_1}{1} = \frac{R_1 R_2}{R_1 + R_2}$$

Electrotechnique I

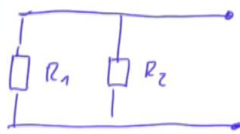
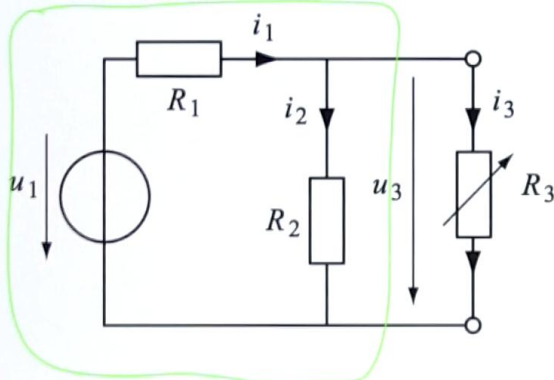
Lets calculate now the short-circuit current. It is the current passing through the terminals a and b , when we short-circuit them, namely when we replace the resistor R_3 by a null resistor. We simply find that i_{cc} is equal to U_1 over R_1 , since the short-circuit cancels out, here, the effect of the resistor R_2 . We have then determined the no-load voltage and the short-circuit current, we can now determine the internal resistor of the circuit, the dipole, the internal resistor is the ratio of U_0 over i_{cc} , and it is equal to, U_0 is R_2 over the sum of these two resistors, multiplied by R_1 , and multiplied by 1 over i_{cc} , namely R_1 over U_1 , which finally gives for the internal resistor, the product of R_1 times R_2 over the sum of R_1 plus R_2 .

Notes

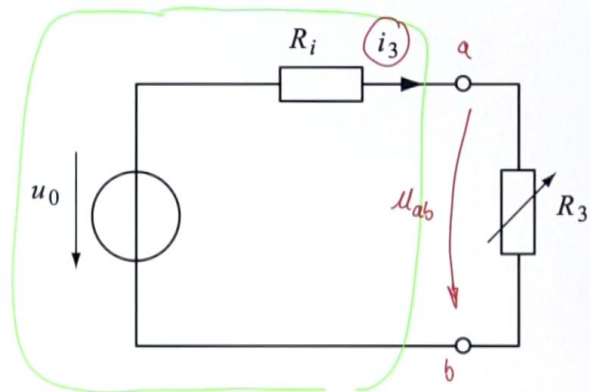
Summary



Exemple



$$R_i = \frac{R_1 R_2}{R_1 + R_2}$$



$$i_3 = \frac{u_0}{R_i + R_3}$$

$$u_{ab} = R_3 \cdot i_3 = \frac{R_3}{R_i + R_3} \cdot u_0$$

Electrotechnique I

Here is our original dipole which is here, replaced by the simplified dipole, found with Thevenin's theorem. We can assure ourselves that the internal resistor seen from outside, R_i which is the resistance seen when all the sources are cancelled out, well, R_1 in parallel with R_2 is equal to the product of $R_1.R_2$ over the sum of R_1 plus R_2 . Now that we have completely determined the three parameters U_0 , i_{cc} and R_i , we can express the current i_3 and the voltage U_3 , or U_{ab} , in function of the resistor R_3 , we know that the current i_3 is equal to U_0 over the sum of R_i plus R_3 , and the output voltage U_{ab} or U_3 , is equal to the product of R_3 times i_3 , which is equal to R_3 over R_i plus R_3 , multiplied by U_0 . We recognise here the expression of the voltage divider.

Notes

Summary





- Simplifications simples, souvent applicables
- Circuits complexes, Théorème de Thévenin et Théorème de Norton
- Autres méthodes

Electrotechnique I

There, the simple simplifications are often applicable, but for complex circuits or sub-circuits, we can usefully apply the theorems of Thevenin and Norton: they guarantee the dipole equivalences. Later, we will see more methods for circuit transformations.

Notes

Summary



10m 11s