ANALYSE ET RÉSOLUTION DE CIRCUITS LINÉAIRES

MÉTHODES D'ANALYSE ET DE RÉSOLUTION – SOURCES DE TENSION ET SOURCES DE COURANT

LEÇON 7

Électrotechnique I
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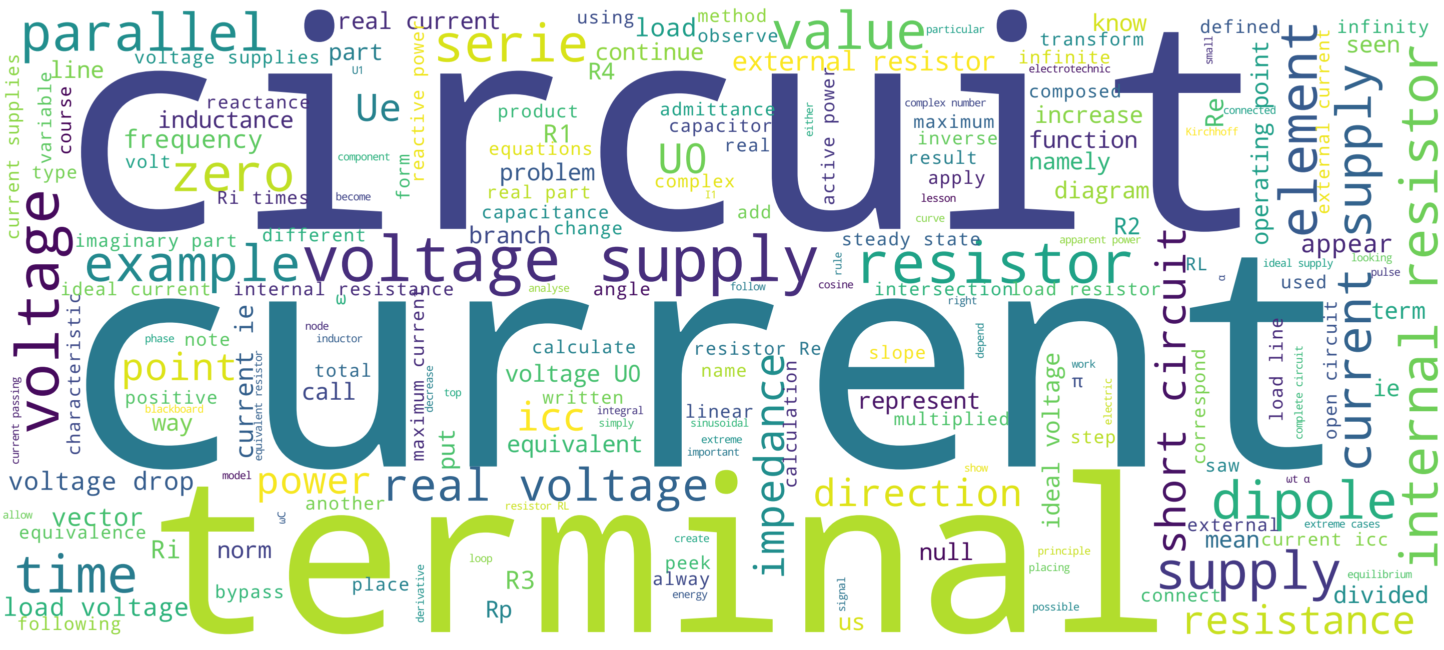


Video

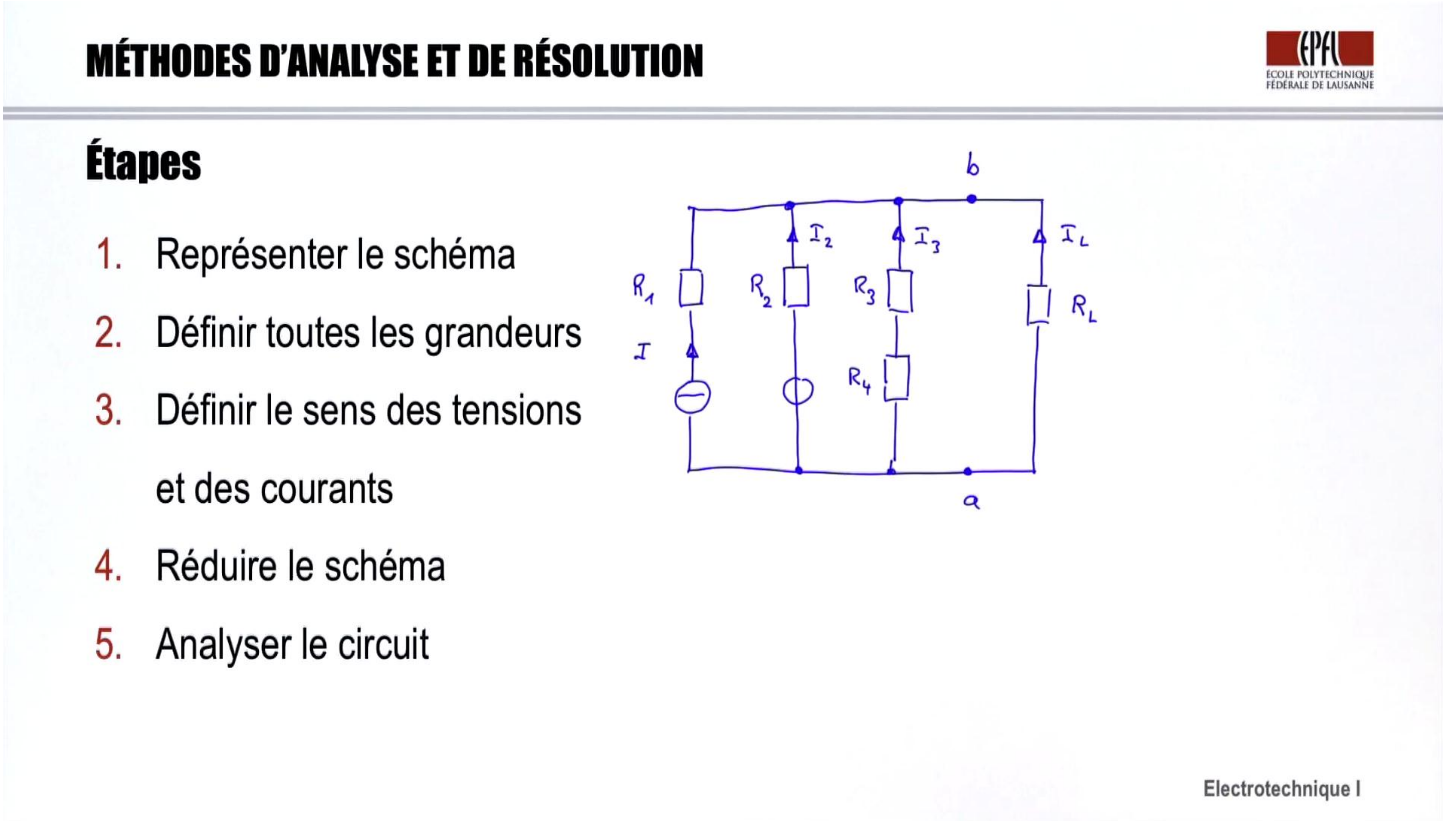
EPFL

Hello. In this course up to now, we have seen the three linear elements that are the resistance, the inductance and the capacitance, as well as voltage supplies, ideal current supplies. We have also seen various methods to combine these elements, in series, in parallel, and we saw a few simple circuits. In today's lesson, we will see resolution methods for a complete circuit. Today we will initially see the methods to analyse a circuit, which is a sequel of a series of steps that let us arrive to the desired result. In a second part, we will see real voltage and current supplies and finally, we will see the equivalence between these two supply types.

Notes

Summary

0m 04s

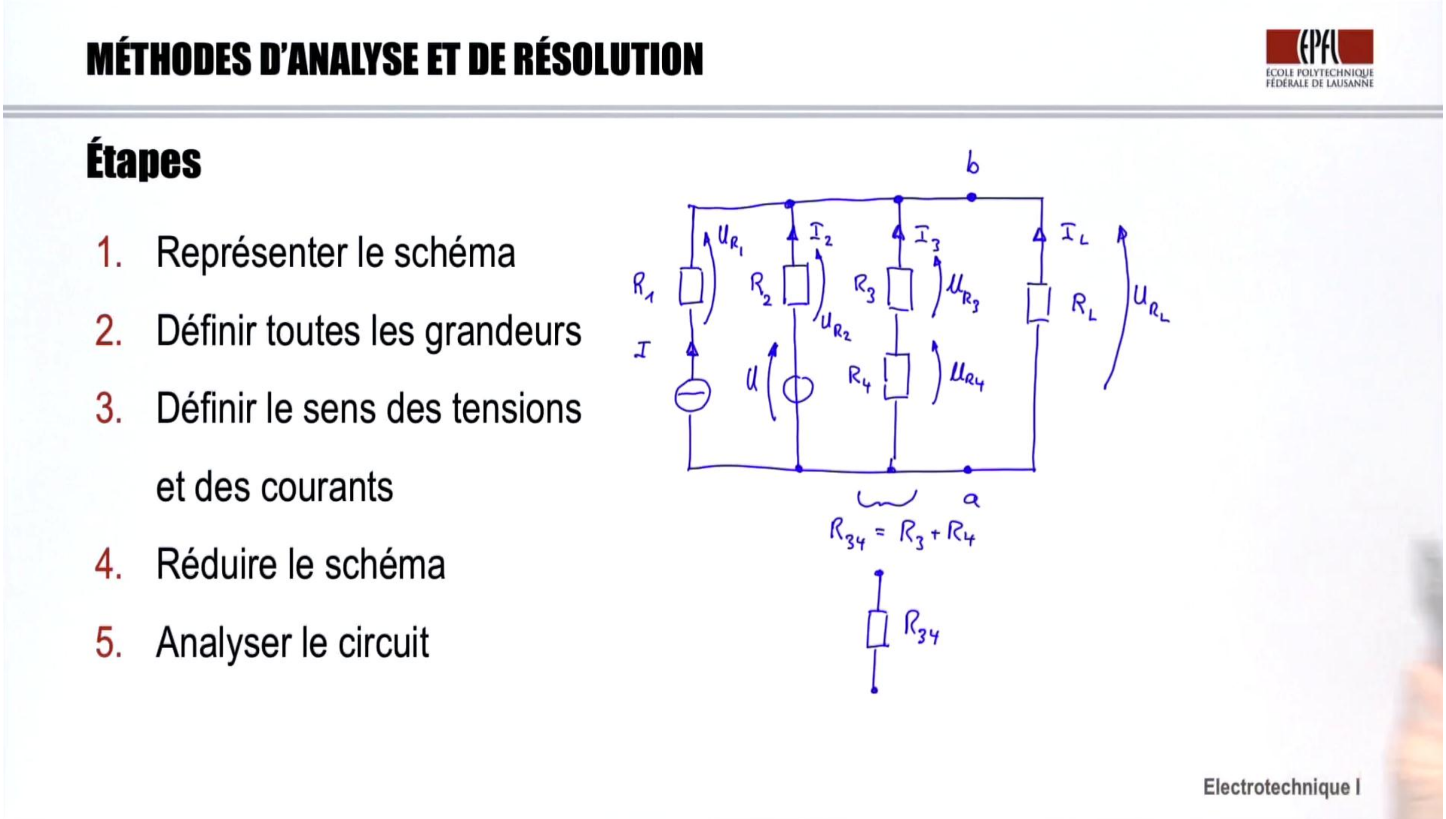


So, the different steps to resolve a circuit are, firstly, to schematically represent the circuit, whether it is given, or transcribed from a wording. We then represent here and example of a diagram with resistances, and voltage supplies. [Silence] So, we represent all the circuit's elements: resistors, current supplies, voltage supplies, and eventually other contact points. We also give a name to these points, for example the circuit's terminals b and a, and once all the elements are represented, we give them a name: the resistor R1, the resistor R2, the resistor R3 and R4, and here, for example, a resistor RL, which is a load resistor. We now define the currents and voltages: current I, for example, here, and then in each branch, we will also give a name to the current that is going through it: I2, I3, and in the last branch, current IL. In principle, before resolving the problem, we do not know in which direction the current, here: the currents I3, I2 and IL, will go through the circuit's branch. So in principle, we choose a direction for the current. If the current is chosen a priori, I would say, as going the wrong way, well we will get a negative numerical value. That is not a problem.

Notes

Summary

0m 57s



However, what is important, is to define in a coherent way the voltage drops at the terminals of each elements, for example here, on the resistance R3, we will have a voltage drop in the same direction as the current I3, so, we are obligated to define here a tension that we will call UR3 in the positive direction like this. Likewise for the voltage UR4. The voltage UR2 is defined as positive in the direction of the current, and the voltage UR1 is also positive in the current's direction. The last voltage, it is the voltage at the terminals of the load, which is going from bottom to top, from terminal a towards terminal a. We are still missing what the wording has given us: the voltage supply U, which imposes a voltage in that direction. So, that is a given problem. Once all the values have been defined with a given direction, we can start reducing the diagram. For example here, we see that we can but the two resistors R4 and R3 in series with an equivalent resistors that we can call R34, which is equal to R3 + R4, and that would replace the two resistors R3 and R4. As a reminder, two resistors or two element are in series when the same current is going through them, [Writes on the blackboard] and two elements are in parallel when they are connected on the same terminals.

Notes

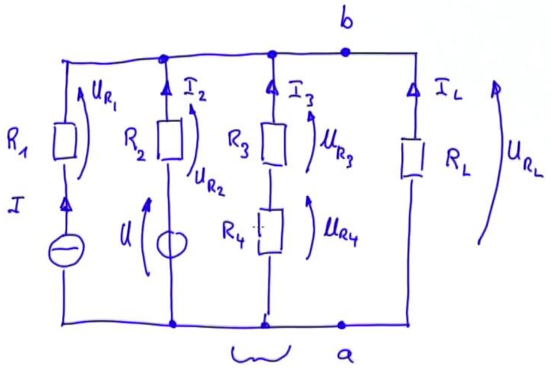
Summary

3m 22s

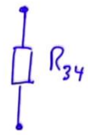
MÉTHODES D'ANALYSE ET DE RÉSOLUTION

Étapes

1. Représenter le schéma
2. Définir toutes les grandeurs
3. Définir le sens des tensions et des courants
4. Réduire le schéma
5. Analyser le circuit



$R_{34} = R_3 + R_4$



Rappel :
En série : parcourus par le même courant
En parallèle : branchés aux mêmes bornes

Electrotechnique I

[Writes on the blackboard] We can see in our example that R3 and R4 are in series since the same current is going through them, and for example, R2 is not in parallel with R1 because this terminal is the same, but this one isn't. On the other hand, the equivalent resistor R34 is in parallel with the resistor RL since they share the same terminals. We talked today about two types of simplifications: in series or in parallel, thereafter we will see other simplification methods.

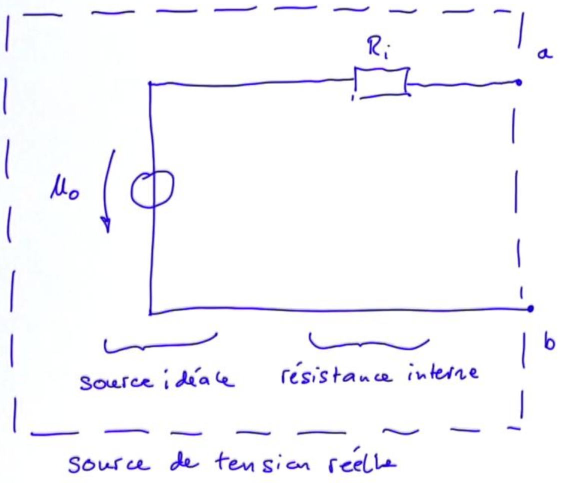
Notes

Summary

5m 37s

SOURCE DE TENSION RÉELLE

Schéma - Définition des grandeurs



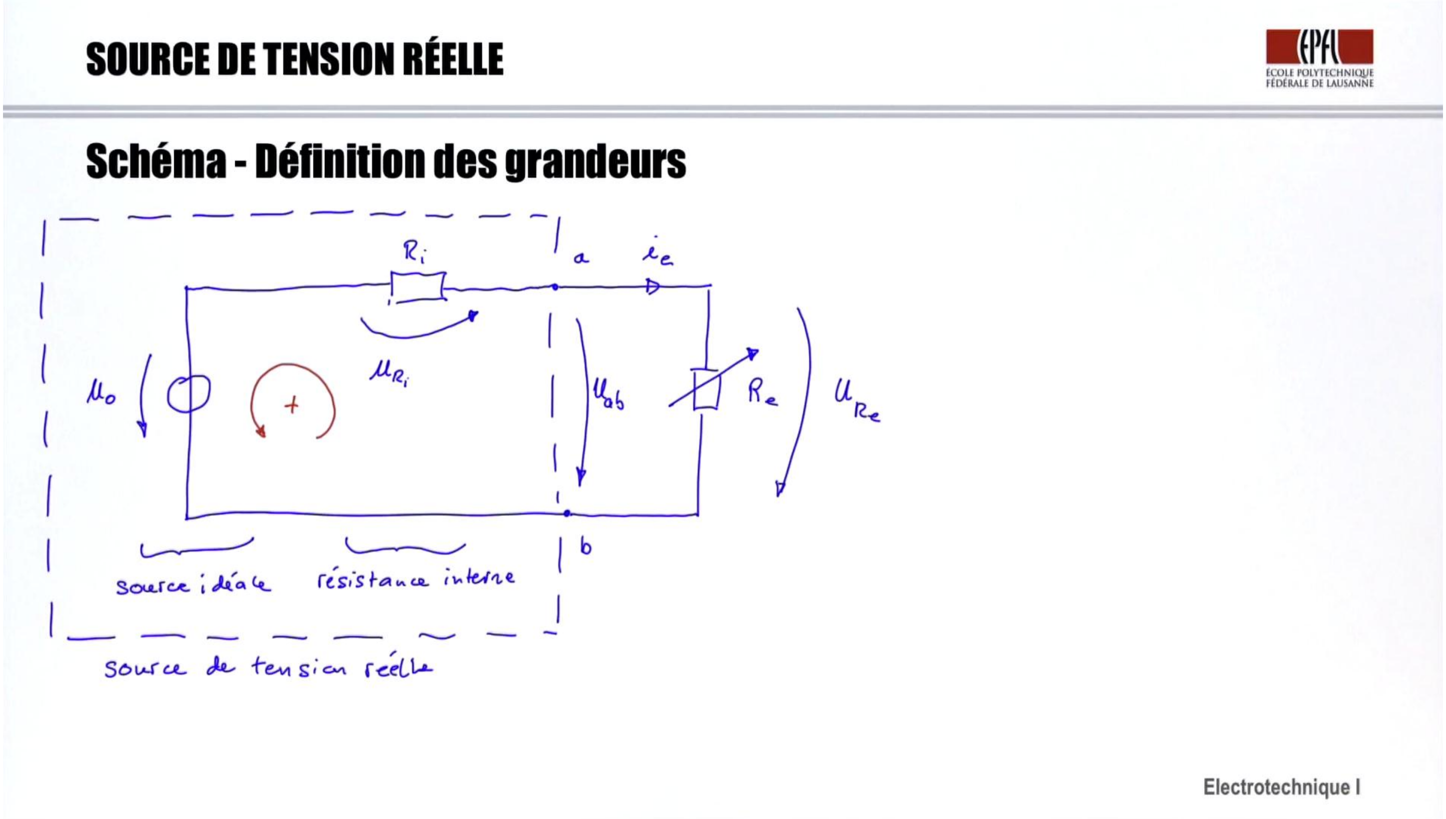
Electrotechnique I

We will now analyse the case of a real voltage supply. It is composed of an ideal voltage supply, like this, with a value U0, but an ideal voltage supply never exists alone. Why? Because we see that if we bypass these voltage supplies, there will be an infinite current in this bypass and so the power provided by the supply would be of U0 times infinity, which has no more physical sense, and moreover, we know that there is a zero voltage at the terminals of a short circuit, we bypass a voltage U0, and a zero voltage, that makes no sense, neither mathematical, nor physical. So this real voltage supply is always combined with a resistance that we call here internal resistor. This internal resistor, we place it in series with the voltage supply. Placing it in parallel would again make no sense because if we bypass the voltage supply, and then the resistor in parallel, we have the same problematic of infinite power, so we place an internal resistor in series. So this is the ideal supply, this is the internal resistance in series, and the whole makes a real voltage supply. We can define two terminals to this real voltage supply, boundary a and boundary b. The voltage supply will also be used to supply a load.

Notes

Summary

6m 28s



We will place here a load resistor that we will call Re, the subscript e being used for external, which means that it is an element that is external to the supply, this resistor can be variable, it can go from zero to infinity, and we show with this arrow here that it is variable. To follow on, there will be a current that settles in the circuit and that we will call ie. We can now define all the voltages at the terminals of our circuit. We have the supply voltage U0, we have a voltage drop here, on the internal resistor that we call URi, a voltage that appears at the terminals of the dipole Uab which is equal to the voltage drop, the voltage that appears on the boundaries of the external resistor URe. Lets define a positive direction for the voltage in this loop. We will now characterise this dipole, this real voltage supply.

Notes

Summary

8m 41s

SOURCE DE TENSION RÉELLE

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Mise en équation - Caractérisation

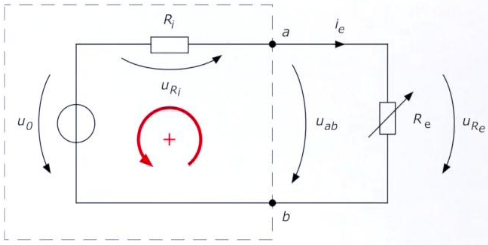
Par Kirchhoff : $\sum u = 0$

$$u_0 - u_{R_i} = u_e$$

$$u_0 - R_i \cdot i_e = R_e \cdot i_e$$

$$u_0 = i_e \left(R_e + R_i \right)$$

$$i_e = \frac{u_0}{R_e + R_i} \quad \text{et} \quad u_e = u_0 \cdot \frac{R_e}{R_e + R_i}$$



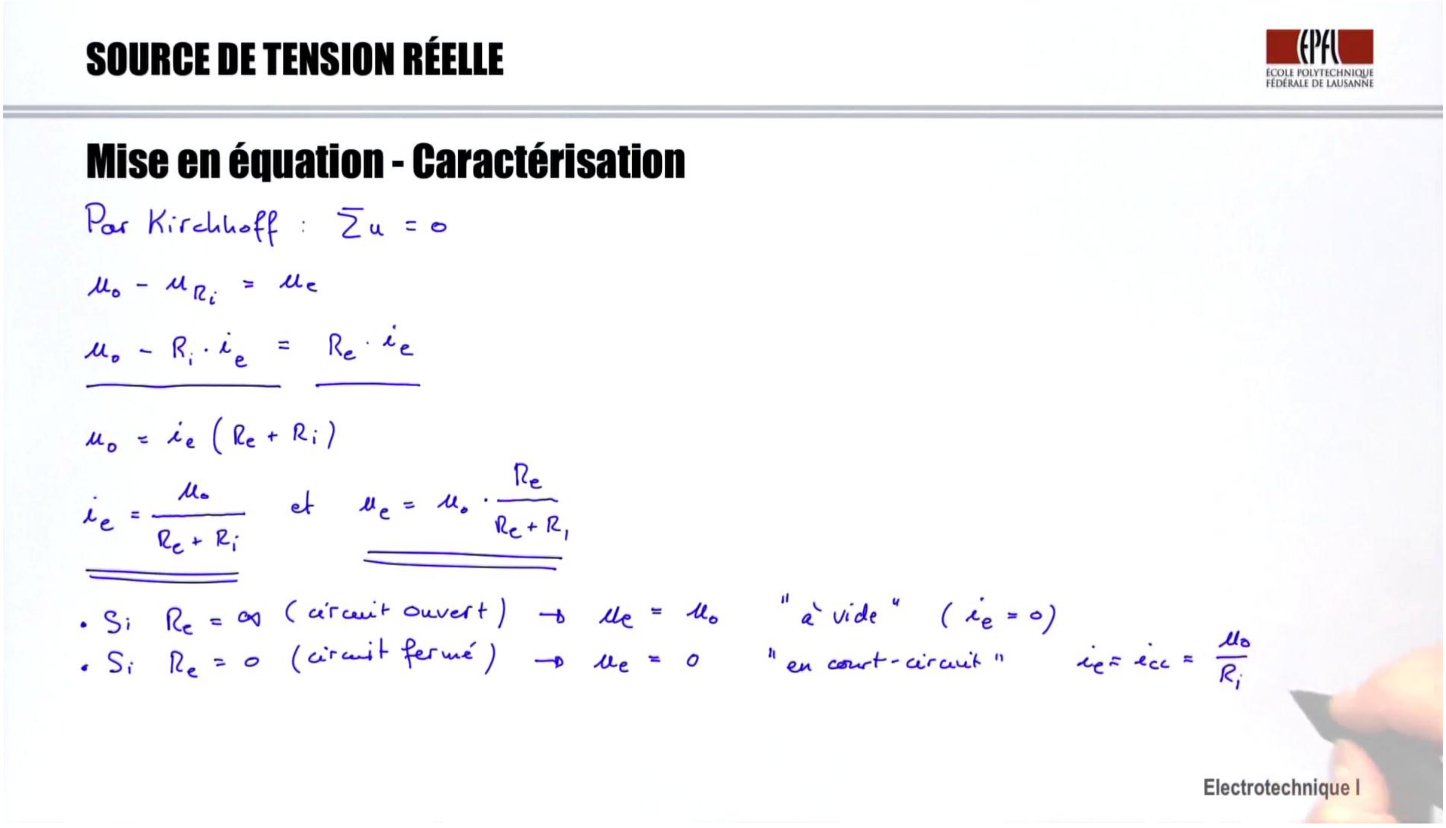
Electrotechnique I

By Kirchhoff, we can write that: the sum of voltages in a loop is equal to zero. We apply this to our circuit and we see that the real voltage supply minus the voltage drop on our internal resistor is equal to the external voltage of the circuit or the dipole. By developing these terms URi and Ue we get that the voltage supply minus Ri times the current going through it is equal to the external resistor times the current. By developing this equation, we get that the voltage is equal to ie, which multiplies Re plus Ri, and this lets us determine the operating point of the circuit, meaning that the current ie, the current passing through the circuit, is equal to the no-load voltage, or the voltage of the ideal supply, divided by Re plus Ri. More still, the voltage at the terminals of the dipole Ue, is equal to U0, the ideal voltage supply, multiplied by Re over Re + Ri. This equation is simply Re times the current ie. ie and Ue give us the operating point. Lets look for the operating limits of this supply, namely, the extreme cases when the internal resistor is infinite, which is the case when we have an open circuit.

Notes

Summary

9m 59s

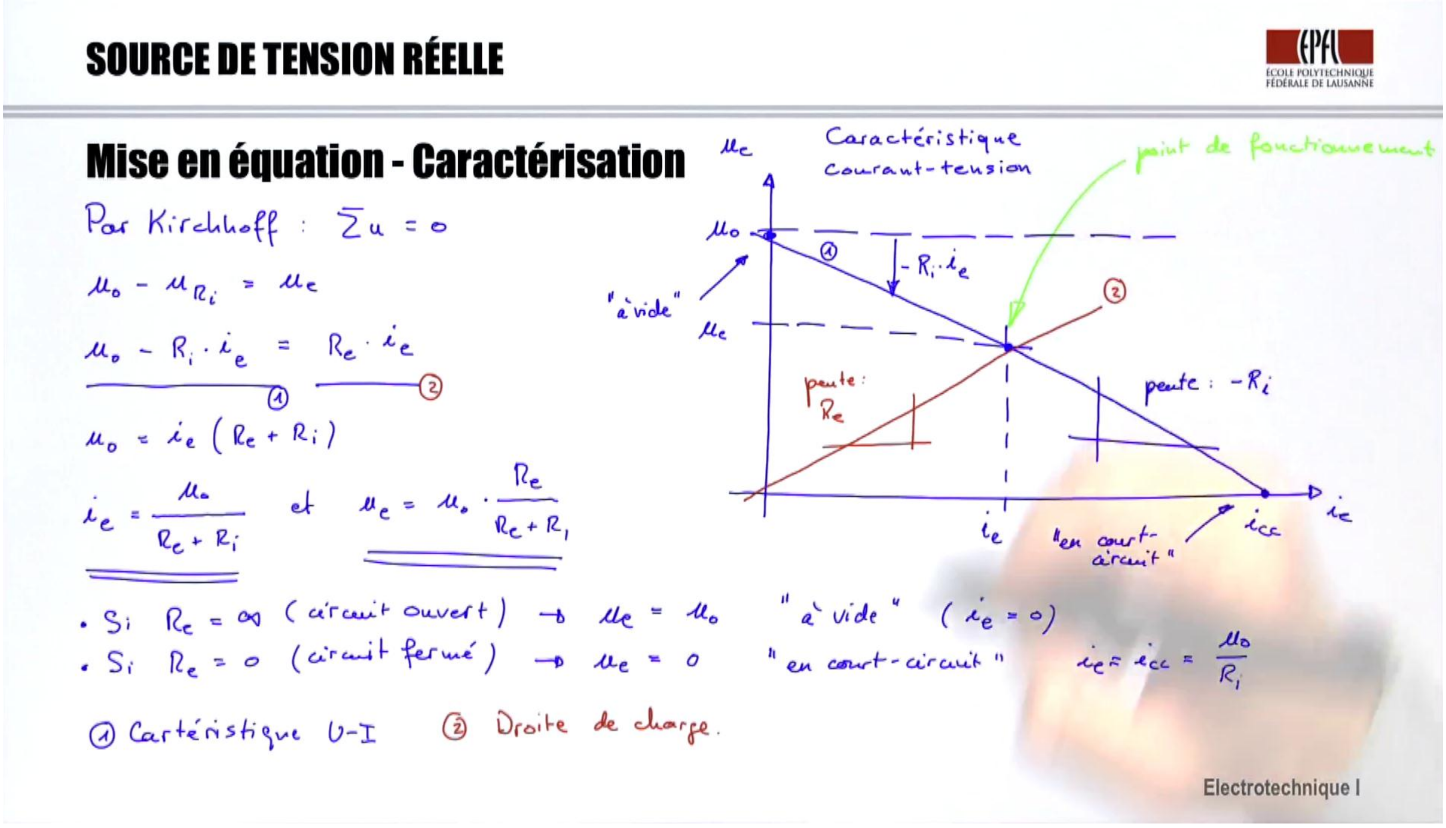


So if Re is equal to infinity, open circuit, well the voltage Ue that appears at the supply's terminals is equal to U0, there is no current in the circuit, so there is no voltage drop at the terminals of the resistance Ri and so we find the voltage U0 at the terminals of the dipole, we take about no-load operation, the external current is equal to zero. Another extreme case, this time if the internal resistor is equal to zero, we take of a closed circuit, well Ue, since the terminals are short-circuited, is equal to zero. We are in a short-circuit state in which the external current is equal to the short-circuit current, which is the maximum current that can be provided by the supply, and that is equal to U0 over Ri. If we observe these equations in a plane or in a current-voltage diagram, we get the following thing: on the abscissa, we have the current circulating through the circuit, on the ordinate, we have the voltage that appears at the terminals of the dipole Ue, at if we take the extreme points, when the external resistor is equal to zero; we are in an open circuit, we have a voltage that appears of value U0, which is the no-load operation.

Notes

Summary

12m 21s



The other extreme case, when the resistor is zero, we have this point here, the resistor is null so the voltage at the terminals of the circuit is null and we have here a maximum current, which is the current icc, it is the short-circuit operating mode. All the elements of the circuit being linear, we can trace a line between these two points, which will give us the characteristics of the circuit. The operating point will be on this line, corresponding to a value of ie, and a voltage at the terminals of the circuit that is equal to Ue. This intersection is the result of the intersection of two lines, the supply characteristic is a line corresponding to the external resistor. This first line corresponds to this equation and the second line to this equation. We see that the blue curve corresponds to a no-load voltage from which we subtract Ri times ie, so the slope is equal to minus Ri. The slope of the second line is equal to Re. The curve 1 is called characteristic of Ui and the curve 2 is called load line.

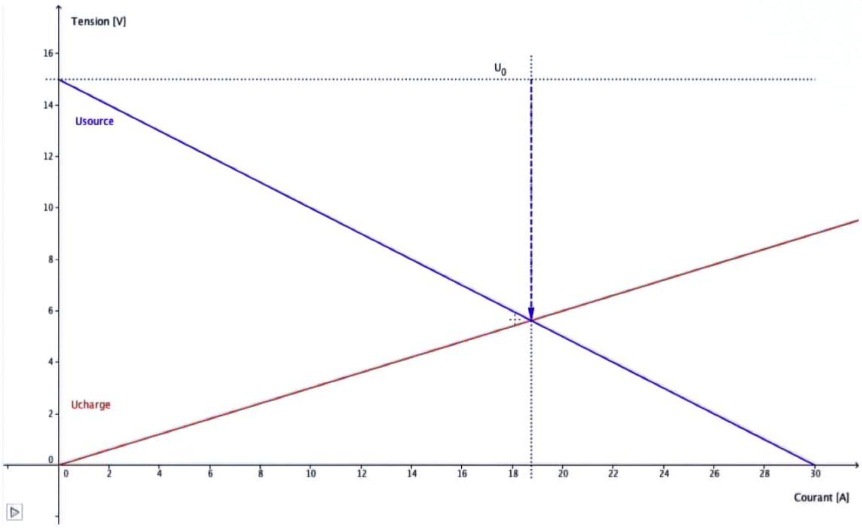
Notes

Summary

14m 27s

SOURCE DE TENSION RÉELLE

Mise en équation - Caractérisation



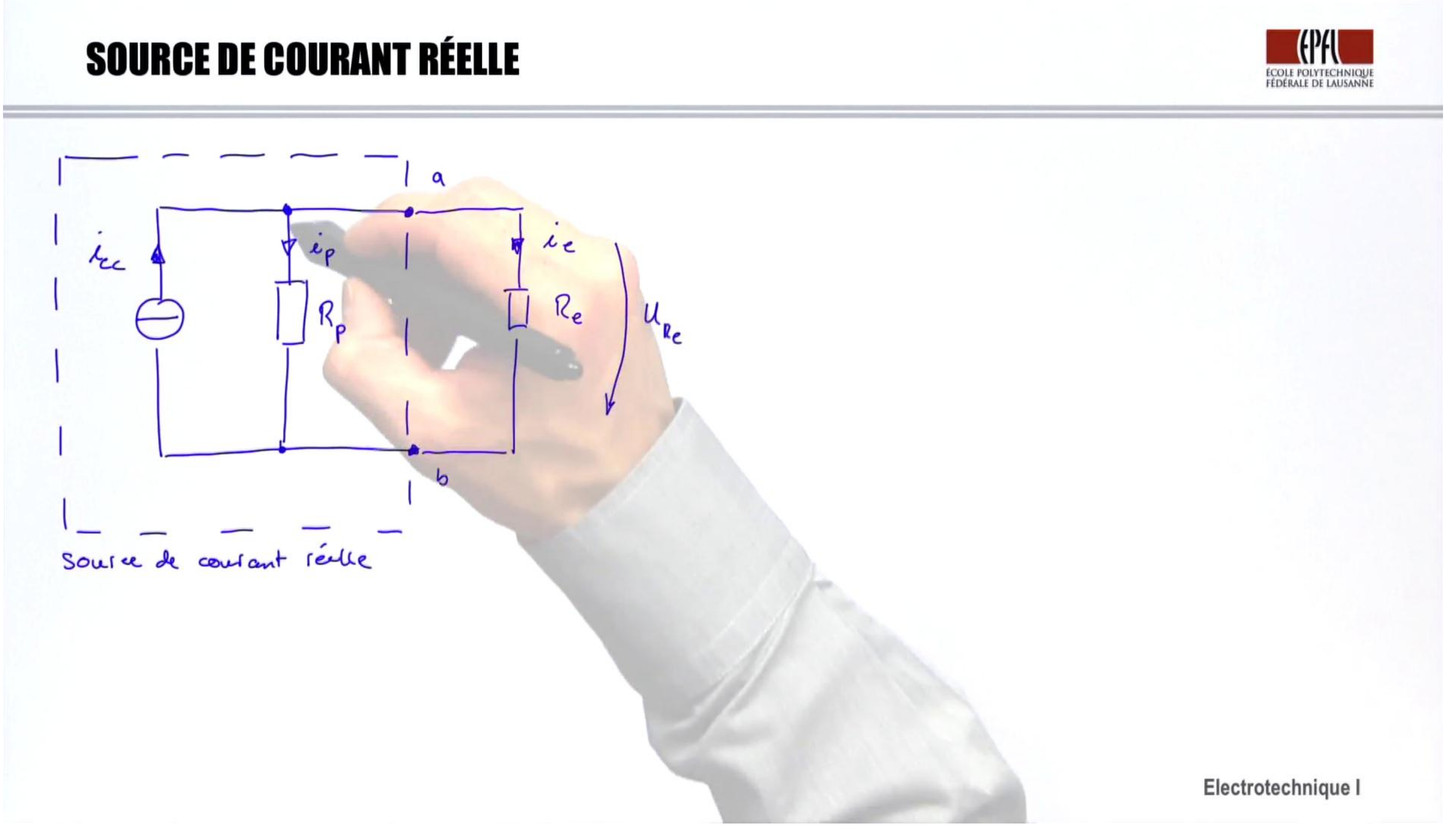
Electrotechnique I

If we observe here the current-voltage characteristic of a given real voltage supply, we see that it is characterised by a no-load voltage U0, here of 15 volts, and a slope, having a characteristic whose slope is given by the internal resistorRi, of a given value. If we improve this supply and we diminish the internal resistor, we get a smaller slope, a smaller internal resistor. If we continue to improve this internal resistor, we see that the characteristic slope of Ui continues to decrease, we remark that the maximal current provided by the supply is increasing. If we represent, for this last voltage supply, the load line that corresponds to the external resistor that we apply to the dipole, we see that the operating point is given by the intersection of the characteristic Ui and the load line, the operating point is here, we observe the output voltage of the dipole and the current passing through this dipole. If we change the load, and so the circuit's resistance, in this example we increase the load, and so we diminish the resistance, we see that the voltage at the terminals of the dipole falls and that the current increases. If we continue to increase the load, so we continue tu decrease the resistance, well the voltage at the terminals of the dipole will continue to fall and the current will continue to increase. We note here the voltage drop on the internal resistor.

Notes

Summary

17m 18s

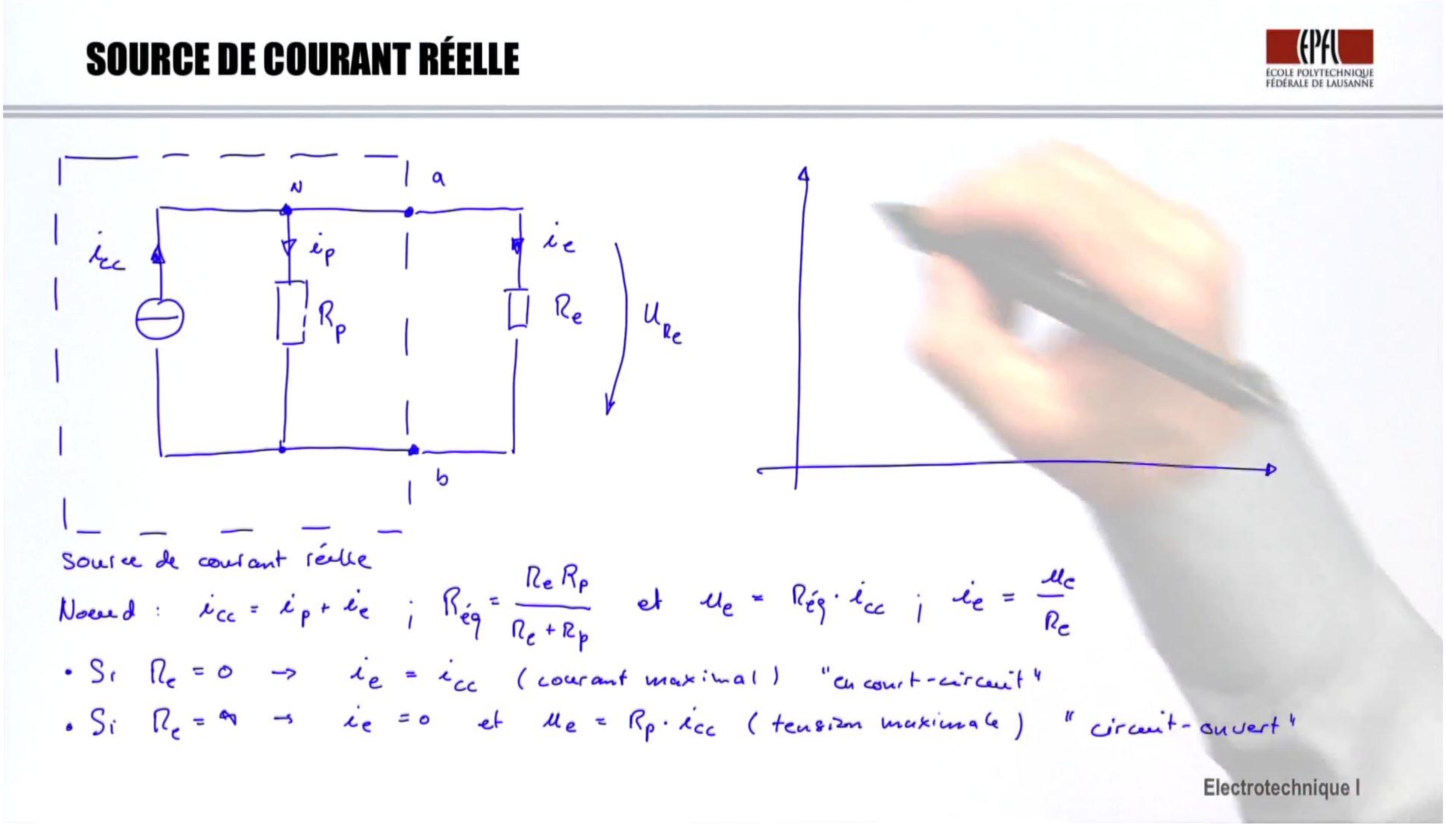


Lets now look at the case of a real current supply. So the real current supply is composed of an ideal current supply in parallel, this time, with an internal resistor, that we call Rp. The value of this current supply is icc. This constitutes a dipole that has two poles a and b, and that we call real current supply. We note this time that the resistor is in parallel; why? Because if we had put it in series, the current supply imposing a current icc, well, in the case of a short-circuit, for example, on the supply, would lead to a physical impossibility of having an infinite power, so we put this resistor Rp in parallel. We can do the same steps as for the real voltage supply, we will just do it a little bit faster by using this time Kirchhoff's current law in one point, this point, where we can write that the current ip going through this branch when we supply an external resistor Re being passed by a current ie, which creates a voltage drop URe on the external resistor, voltage that we will also find between these points and these points. On this knot n here, we can say that icc is equal to ip plus ie.

Notes

Summary

19m 18s

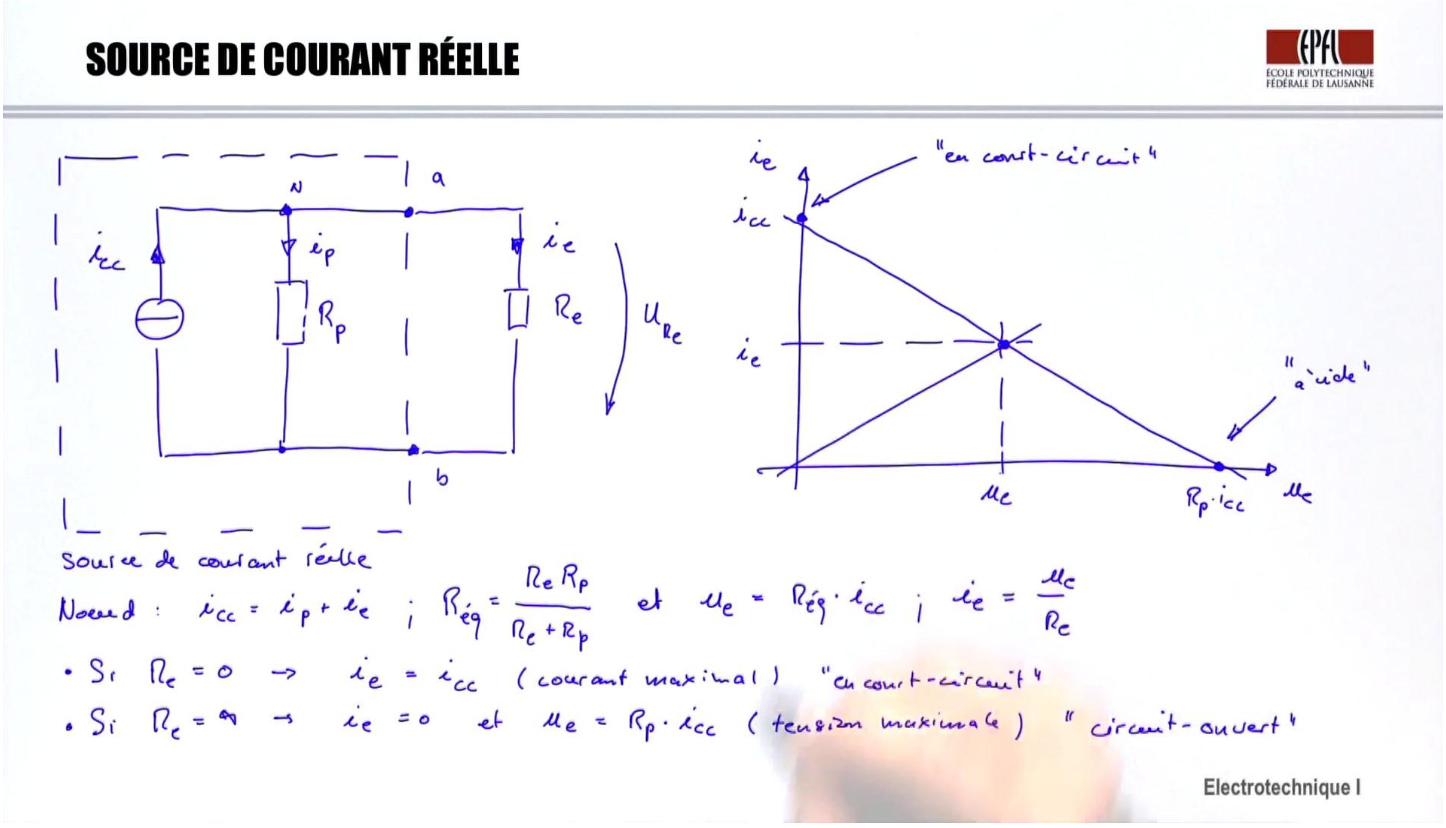


We can put the resistors Rp and Re, in parallel, which gives us a resistance, that we call Req for equivalent, which is the product of the sum, and which make the voltage at the terminals of the circuit, the voltage Ue, equal to the current icc that passes through the resistorReq, which is then equal to Req multiplied by icc. We can also write that ie is equal to Ue over Re. Once again, to characterise this supply, if we take the extreme cases, namely if the load resistor is equal to zero, we have a current ie which is equal to the current icc, we are in the case of a maximal current that the supply can provide, we are once again in short-circuit. The other extreme case, if the external resistor is equal to infinity, well we have an external current equal to zero and a voltage at the terminals of the dipole ab, where Ue which is equal to Re multiplied by icc, is also the maximum voltage that can appear at the terminals of the dipole, and we are in the case of an open circuit. Lets graphically represent the characteristic Ui of this supply. Of we represent Ue on the abscissa and ie on the ordinate, we have a maximum current, equal to icc, a maximal voltage which is given here and is equal to Rp times icc, we have the no-load mode and here the short-circuit mode.

Notes

Summary

21m 37s

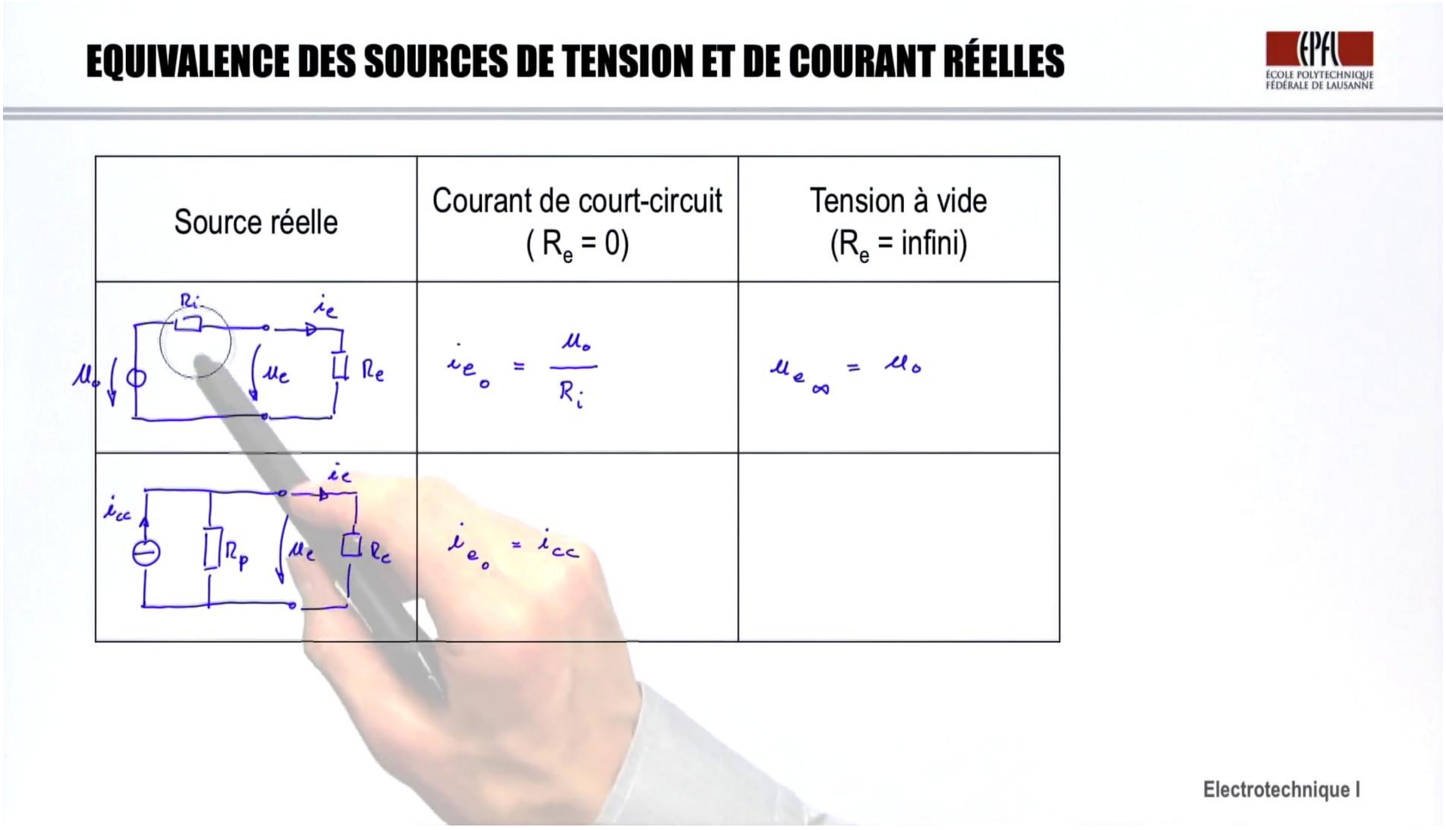


Once again, everything being linear, we can connect these two points to get the current-voltage characteristic with an intersection point with the external resistor which is given by the intersection of the two curves, the characteristic and the load line, for the operating points Ue and ie.

Notes

Summary

24m 32s

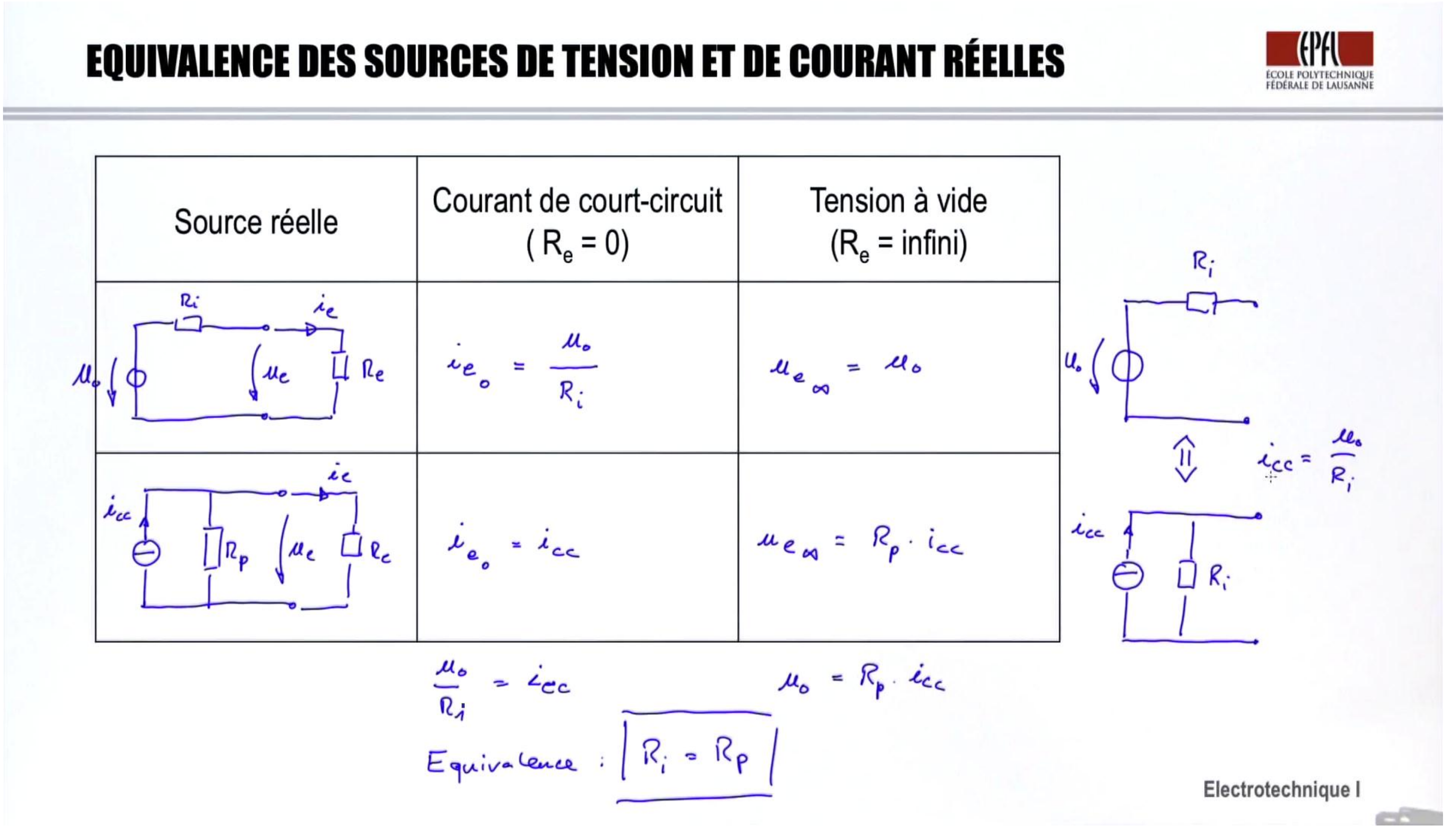


We will now see what we can do to make these two types of supplies equivalent. I redraw the diagram of the real voltage supply here, with a no-load voltage U0, and an internal resistance Ri, a voltage at the terminals of the circuit equal to Ue, and a load Re which means that a current ie is passing through the circuit. For the current supply, we have here a current supply equal to icc, an internal resistor in parallel, which form the supply's dipole, at which appears a voltage at the terminals when we connect an internal resistor Re and a current ie passing through. If we look at the short-circuit current in both cases, well the short-circuit current, that we call ie0 for the voltage supply, this current ie is equal to U0 divided by the internal resistor, when this resistor Re is equal to zero. In the case of the current supply, when the resistor Re is equal to zero, well the short-circuit current is equal to icc, all the current passes through the external resistor. When looking at the no-load voltages, so when the external resistor is infinite, well the no-load voltage that we call infinite Ue, is equal to U0 in the case of a real voltage supply. Why?

Notes

Summary

25m 00s



Because the current ie being null, there is no voltage drop at the terminals of Ri and we will find again the voltage U0 at the terminals of the circuit Ue. In the case of the current supply, the no-load voltage is equal to Rp times icc, it is the voltage that appears at the circuit terminals when the resistor is infinite, namely when there is no current here, all the current passes through the resistor Rp and so the no-load voltage at the terminals of Rp is equal to icc times Rp. So for these two supplies to be equivalent, the two short-circuit current must be equal, first condition, so U0 over Ri must be equal to icc, second condition, the two no-load voltages must also be equal, so U0 must be equal to Rp to icc. In summary, we get the equivalence if Ri is equal to Rp. This means that when we have an ideal voltage supply, with an internal resistor Ri, ideal voltage supply U0, that form a real voltage supply, well we can transform in, and conversely, with an ideal current supply of value icc with an internal resistor Ri by applying the following relation: icc is equal to U0 over Ri. Inversely, if we have a real current supply, we can transform it into a real voltage supply by placing a resistor in parallel and a resistance in series and by transforming this current supply in a voltage supply of value Ri times ic.

Notes

Summary

27m 10s

CONCLUSIONS

- Maîtrise d'un circuit complet
- Traitement du circuit
- Analyse

- Autres méthodes

Electrotechnique I

There, after having seen all the simple elements individually, we have the tools to master a complete circuit. We have seen the method to process a circuit and the analysis that are possible to do. Thereafter, we will see new circuit transformation methods.

Notes

Summary

29m 37s