ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

ANALYSE ET RÉSOLUTION DE CIRCUITS LINÉAIRE

COMBINAISON SIMPLES D'ÉLÉMENTS LINÉAIRE

LEÇON 5

Électrotechnique I
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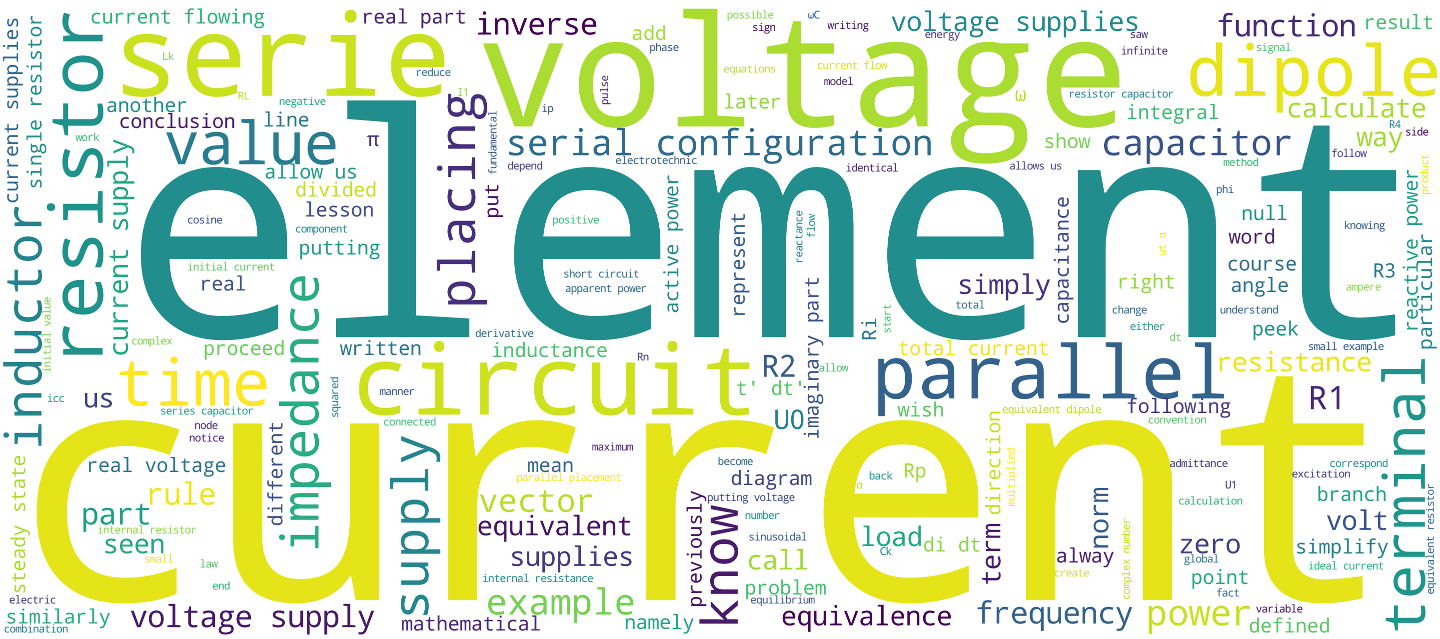


Video

EPFL

Hello and welcome to this lesson dedicated to the analysis and resolution of linear circuits. In this lesson, we will study the combination of simple linear elements. First of all what is am element, or elements, in series like a network. This elements in series for the resistor for the capacitor, for the inductor. We will do the same for these elements in parallel. And finally, we will make a conclusion of this lesson and chapter.

Notes

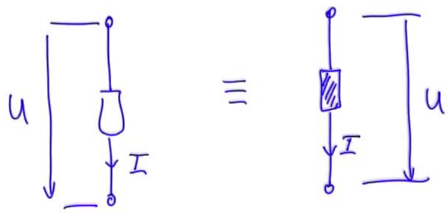
Summary

0m 03s

COMBINAISON SIMPLES D'ÉLÉMENTS LINÉAIRE

Définition : Dipôles équivalents

Deux dipôles qui ont en tout temps le même courant lorsqu'ils sont soumis à la même tension



Electrotechnique I

Before starting with this combination in series or in parallel, I would like to define here, what an equivalent dipole is : Two equivalent dipoles have, at all times, by definition, the same current when they are submitted to the same voltage. It is important to define this to later on allow us to render objects equivalent in a Kirchhoff model. For example, we will have here an element, we know together that it's a resistor, but let's assume that we don't know that, a different element, that I write like so; well, these two elements, we can say that they are equivalent if, submitted to a voltage u and similarly for your first dipole u, we have a current I which is identical on both sides. In which case, we will say that these dipoles are equivalent.

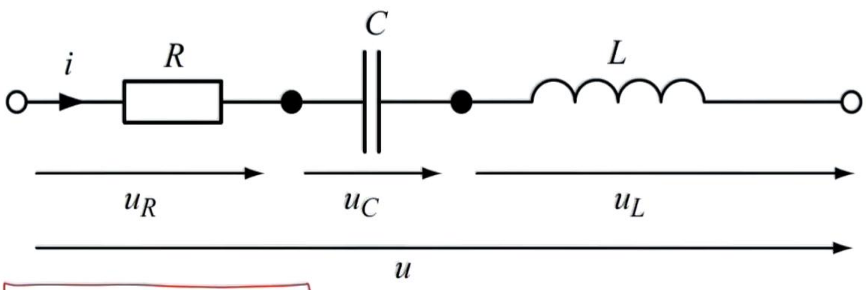
Notes

Summary

0m 28s

ÉLÉMENTS EN SÉRIE

Propriété fondamentale



$$\boxed{i = i_R = i_C = i_L}$$

$$u = u_R + u_C + u_L$$

Electrotechnique I

Now, the series connection of elements. It is fundamental to understand, in a more complex circuit, the elements that can be put, or which are, in a serial configuration, or elements that are in a parallel configuration to, later on, reduce the circuit, and to know how to reduce it so as to not create more problems than we are solving. You have here the fundamental properties of putting elements in a serial configuration. We have on this line, in this model, put a resistor, a capacitor, and an inductor. The fundamental definition of putting in series, is to be able to say that the currents, the three currents of these three elements, are the same, or are identical. Thus, placing elements in series, means that the current flowing through each element is the same. And then, another property derived from Kirchhoff's equations, is that if these elements are in series and have the same current flowing through, the sum of the voltages, which is here between the resistor, capacitor and inductor this sum of voltages is also the voltage at the terminals of the complete dipole, Between this part of the dipole and this part of the dipole.

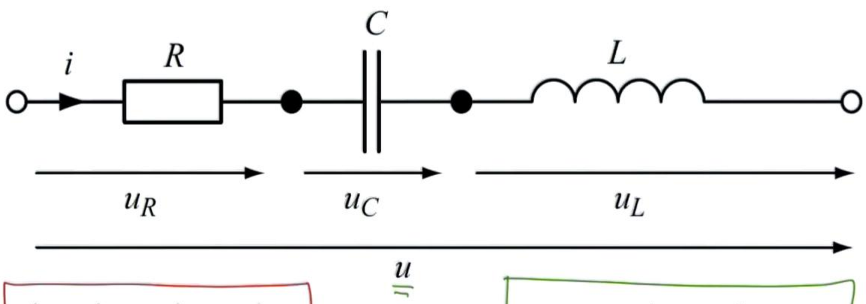
Notes

Summary

1m 28s

ÉLÉMENTS EN SÉRIE

Propriété fondamentale



$$\boxed{i = i_R = i_C = i_L}$$

$$\boxed{u = u_R + u_C + u_L}$$

Electrotechnique I

We will thus have the voltage u present here which is equal to uR + uC + uL Now that we've defined what is the placing of elements in a serial configuration, we will focus on the three elements R, L, C to separately see what it means for each of these elements.

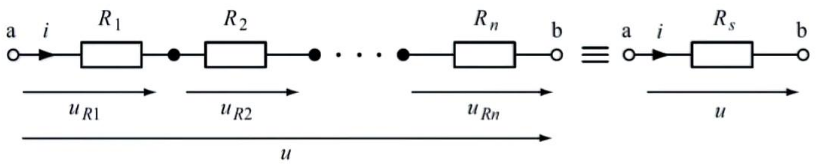
Notes

Summary

2m 48s

ÉLÉMENTS EN SÉRIE

Mise en série de résistances



$$u = \sum_{k=1}^{n} u_{R_k} = \sum_{k=1}^{n} R_k \cdot i = R_s\, i \quad \Rightarrow \quad R_s = \sum_{k=1}^{n} R_k$$

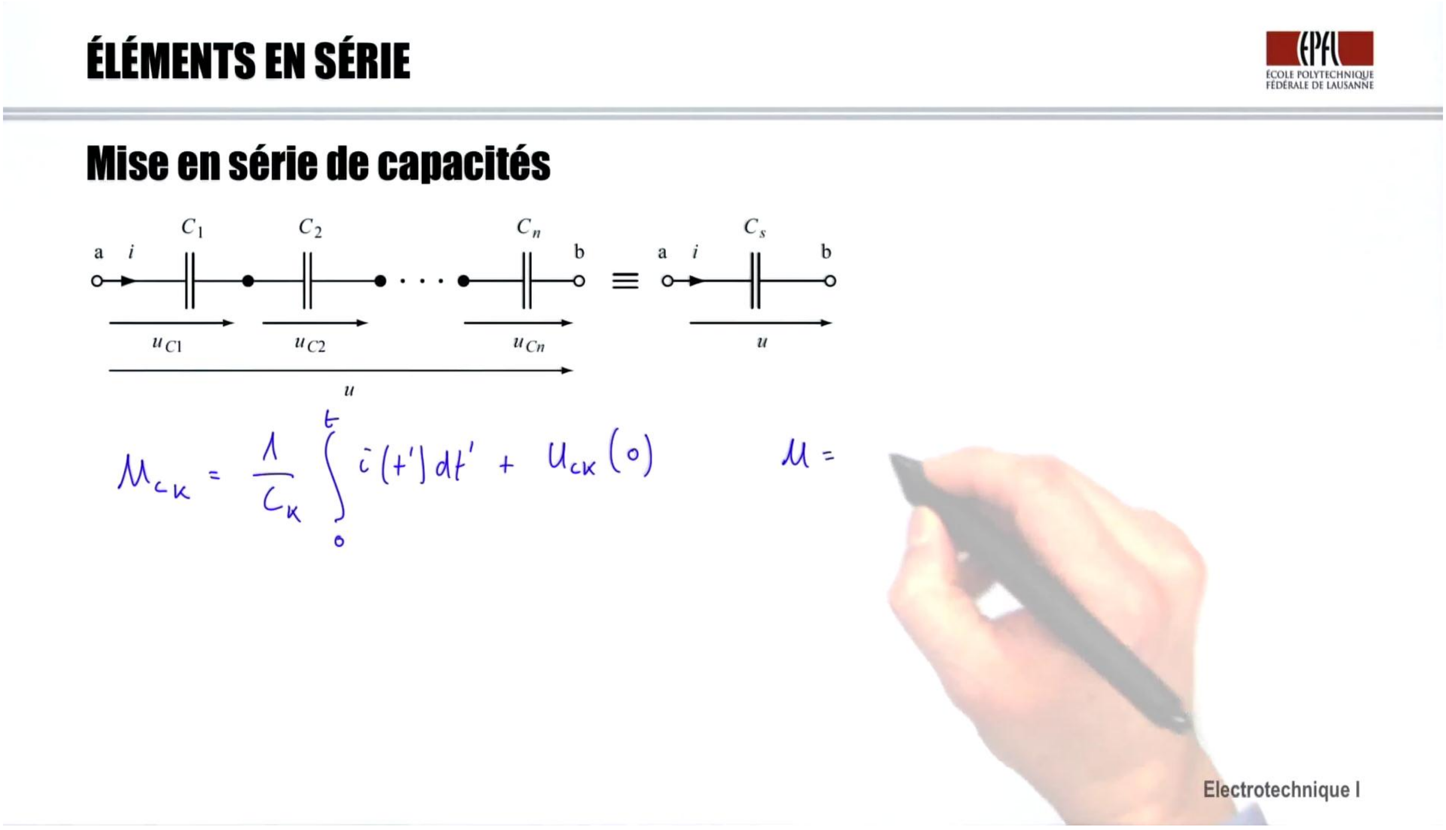
Electrotechnique I

Let's start with the resistor and placing resistors in series. What we wish for, in the end, when we have resistors in series with the same current flowing through R1 to Rn, is to replace this set of resistors with a single resistor known as equivalent, Rs, through which the same current flows and with the same voltage at the terminals, so, it is the equivalence of dipoles that we've seen earlier, and in that instant, what can we say about this new equivalent resistor ? Well, knowing that the voltage u is the sum of the partial voltages from k to 1 up to n of uRk, that this sum is equal to the complete u, but that we can also replace this uRk by this sum from 1 to n, of the Rk times I, the current being the same all the way through, and we want this to be equal to the Rs resistor times I, from this, ensues an obvious fact that most of you are already aware of, that this series resistor in a series, of several resistors in a series, is simply the mathematical sum from K 1 to n of the separate resistors Rk.

Notes

Summary

3m 10s



We will do the same for capacitors in a serial configuration. So once again, we have an equivalence of dipoles, we which to render equal these two dipoles. We have a dipole of several capacitors in a serial configuration, through which the same current flows, we have at the terminals of this complete dipole a voltage u and we which to replace this dipole with a single capacitor, in other words, a capacitor equal to the set of all the capacitors in the serial configuration, here. How do we do this ? we know that a voltage at the terminals of the capacitor k, is 1/Ck integral(from 0 to t) i(t')dt' + the initial condition : Uck(0) So know, we will sum all of these voltages, so we will have the global u of the sum of all the voltages. What is it ? It's : 1/C1 + 1/C2 + etc until 1/Cn of the integral(from 0 to t) of i(t')dt' + a set of initial conditions [Uc1(0) etc] And we want this to be equal to 1/C, the series capacitor Cs, integral(from 0 to t) i(t')dt' + u(0) it of course follows that 1/Cs is equal to the sum of k=1 to n of the 1/Ck This is the same as writing that Cs is equal to 1 / sum of k=1 to n of the 1/Ck This is the rule of putting capacitors in series.

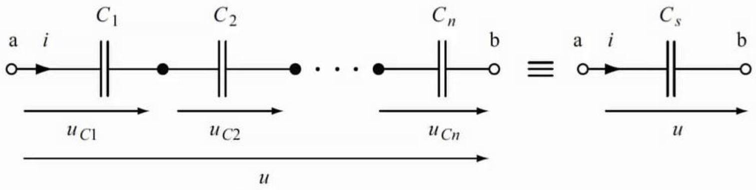
Notes

Summary

4m 43s

ÉLÉMENTS EN SÉRIE

Mise en série de capacités



$$u_{CK} = \frac{1}{C_K}\int_0^t i(t')\,dt' + u_{CK}(0)$$

$$u = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots \frac{1}{C_n}\right)\int_0^t i(t')\,dt' + [u_{C1}(0)\dots]$$

$$= \frac{1}{C_S}\int_0^t i(t')\,dt' + u(0)$$

$$\boxed{\frac{1}{C_S} = \sum_{K=1}^{n} \frac{1}{C_K} \equiv C_S = \frac{1}{\sum_{K=1}^{n} 1/C_K}}$$

$$u(0) = \sum_{K=1}^{n} u_{CK}(0)$$

Electrotechnique I

We say one more thing about the initial value of the voltage at the terminals of the circuit. This initial value, u(0) is simply equal to the sum of all the values of uck(0) We can also say that the placing of capacitors in a serial configuration will insure that the final series capacitor of all the elements will in general, necessarily, according the equations here, be smaller than each of the partial capacitors.

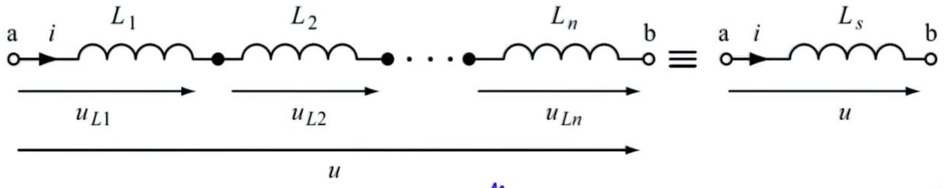
Notes

Summary

6m 56s

ÉLÉMENTS EN SÉRIE

Mise en série d'inductances



$$u = L \frac{di}{dt} \qquad u = \sum_{k=1}^{n} u_{Lk} = (L_1 + L_2 + \dots L_n) \frac{di}{dt} = L_s \frac{di}{dt}$$

$$L_s = \sum_{k=1}^{n} L_k$$

Electrotechnique I

Now, here is the placing of inductors in a serial configuration. So just like the previous ones, you have understood it now, between the resistor and capacitor, we have here an equivalence of two dipoles. We want this set of inductors in series to be equal to this single equivalent inductor. So then, we can write that, knowing u is equal to L di/dt in a general manner for an inductor. we can write that the global u of placing in a serial configuration, is the sum of k=1 to n of the uLK, which is equal to (L1 + L2 + etc Ln) multiplied by di/dt, the i being common to all, it's the same, and we want this to be equal to Ls di/dt It follows that the rules are exactly the same as for the resistors, that Ls is the mathematical sum of k from 1 to n of the Lk.

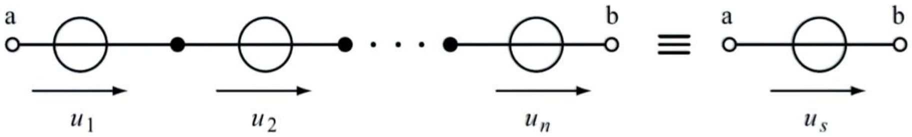
Notes

Summary

7m 28s

ÉLÉMENTS EN SÉRIE

Mise en série de sources de tension



$$u_s = \sum_{k=1}^{n} u_k$$

Electrotechnique I

An important element now is putting voltage supplies in a serial configuration. What can we say when we put different supplies in series and that we want to, like previously for the equivalence of two dipoles, we wish to replace the set of supplies by a single equivalent supply Us ? Well, here like previously, it's actually relatively obvious, this global us, this supply which replaces all the partial supplies in series, is the sum of k from 1 to n, of the partial supplies uk.

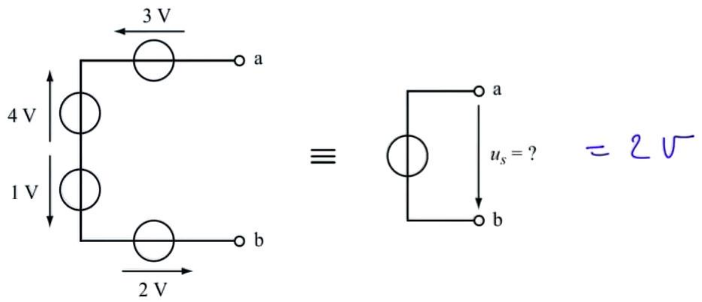
Notes

Summary

8m 40s

ÉLÉMENTS EN SÉRIE

Mise en série de sources de tension - exemple



Electrotechnique I

We can make a small example to convince ourselves, by saying : well here we are, we have an example of 4 supplies in a serial configuration. We wish to replace this element of 4 supplies in series by a single supply to simplify the diagram. What is the value of Us, knowing that we have 3 volts, we have 4 volts, we have 1 volt and 2 volts, but with different directions. Well, we will have 3V - 4V + 1V + 2V and this gives us a Us = 2V. So of course, take care with the direction of the mathematical voltages when we proceed with the sum and the placing in series of voltage supplies.

Notes

Summary

9m 19s

ÉLÉMENTS EN SÉRIE

Mise en série de sources de courant

Impossible sauf si toutes les sources ont même valeur et signe

Electrotechnique I

It is obvious that it is not possible to consider placing several current supplies in series without violating Kirchhoff's first law, except if all the individual supply produce exactly the same current. In which case, the equivalent dipole will simply be equal to any one of of the individual supplies. Thus, what we can say here is that the placing in series here is impossible, except if all the supplies are of the same value and sign. In this very particular case, it is possible to put current supplies in series.

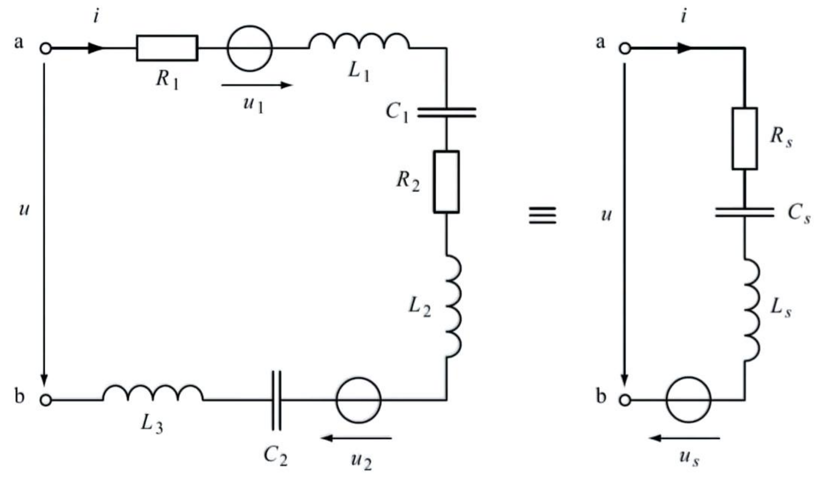
Notes

Summary

10m 01s

ÉLÉMENTS EN SÉRIE

Mise en série de plusieurs éléments de chaque type



Electrotechnique I

And finally, an example of placing several elements in series. Once again, we notice that all of these elements, it's true that it's pretty obvious here, having the same current flowing through, all of these elements in series. We group them based on families, in other words, all the resistors together, all the capacitors together and all the inductors together. We can then already proceed to a simplification of this model on the left through this equivalent model on the right with 3 elements which represent the placing in series of the all separate elements.

Notes

Summary

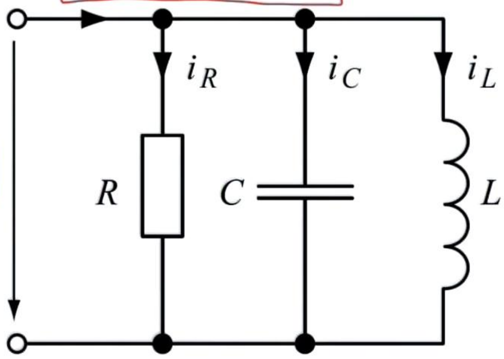
10m 45s

ÉLÉMENTS EN PARALLÈLE

Propriété fondamentale

$$\boxed{i = i_R + i_C + i_L}$$

$$\boxed{u = u_R = u_C = u_L}$$



Electrotechnique I

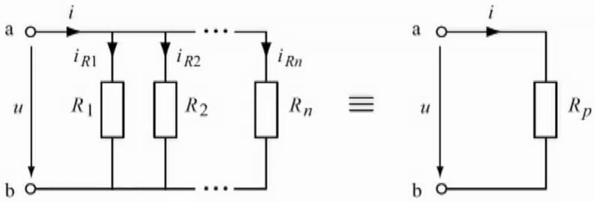
Let us now proceed to parallel placement of elements. What is the definition of putting elements in parallel ? You have a diagram with a resistor, capacitor and inductor which are in parallel. The definition, is that the 3 elements R, L and C, have the same voltage at their terminals. So this is the definition. R and C are parallel, similarly R and L are parallel, similarly C and L are parallel. Because the voltage at their terminals is strictly equal. Thus, the three currents that flow through these elements are different, but, through Kirchhoff, the sum of these three currents is necessarily equal to the total current output here of the dipole, so we can also write that this current here, is the sum of iR, of iC and of iL. We will now go back to R, C then L, and for each of these elements, see what are the rules to be able to simplify elements in parallel of R, of C or of L.

Notes

Summary

11m 19s

Mise en parallèle de résistance



$$i = i_{R_1} + i_{R_2} + \cdots i_{R_n} = i$$

$$= \frac{u}{R_1} + \frac{u}{R_2} + \cdots \frac{u}{R_n} = \frac{u}{R_p}$$

$$\Rightarrow \frac{1}{R_p} = \sum_{k=1}^{n} \frac{1}{R_k} \equiv R_p = \frac{1}{\sum_{k=1}^{n} 1/R_k}$$

First of all, putting resistors in parallel. Once again, we decide to define 2 dipoles : the dipole with the number of resistors at n resistors placed parallel to each other, because, as you can see here, connected to the same voltage uAB, and we wish to replace this with a single resistor equivalent to the previous dipole, and this parallel equivalent resistor, with a single resistor Rp. How do we do it ? We know that the current as written previously, is the current in R1 + the current in R2 + etc the current in Rn. It's the total current of the dipole that we have, i. We also know that this current must be equal to i of the right dipole, as written already here. If we replace iR1, iR2, iRn and so on, using Ohm's law, we have u/R1 u/R2 etc u/Rn and this must be equal to u/Rp We define then the rules of placing resistors in parallel, which is the following : it's that 1/Rp is equal to the sum of k from 1 to n of the 1/Rk In other words, we can also write this in another way, it's that Rp is equal to 1 over the sum of the inverses of Rk for k from 1 to n.

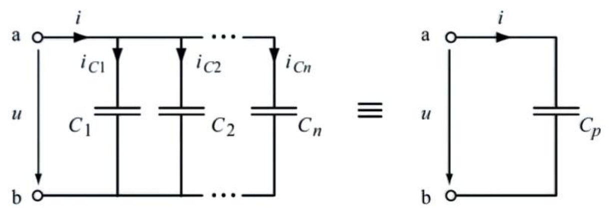
Notes

Summary

12m 26s

ÉLÉMENTS EN PARALLÈLE

Mise en parallèle de capacités



$$i = \sum_{k=1}^{n} i_{Ck} = (C_1 + C_2 + \dots C_n) \frac{du}{dt} = C_p \frac{du}{dt}$$

$$C_p = \sum_{k=1}^{n} C_k$$

Electrotechnique I

We will now look at placing capacitors in parallel. We apply exactly the same rule. We know that the total current output here in this dipole, is the sum of each of the currents of each branch, We can thus write i is the sum of kfrom 1 to n of the ick in other words (C1 + C2 + etc until Cn) of the voltage variation over time. And we want this to be equal to Cp times du/dt Thus, we can write that Cp is equal to the sum of k from 1 to n of the Ck. So it's pretty simple to calculate the placing of capacitors in parallel. It's simply the sum.

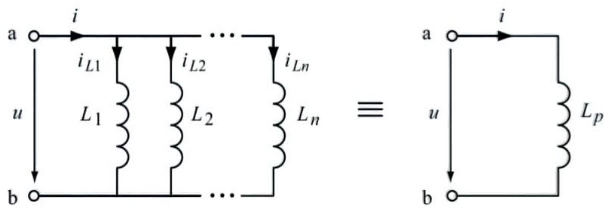
Notes

Summary

14m 24s

ÉLÉMENTS EN PARALLÈLE

Mise en parallèle d'inductances



$$\boxed{\frac{1}{L_p} = \sum_{k=1}^{n} \frac{1}{L_k} \equiv L_p = \frac{1}{\sum_{k=1}^{n} 1/L_k}}$$

$$i_{Lk}(t) = \frac{1}{L_k}\int_0^t u(t')\,dt' + i_{Lk}(0)$$

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \dots \frac{1}{L_n}\right)\int_0^t u(t') + \left[i_{L1}(0) \dots i_{Ln}(0)\right]$$

$$= \frac{1}{L_p}\int_0^t u(t')\,dt' + i(0)$$

$$i(0) = \sum_{k=1}^{n} i_{Lk}(0)$$

Electrotechnique I

Finally, we need to calculate the placing in parallel elements of inductors before proceeding to supplies. So we proceed once again in the same manner. We know that a current iLK as a function of time is 1/Lk integral(from 0 to t) of u(t')dt' + an initial current iLK. We can then write that this current i, that we want in the dipole on the right, is 1/L1 + 1/L2 and so on until 1/Ln all of this in brackets, times the integral of u(t') + all the initial currents, [iL1(0) etc until iLn(0)]. And this, we want it to be equal to 1/Lp integral(from 0 to t) u(t')dt' + i(0). What we can say and what we can see, is that placing in parallel will allow us to compute the equivalent L, Lp, 1/Lp is equal to the sum of k from 1 to n of the 1/Lk In other words, Lp is the inverse of the sum of inverses, from k=1 to n of Lk And similarly, we will be able to say, just like the resistor, it's the inverse of the sum of inverses, for inductors in parallel, and add to this the fact that initial value of the current flowing through the circuit, and simply equal to the mathematical sum of the initial currents. So we will have i(0) equal to a sum from k=1 to n of the iLk(0). So now we have the rules of putting in parallel elements of R, L and C.

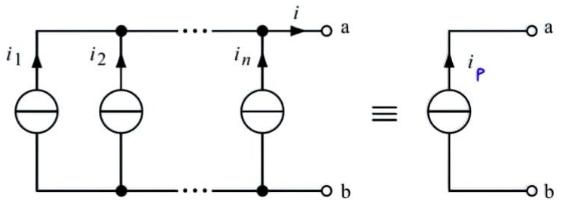
Notes

Summary

15m 18s

ÉLÉMENTS EN PARALLÈLE

Mise en parallèle de sources de courant



$$i_p = \sum_{k=1}^{n} i_k$$

Electrotechnique I

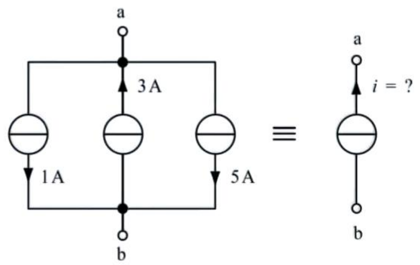
We will now see the placing in parallel of current supplies. So here also, it's pretty simple since we apply Kirchhoff. By wanting to equalize the left dipole with the right dipole, we can see that i, which is defined here, is we sum the currents, we will discover that ip, we will call like this, is equal to the sum of ik currents by taking k=1 to n, and we will thus define this i as ip. So we have an equivalence here, which allows us to, once again, do a small example, to show you in a practical manner how this happens.

Notes

Summary

17m 42s

Mise en parallèle de sources de courant - exemple



$i = -3A$

Electrotechnique I

So if you have here a dipole ab with 3 current supplies in parallel that we wish to replace with a single current supply. What is the value of the iA current ? We will have 3 amperes which go up - 1 ampere, 2 remain, - 5 amperes. Well, the current i is of - 3 amperes. So this i is equal to -3A.

Notes

Summary

18m 29s

Mise en parallèle de sources de tension

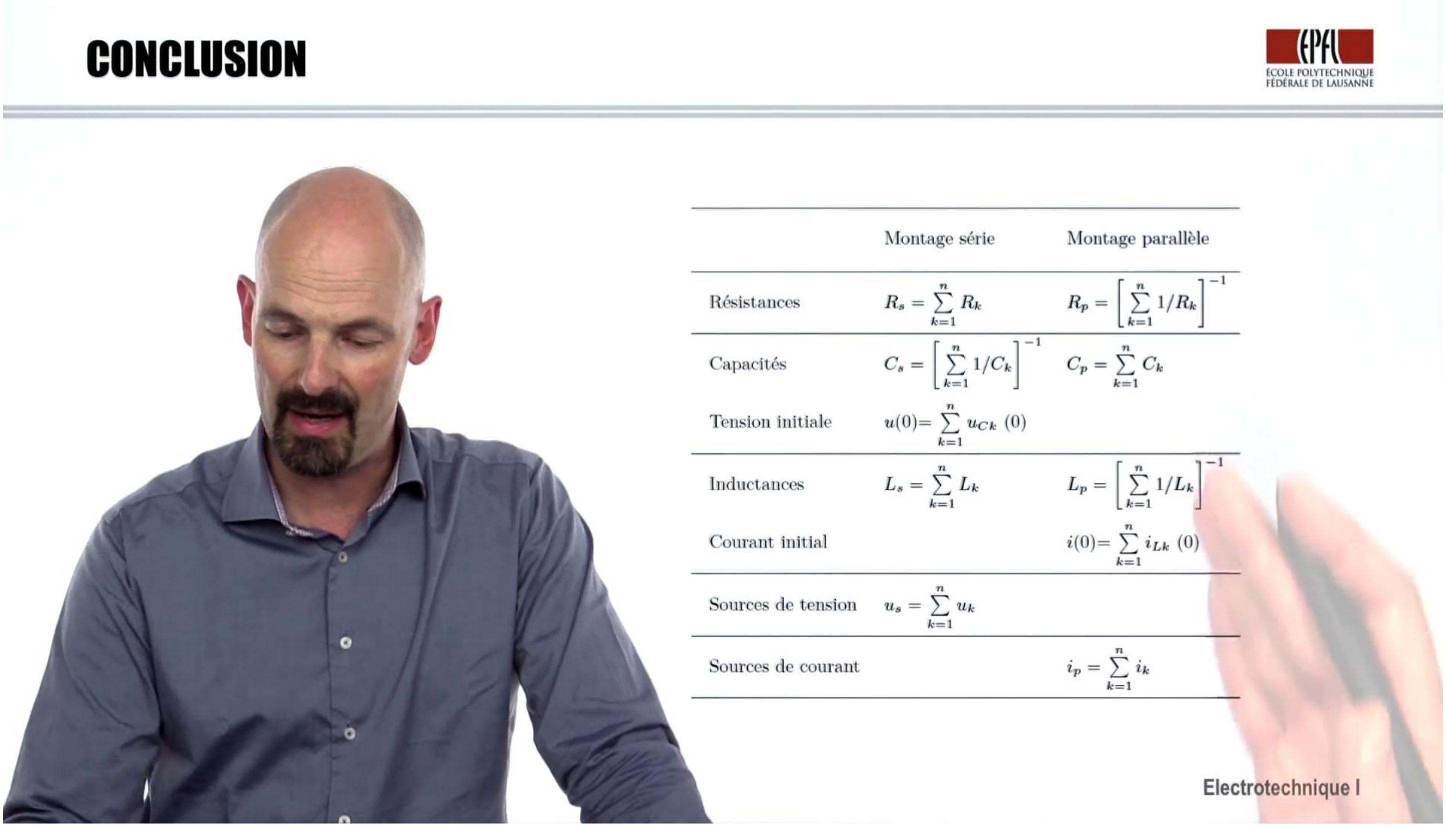
Impossible, Sauf si elles ont toutes la même valeur et le même Signe

We continue with parallel placement of voltage supplies, and similarly, we have a problem with putting voltage supplies in parallel. We cannot do this parallel placement without violating Kirchhoff's second law and once again, here as well, except if they are all the same voltages. If they all have the same voltages, in this case we can simplify it to a single supply, which will be the same supply that each of these defines. And thus we can write that putting voltage supplies in parallel is impossible except if they all have the same value and sign.

Notes

Summary

19m 00s



To conclude, we've seen the parallel and serial placement of all the elements R, L, C, of ideal current and voltage supplies. As a conclusion, I will show you here a table which sums up what we've just seen with serial configurations, the definition of resistors in series, capacitors in series, inductors in series, voltage supplies in series, and parallel configurations with the resistor, capacitor, inductor, and similarly for the ideal current supply. Thank you

Notes

Summary

19m 42s