ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

ANALYSE ET RÉSOLUTION DE CIRCUITS LINÉAIRE

CIRCUITS COMBINÉS SÉRIE-PARALLÈLE

LEÇON 6

Électrotechnique I
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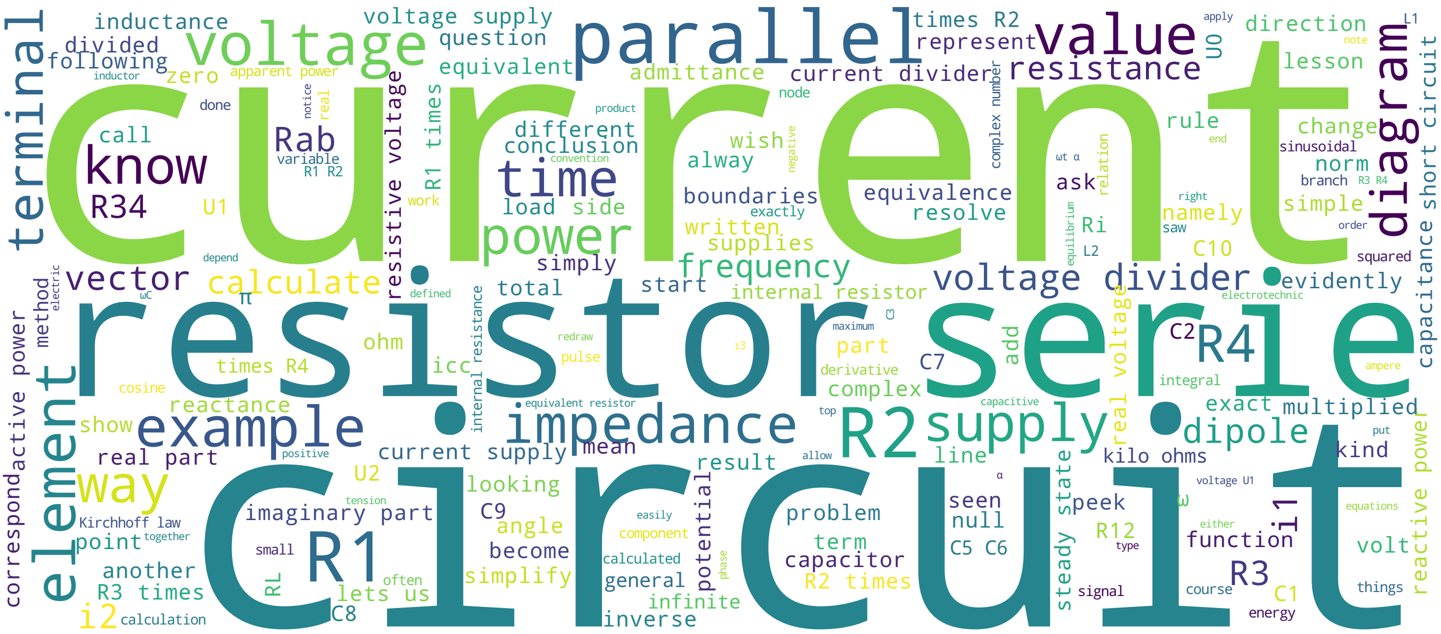
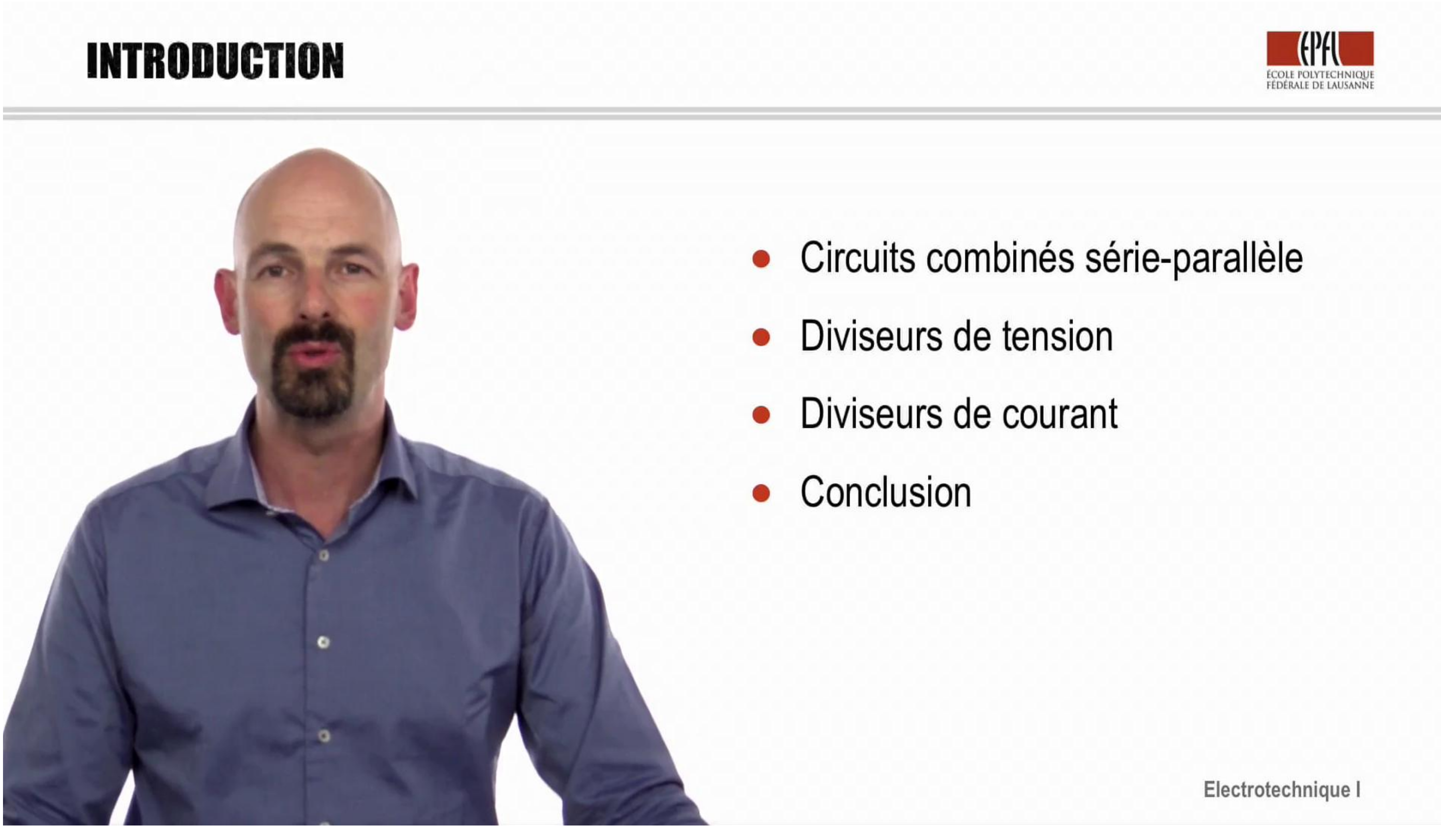


Video

EPFL



Hello and welcome to this MOOC or in this lesson dedicated to the analysis and resolution of linear circuits, the second part of our chapter. We will see in this second part combined, series and parallel circuits when we have multiple elements that are together, in series and in parallel. We will then see these two elements or two very special models that are finally very very common in the electrical circuits that are tension and current dividers.

Notes

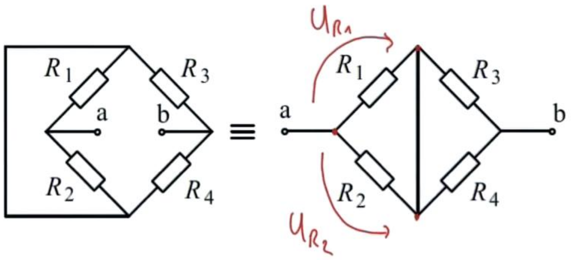
Summary

0m 04s

CIRCUITS COMBINÉS SÉRIE-PARALLÈLE

Exemple avec des résistances

$U_{R_1} = U_{R_2}$



Electrotechnique I

Finally, we will have a conclusion for this lesson. We will start with an example. An example here with only resistors, an extremely clear example, to see that it is very difficult in this diagram to know if R1, R2, R3 and R4 are in series or in parallel knowing that we wish to calculate an equivalence of this dipole between a and b. When we are in such a case, the first thing to do is to redraw the circuit differently, to play with it, to inverse it, in order to find a more clear vision of the diagram. That is what we do here, by extracting a and b and by redrawing the same diagram. We still have the circuit between R1 R3, you have here between R1 R3, and R2 R4 this diagram that closes and so strictly the same thing. We can then notice a certain number of things. We note that the voltage that is here on the boundaries of R1, this voltage that we will call UR1, is the same as this voltage here, UR2. Why? Because this potential is the same as this one and this potential is the same for R1 and for R2. So UR1 UR2 being equivalent, we know that R1 R2 is in parallel. We can do the same thing with R3 and R4 that are in parallel and we can then write that this diagrams transforms into R12 and R34 and these two are in series since the current that circulates between these two blocs will now be equivalent.

Notes

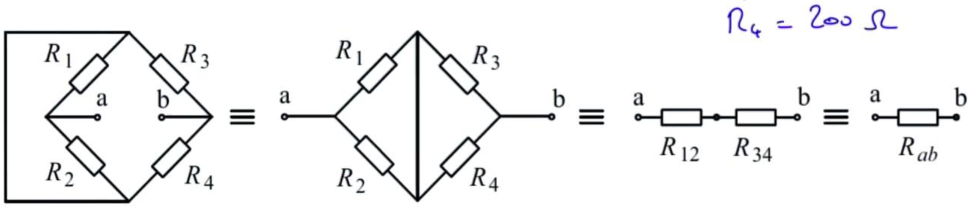
Summary

0m 34s

CIRCUITS COMBINÉS SÉRIE-PARALLÈLE

Exemple avec des résistances

$R_1 = 2\ \mathrm{k}\Omega$
$R_2 = 8\ \mathrm{k}\Omega$
$R_3 = 5\ \mathrm{k}\Omega$
$R_4 = 200\ \Omega$



$$R_{12} \Rightarrow \quad R_{12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \equiv R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} \qquad R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4}$$

$$R_{ab} = R_{12} + R_{34} = 1792\ \Omega$$

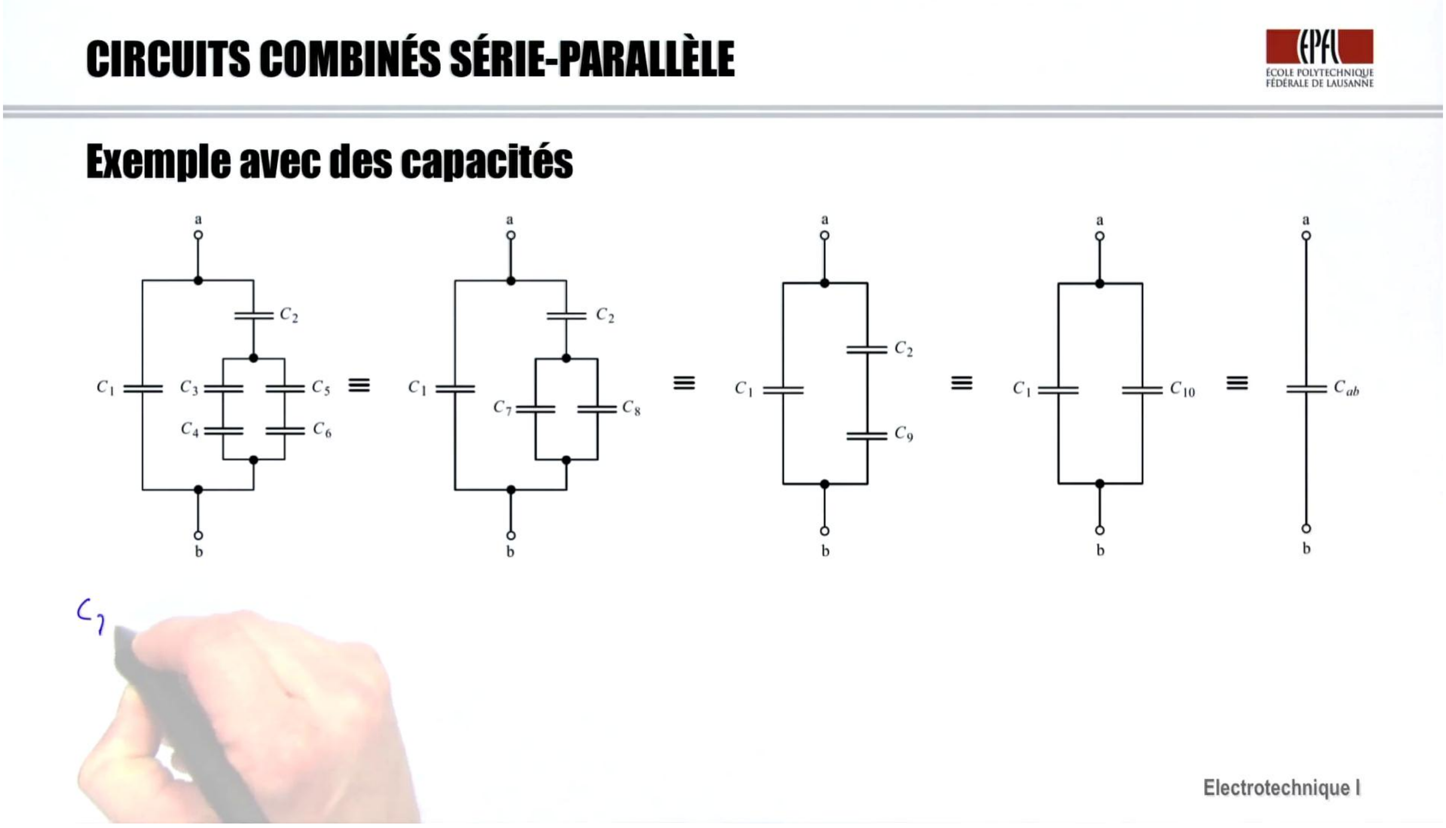
Electrotechnique I

And to finish, we have the equivalence between a and b, Rab. So how do we calculate this Rab? Well, for example for R12. First, R12. We know that to calculate R12, R12 is 1 over the inverse of the resistor's sum so 1 over R1 plus one over R2. This lets us writes that R12 is equal to R1 times R2 over R1 plus R2. We can evidently write the same thing for R34 that will, in the same way, be written as R3 times R4 over R3 plus R4. And finally, Rab being R12 and R34 in series, it becomes the sum of R12 plus R34, so Rab is R12 plus R34. We can now do a numerical example, we give for example R1 equal to 2 kilo-ohms, R2 to 8 kilo-ohms, R3 to 5 kilo-ohms and finally R4 equal to 200 ohms. We get a Rab, if you do the whole calculation with all that was shown, you will find 1792 ohms.

Notes

Summary

2m 22s



We can now do the exact same things with capacitances and I will show you once again a quite complete diagram. How do we resolve such a diagram? To start with, here is the first question. Which elements will be in series and which will be in parallel? It is also very good to ask yourself the questions "Is this parallel?", "Is this in series?". First, we have a tension that we will call U1 that is exactly the same as U1 here. This means that C3 C4 are in parallel with C5 C6. We also see that the current that circulates here, lets call it i3, will be the same as the current passing here, i4, so C3 and C4 are in series. In the same way, C5 C6 are in series. Then, we ask ourselves these questions. We can then write that C3 C4 being in series, we can replace them by C7, C5 C6 can be replaced by C8, we then have C7 C8 in parallel that will become C9 and that will be in series with C2 and then in parallel with C1 and this finally gives us our capacitance Cab. Then, we can evidently give the final elements. First, this C7 that we have written before. C7 being in series, so we have the inverse of the capacitance that is equal to the sum of the inverses; we find the same rules again, we have C3 times C4 over C3 plus C4.

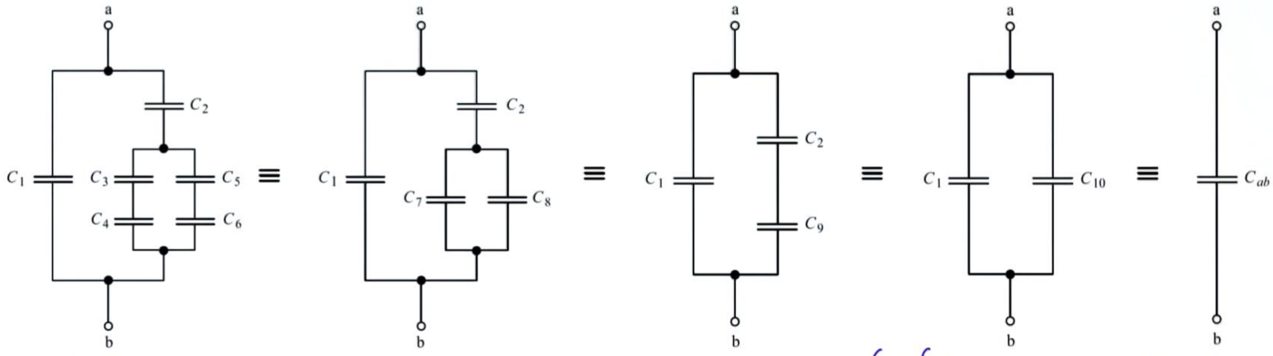
Notes

Summary

4m 08s

CIRCUITS COMBINÉS SÉRIE-PARALLÈLE

Exemple avec des capacités



$$C_7 = \frac{C_3 \cdot C_4}{C_3 + C_4} \qquad C_8 = \frac{C_5 \cdot C_6}{C_5 + C_6} \qquad C_9 = C_7 + C_8 \qquad C_{10} = \frac{C_2 \cdot C_9}{C_2 + C_9} \qquad C_{ab} = C_1 + C_{10}$$

Electrotechnique I

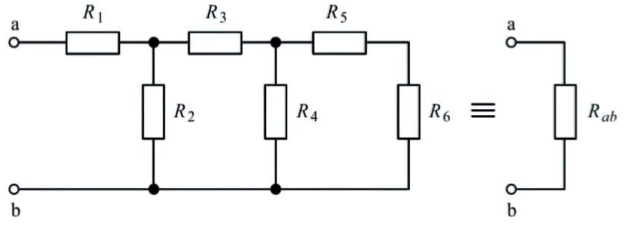
Then, we have C8, which in the same way can be calculated as being C5 times C6 over C5 plus C6. We then have C9. If you put C7 and C8 in parallel, you get C9, so it is simple the sum of C7 and C8. Finally, we have C10. How do you find C10? You put C2 and C9 in series. We will then have C2 times C9 divided by C2 plus C9. And finally, we put C1 and C10 in parallel to get Cab and it is simply the sum of C1 plus C10 that we have just calculated. And that is how we simplify a diagram when we put various elements in parallel or in series like the capacitance here.

Notes

Summary

5m 49s

Exemple de circuit en échelle



$$R_{ab} = R_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{\cfrac{1}{R_3} + \cfrac{1}{\cfrac{1}{R_4} + \cfrac{1}{R_5 + R_6}}}}$$

I will also show you a specific example of a circuit that can make you think that we are neither in series nor in parallel. So, yes and no is what we call a ladder circuit. We can see here that R5 R6 are in series and then in parallel with R4 which is itself in series with R3 and then in parallel with R2, and so forth. We can then calculate this resistance Rab in a successive way, beginning with R1, R1 to which we add all of this, so we have 1 over R2 plus 1 over 1 over R3 plus 1 over 1 over R4 plus 1 over R5 plus R6. And finally there is that scaling of the different elements in series, which lets us simplify the problem and have only one equivalent resistor.

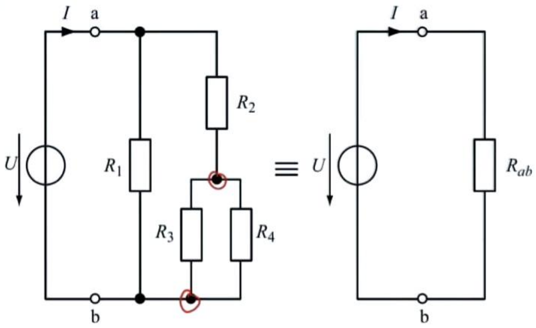
Notes

Summary

6m 47s

CIRCUITS COMBINÉS SÉRIE-PARALLÈLE

Exemple de circuit complet



$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4}$$

$$R_{234} = R_2 + \frac{R_3 \cdot R_4}{R_3 + R_4}$$

$$R_{ab} = \frac{R_1 \cdot R_{234}}{R_1 + R_{234}}$$

$U = 73.45\ V$

$R_1 = 22\ \Omega$

$R_2 = 36\ \Omega$

$R_3 = 18\ \Omega$

$R_4 = 15\ \Omega$

Electrotechnique I

Here is another example of a complete circuit with resistors this time. Once again in this diagram, which resistors are in parallel and which of them are in series? We clearly have here R3 and R4 in parallel, then are at the same voltage; these two points here have the same potential for R3 R4 and this one the same potential for R3 R4. Then, if we simplify this by R34, this will be in series with R2. Then the whole will be in parallel with R1. So this is the way to resolve such a circuit. We can then summarise in this way, that we will have R34 which is equal to R3 times R4 over R3 plus R4. We then have R2 in series with R34, that we will call R234, namely R2 plus what we have just calculated, R3 times R4 over R3 plus R4. Finally, Rab will be R1 and R234 in parallel. So Rab finally is R1 times R234 over R1 plus R234. We can now do a numerical calculation to give us an idea. We have, if we take a numerical example, U equal to 73,45 volts for example. A measurement example that was done lately on this type of circuit with R1 equal to 22 ohms, R2 equal to 36 ohms, R3 equal to 18 ohms et R4 equal to 15 ohms. So if we first ask ourselves, as suggested earlier, what is Rab equal to, we find the final Rab equal to 14,69 ohms.

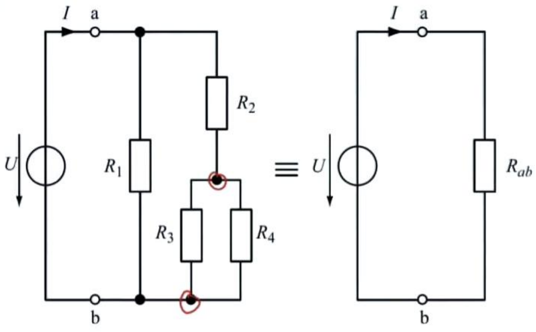
Notes

Summary

7m 51s

CIRCUITS COMBINÉS SÉRIE-PARALLÈLE

Exemple de circuit complet



$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4}$$

$$R_{234} = R_2 + \frac{R_3 \cdot R_4}{R_3 + R_4}$$

$$R_{ab} = \frac{R_1 \cdot R_{234}}{R_1 + R_{234}}$$

$$U = R_{ab} \cdot I$$

$$I = \frac{U}{R_{ab}} = 5\ A$$

$U = 73.45\ V$

$R_1 = 22\ \Omega$

$R_2 = 36\ \Omega$

$R_3 = 18\ \Omega$

$R_4 = 15\ \Omega$

$R_{ab} = 14.69\ \Omega$

Electrotechnique I

It is now evident to answer an initial question that we could ask ourselves: "What is the value of the current delivered by the supply?". We know that U is equal to Rab times i. And, if we are looking for the current, i is equal to U over Rab, we then find that this current is simply equal to 5 amperes.

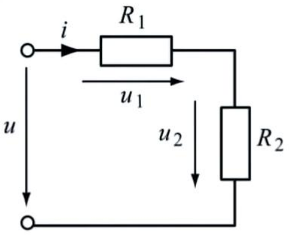
Notes

Summary

10m 07s

CIRCUITS DIVISEURS DE TENSION ET DE COURANT

Diviseur de tension résistif



$$U_1 + U_2 = U$$

$$R_1 \cdot i + R_2 \cdot i = U \quad \Rightarrow \quad i = \frac{U}{R_1 + R_2}$$

Electrotechnique I

We now pass to a familiar diagram that we often find in a more complete ensemble of an electrical model which is the resistive voltage divider. Why do we call this the resistive voltage divider? Well because if we have an initial voltage U with 2 resistors in series, at the boundaries of the second resistor, here, as it happens, U2, we will have a U2 small than the initial U so this lets us uncouple, divide, separate, -- see it as you wish -- the initial voltage in a smaller voltage, more interesting for such and such a circuit. For example, you have 24 volts arriving from a supply but you would like to get 5 volts for this or that reason, well, with the judicious choice of R1 and R2, we can calculate U2 in such a way that we will get exactly the 5 volts we are looking for. How do we define or how do we calculate this kind of thing? Well, we simply use Ohm's law. We know that the sum of the voltage U1 plus U2 is equal to U. We know that U1 is R1 times i and that U2 is R2 times i. All this being always equal to U, so i is equal to U over R1 plus R2. Since U1 is equal to R1 times i and U2 is equal to R2 times i, we finally get that for U2, which is here, and is equal to R2 times i, it is also equal to U times R2 over R1 plus R2.

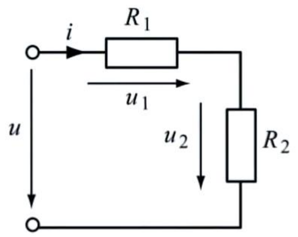
Notes

Summary

10m 31s

CIRCUITS DIVISEURS DE TENSION ET DE COURANT

Diviseur de tension résistif



$$u_1 + u_2 = u$$

$$R_1 \cdot i + R_2 \cdot i = u \quad \Rightarrow \quad i = \frac{u}{R_1 + R_2}$$

$$\boxed{u_2 = R_2 \cdot i = u \cdot \frac{R_2}{R_1 + R_2}}$$

Electrotechnique I

This lets us calculate, in a general and fast way, that this voltage U2, if I have chosen rightly R1 and R2, I can do a voltage divider. And this can evidently be done not only with resistors -- so it is here a resistive voltage divider -- but also with a capacitive voltage divider.

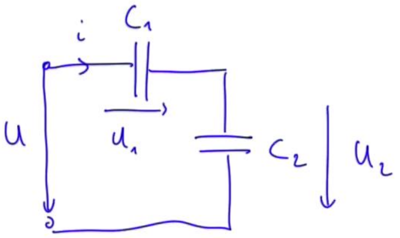
Notes

Summary

12m 21s

CIRCUITS DIVISEURS DE TENSION ET DE COURANT

Diviseur de tension capacitif



$$\boxed{U_1 = u \frac{C_2}{C_1 + C_2}}$$

Electrotechnique I

In this case, we take the exact same diagram as before so we will have here a voltage U, on who's boundaries we will have two capacitances that will let us, C1 and C2, do the exact same thing then before, with still U1 and here U2, and to divide this voltage here between C1 and C2. In such a case, we can also easily convince ourselves that U1, using Kirchhoff's law as we did before, U1 will be equal to U times C2 over C1 plus C2. This will be the rule for a capacitive voltage divider.

Notes

Summary

12m 44s

CIRCUITS DIVISEURS DE TENSION ET DE COURANT

Diviseur de tension inductif

$$U_1 = U \frac{L_1}{L_1 + L_2} \qquad U_2 = U \frac{L_2}{L_1 + L_2}$$

Electrotechnique I

Finally, the inductive voltage divider. Once again, we have the same diagram, I will not redraw it here, but we will find it again, since the rules to put things in series are the same as those for resistances with inductance, in the same way we see that U1 is equal to U times L1 over L1 plus L2, and from there we can also write that U2 is equal to U times L2 over L1 plus L2. It is then easy to remember than the final rule for the inductive voltage divider follows the same kind of relation as the resistive voltage divider.

Notes

Summary

13m 34s

CIRCUITS DIVISEURS DE TENSION ET DE COURANT

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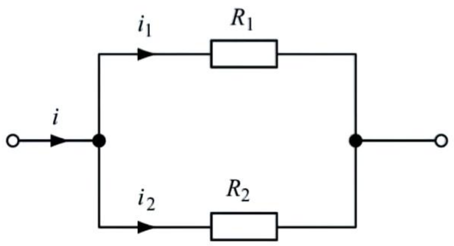
Circuits diviseurs de courant



$$i = i_1 + i_2$$

$$u_1 = u_2 = R_1 \cdot i_1 = R_2 \cdot i_2$$

$$i_1 = i \frac{R_2}{R_1 + R_2} \qquad i_2 = i \frac{R_1}{R_1 + R_2}$$

Electrotechnique I

Now the current divider. You have here another phenomenon, so the current arrives at the boundaries of this dipole, separates in two on two resistors. We very often have this kind of problematic in a circuit. We have R1 and R2 in parallel, but if we wish to calculate i1 or i2, it is excluded to resolve this circuit by replacing R1 and R2 by a single resistor. We would then have i1 and i2 "drowned in the equivalent resistor". We must then keep these two resistors et calculate separately the values of i1 and i2. Once again, we will do this in a very simple manner. We can apply here Kirchhoff's laws. We know that i is equal to i1 plus i2. We also know that U1 which is equal to U2 is equal to R1 times i1 but it is also equal to R2 times i2. From all this, we can easily find that i1 is equal to i total that multiplies R2 over R1 plus R2 and we also find that i2 is equal to i that multiplies R1 over R1 plus R2. Then, in a very fast way, if we wonder how these two currents and two resistors finally separate, we can evidently see how to calculate the current i1 or the current i2. A special case that we can see here, would be if R1 and R2 were equal.

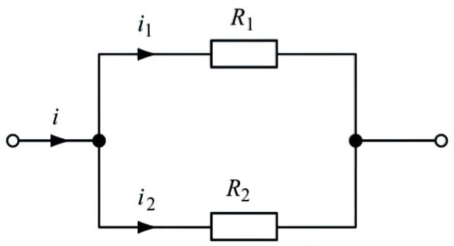
Notes

Summary

14m 20s

CIRCUITS DIVISEURS DE TENSION ET DE COURANT

Circuits diviseurs de courant



$$i = i_1 + i_2$$

$$u_1 = u_2 = R_1 \cdot i_1 = R_2 \cdot i_2$$

$$i_1 = i \frac{R_2}{R_1 + R_2} \qquad i_2 = i \frac{R_1}{R_1 + R_2}$$

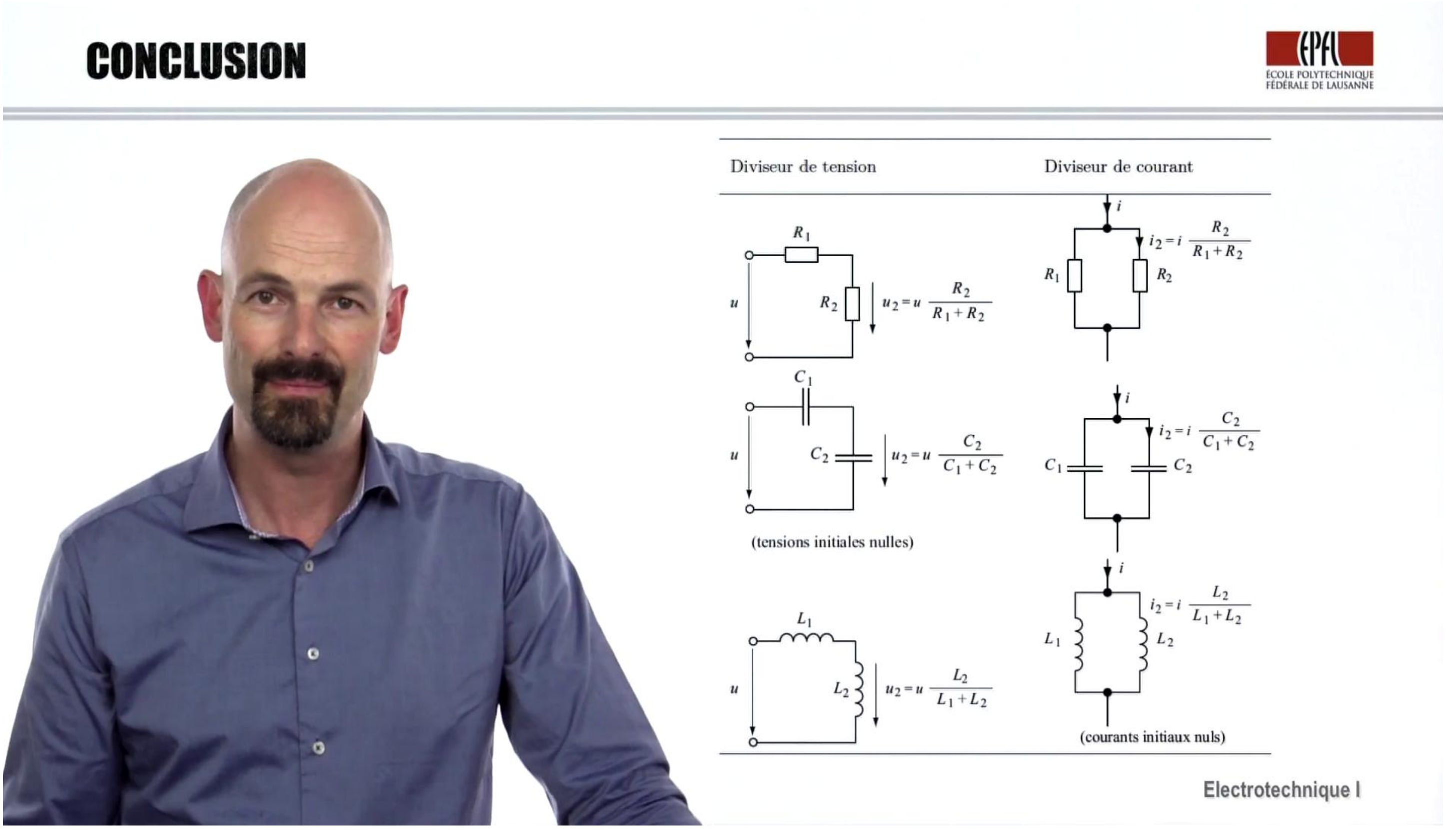
Electrotechnique I

For example, you take R1 and R2 equal to 100 ohms, you will have 100 divided by 200, which will be equal to 0,5, it is then logical that i1 and i2 are equal to 0,5 times the current. The current is then divided in a uniform manner on either side of these equal resistors. We can make an analogy with hydraulic circuits. If we have two pipes of same section, so of same hydraulic resistance, with only one tap bringing water, well we will get the same amount of water, an equivalent flow on both sides since the resistance is the same. Another special case that we can see here is the case where R1 is equal to 0. In such a case, we have a wire on one side and a resistor on the other. Now we have the question then where does the current go? You see straight away here in i1. i1 is equal to i times R2 over R2. If R2 is equal to 0, we have 1. This means that all the current i, whatever the resistor R2, if R1 is equal to 0, all the current goes towards the top and so i1 is equal to i, i2 is equal to 0.

Notes

Summary

16m 00s



In conclusion, I don't give you here the rules for the capacitive current divider and the inductive current divider, I prefer to give you a recapitulation with voltage dividers, current dividers, with the resistance, the capacitance and the inductance, and so in this table, you have all the necessary elements for the case where these elements appear in a circuit, to faster calculate the current division where you need it, the voltage U2 or the voltage U1 in general. Thank you.

Notes

Summary

17m 09s